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On Properties of the Graph of Fuzzy Topographic Topological Mapping

Mohamed Sayed[1,3] and Tahir Ahmad[1,2]

[1]*Department of Mathematical Science, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia*
[2]*IbnuSina Institute for Fundamental Science studies, Universiti Teknologi Malaysia*
[3]*International University of Africa, Department of Mathematical Science, Sudan*

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ABSTRACT

Fuzzy Topographic Topological Mapping (FTTM) is a novel noninvasive technique to determine the location of epileptic foci in epilepsy disorder patients. The model which consists of topological and fuzzy structure is composed into three mathematical algorithms. There are two FTTMs, namely FTTM Version 1 and FTTM Version 2. FTTM Version 1 is homeomorphic to FTTM Version 2. The homeomorphism of FTTM can be presented using graphs. This paper presents some properties of the graph of FTTM.

| Fuzzy Topographic Topological Mapping | New element of order *k* | Graph of *FTTM* |

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1. INTRODUCTION

Fuzzy Topological Mapping (FTTM) was developed by Fuzzy Research Group (FRG) at UTM in 1999 [1]. *FTTM* was built to solve neuromagnetic inverse problem to determine the location of epileptic foci in epilepsy disorder patient [2].

FTTM Version 1 was developed to present a 3-D view of unbounded signal current source [3]& [4]. Besides that, $FTTM$ Version 2 can process image data of magnetic field. $FTTM$ Version 1 as well as $FTTM$ Version 2 is specially designed to have equivalent topological structures between its components [5]. In other words, there are homeomorphisms between each element of *FTTM* Version 1 and *FTTM* Version 2 [5].

Liau, [5] defined *FTTM* as a set consisting of models with four components and three algorithms. The four components are homeomorphic.

The fact that *FTTM* Version 1 and *FTTM* Version 2 are homeomorphic component wise , there are at least another 14 elements of *FTTM* that can be identified. In general, Liau, [5] has proposed a conjecture as follows:

If there exist$k$ elements of *FTTM*, then the numbers of generating elements are

$$k^4 - k$$

The conjecture was proven by Suhana [6].

2. MATERIALS AND METHOD

In the first part of this paper a mathematical formula is suggested to present the number of the new elements of FTTMwith$n$ vertices and k versions. And the second part of the paper deals with the elements which present a graph of degree zero.

2.1 The new elements of order *k*

Definition 1

Let $F_{n,1} = \{a_{1,1}, a_{2,1}, a_{3,1}, \ldots, a_{n,1}\}$ to be $FTTM$ Version 1 , where n present the number of vertices. Then $\{a'_{1,i}, a'_{2,i}, a'_{3,i}, \ldots, a'_{n,i}\}$ is said to be a new element of $FTTM$ where $a'_{1,i} \cong a'_{2,i}$, $a'_{2,i} \cong a'_{3,i}$,$a'_{3,i} \cong a'_{4,i} \cdots a'_{n-,i} \cong a'_{n,i}$

Definition 2

The new element is said to be an element of order k if its components appear in exactly k versions of $FTTM$.

Notations

- $F_{n,i} = \{a_{1,i}, a_{2,i}, a_{3,i}, \ldots, a_{n,i}\}$presents $FTTM$version i , where $FTTM$ consists of n Vertices.
- $F(n,k) = \{F_{n,1}, F_{n,2}, F_{n,3}, \ldots, F_{n,k}\}$presents the combination of k versions of FTTM.
- $\# F\,(n,k)$ denotes the number of the new elements of order k which can be generated from $F\,(n,k)$, such that $k \leq n$.

*Corresponding author. E-mail: tahir@ibnusina.utm.my

- $G_0(\# F(n,k))$ denotes the number of the new elements which produce a graph of degree zero.

Conjecture 1

Let $F(n,k) = \{F_{n,1}, F_{n,2}, F_{n,3}, \dots, F_{n,k}\}$, such that $F_{n,i} = \{a_{1,i}, a_{2,i}, a_{3,i}, \dots, a_{n,i}\}$, $i = 1,2,3,\dots,k$, and $k \leq n$.
Then

$$\# F(n,k) = k^n - (k-1)^n \binom{k}{k-1} + (k-2)^n \binom{k}{k-2} - \cdots + (-1)^{k-1}(1)^n \binom{k}{1}$$
$$= \sum_{j=0}^{k-1} (-1)^j \, (k-j)^n \binom{k}{k-j}$$
$$\forall \; n = 1,2,3,\dots \quad \text{and} \;\; k \leq n$$

Illustration

If $k = 1$, then $\# F(n,k) = 1$ since we have only one version of FTTM.
If we have two versions $F_{n,1}$ and $F_{n,2}$ of $FTTM$, such that $F_{n,1} = \{a_1, a_2, a_3, \dots a_n\}$ and $F_{n,2} = \{b_1, b_2, b_3, \dots b_n\}$ as shown bellow.

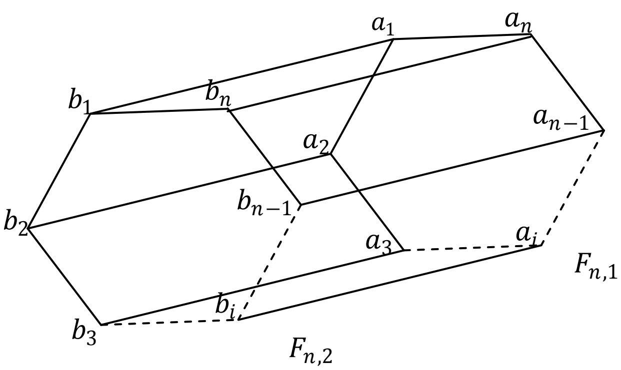

Fig. 1 Combination of $F_{n,1}$&$F_{n,2}$

The new elements of order two will be of the form

$\{a_1 \; or \; b_1 \;, \;\; a_2 \; or \; b_2 \;\; , a_3 \; or \; b_3 \;, \dots, \;\; a_n \; or \; b_n\}$

From the product principle we can generate $\underbrace{2 \times 2 \times 2 \times \cdots \times 2}_{n \; terms} = 2^n$ elements of order two, but we have to exclude $F_{n,1}$ and $F_{n,2}$, since its order is one. Thus

$$\# F(n,2) = 2^n - 2$$

By using the same procedure we can find $\# F(n,3)$.

Let $F_{n,1} = \{a_1, a_2, a_3, \dots a_n\}$, $F_{n,2} = \{b_1, b_2, b_3, \dots b_n\}$ and $F_{n,3} = \{c_1, c_2, c_3, \dots, c_n\}$to be three versions of FTTM with n vertices, as shown bellow

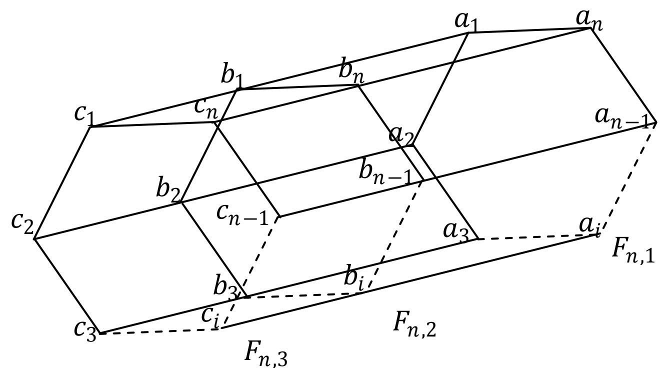

Fig. 2 Combination of $F_{n,1}, F_{n,2}$&$F_{n,3}$

The new elements of order three will be of the form
$\{a_1 \,, b_1 \; or \; c_1 \;\; , \;\; a_2, b_2 \; or \; c_2, a_3, b_3 \; or \; c_3, \dots, \;\; a_n, b_n or \; c_n\}$
From the product principle we we can generate $\underbrace{3 \times 3 \times 3 \times \cdots \times 3}_{n \; terms} = 3^n$ elements . But we have to exclude the elements of order 1 and 2. So we have to subtract $2^n \binom{3}{2}$ from 3^n.

$$3^n - 2^n \binom{3}{2}$$

But we can observe that, $F_1$and F_2 are excluded twice, so we have to include it again. Thus

$$\# F(n,3) = 3^n - 2^n \binom{3}{2} + 1^n \binom{3}{1}$$

We can use the same argument with the principle of inclusion and exclusion to find $\# F(n,k)$, $\forall$ $k \leq n$ .So

$$\# F(n,k) = k^n - (k-1)^n \binom{k}{k-1} + (k-2)^n \binom{k}{k-2} - \cdots + (-1)^{k-1}(1)^n \binom{k}{1}$$
$$= \sum_{j=0}^{k-1} (-1)^j \, (k-j)^n \binom{k}{k-j}$$
$$\forall \; n = 1,2,3,\dots \quad and \;\; k \leq n$$

Lemma.1

$$k(k-a)^{n-1}\left(\binom{k}{k-a} - \binom{k-1}{k-a}\right) = (k-a)^n \binom{k}{k-a}$$
$$0 \leq a \leq k \,, n \geq 1$$

Proof

$$k(k-a)^{n-1}\left(\binom{k}{k-a} - \binom{k-1}{k-a}\right)$$

$$= k(k-a)^{n-1}\left(\frac{k!}{(k-a)!\,a!} - \frac{(k-1)!}{(k-a)!\,(a-1)!}\right)$$
$$= k(k-a)^{n-1}\left(\frac{k! - a(k-1)!}{(k-a)!\,a!}\right)$$
$$= k(k-a)^{n-1}\left(\frac{k(k-1)! - a(k-1)!}{(k-a)!\,a!}\right)$$
$$= k(k-a)^{n-1}\left(\frac{(k-a)(k-1)!}{(k-a)!\,a!}\right)$$
$$= k(k-a)^{n}\left(\frac{(k-1)!}{(k-a)!\,a!}\right)$$
$$= (k-a)^{n}\left(\frac{k(k-1)!}{(k-a)!\,a!}\right)$$
$$= (k-a)^{n}\binom{k}{k-a}$$

□

Theorem.1

$$\# F(n,k) = k(\# F(n-1,k) + \# F(n-1,k-1))$$

Proof

$$\# F(n,k) = \sum_{j=0}^{k-1}(-1)^j (k-j)^n \binom{k}{k-j}$$
$$= k^n - (k-1)^n\binom{k}{k-1} + (k-2)^n\binom{k}{k-2} - \cdots + (-1)^{k-1}(1)^n\binom{k}{1}$$
$$= k^n\binom{k}{k} - \underbrace{(k-1)^n\binom{k}{k-1}}_{from\ lemma\ 1} + \underbrace{(k-2)^n\binom{k}{k-2}}_{from\ lemma\ 1} - \cdots + (-1)^{k-1}\underbrace{(1)^n\binom{k}{1}}_{from\ lemma\ 1}$$
$$= k(k)^{n-1} - k(k-1)^{n-1}\left(\binom{k}{k-1} - \binom{k-1}{k-1}\right) + k(k-2)^{n-1}\left(\binom{k}{k-2} - \binom{k-1}{k-2}\right) - \cdots + (-1)^{k-1}k(1)^{n-1}\left(\binom{k}{1} - \binom{k-1}{1}\right)$$
$$= k\Bigg((k)^{n-1} - (k-1)^{n-1}\binom{k}{k-1} + (k-2)^{n-1}\binom{k}{k-2} - \cdots + (-1)^{k-1}(1)^{n-1}\binom{k}{1} + (k-1)^{n-1}\binom{k-1}{k-1} - (k-2)^{n-1}\binom{k-1}{k-2} + \cdots + (-1)^{k-2}(1)^{n-1}\binom{k-1}{1}\Bigg)$$
$$= k\left(\sum_{j=0}^{k-1}(-1)^j (k-j)^{n-1}\binom{k}{k-j} + \sum_{j=0}^{(k-1)-1}(-1)^j (k-j)^{n-1}\binom{k-1}{k-j}\right)$$
$$= k\big(\# F(n-1,k) + \# F(n-1,k-1)\big)$$

□

Corollary.1

$$\# F(n,2) = 2 + 2^2 + 2^3 + \cdots + 2^{n-1} = \sum_{j=1}^{n-1} 2^j$$

Proof

$$\# F(n,2) = 2\big(\# F(n-1,1) + \# F(n-1,2)\big)$$
$$\# F(2,2) = 2\Big(\# F(1,1) + \underbrace{\# F(1,2)}_{=0}\Big) = 2$$
$$\# F(3,2) = 2\big(\# F(2,1) + \# F(2,2)\big) = 2(1+2) = 2 + 2^2$$

Assume the result is true up to $n = m$

$$\# F(m,2) = 2 + 2^2 + 2^3 + \cdots + 2^{m-1}$$

(Induction hypothesis)

$$\# F(m+1,2) = 2\big(\# F(m,1) + \# F(m,2)\big) = 2(1 + 2 + 2^2 + 2^3 + \cdots + 2^{m-1}) = 2 + 2^2 + 2^3 + \cdots + 2^m = \sum_{j=1}^{m} 2^j$$

Since the result is true for $n = m$ and for $n = m+1$, then it is true for all n. □

Theorem.2

$$G_0\big(\# F(m,2)\big) = \begin{cases} 2, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

Proof

Let n to be an even integer, such that, $F_{n,1} = \{a_1, a_2, a_3, \ldots a_n\}$ and $F_{n,2} = \{b_1, b_2, b_3, \ldots b_n\}$.
Define $\# a$ and $\# b$ to present the number of a's and b's in the string of the new element respectively.
Since n is even, $\# a = \# b$ and they must appear alternating.
So to produce a graph of degree zero, the new element must be of the form

$$\{a_1, b_2, a_3, b_4, \ldots, a_{n-1}, b_n\}$$

or

$$\{b_1, a_2, b_3, a_4, \ldots, b_{n-1}, a_n\}$$

Thus

$G_0\big(\# F(n,2)\big) = 2.$ if n is an even integer.

Let n to be an odd integer, this implies that $\# a \neq \# b$.

If $\# a > \# b$.

To produce a graph of degree zero the new element must be of the form

$$\{a_1, b_2, a_3, b_4, \dots, b_{n-1}, a_n\}$$

But the degree of the above element is 2. So if n is an odd integer and $\# a > \# b$, no new element will produce a graph of degree zero.

If $\# a < \# b$.

To produce a graph of degree zero the new element must be of the form

$$\{b_1, a_2, b_3, a_4, \dots, a_{n-1}, b_n\}$$

But the degree of the above element is 2. So if n is an odd integer and $\# a < \# b$, no new element will produce a graph of degree zero.
Thus, if n is an odd integer $G_0\big(\# F(n,2)\big) = 0.$

□

3. CONCLUSION

The paper suggests that :

$$\begin{aligned} \# F(n,k) &= k^n - (k-1)^n \binom{k}{k-1} + (k-2)^n \binom{k}{k-2} \\ &\quad - \cdots + (-1)^{k-1}(1)^n \binom{k}{1} \\ &= \sum_{j=0}^{k-1} (-1)^j \, (k-j)^n \binom{k}{k-j} \\ &\forall\, n = 1,2,3,\dots \quad and\ \ k \le n \end{aligned}$$

and

$$\# F(n,k) = k(\# F(n-1,k) + \# F(n-1,k-1))$$

The second part of this paper deals with the elements which present a graph of degree zero. And it is shown that:

$$G_0\big(\# F(m,2)\big) = \begin{cases} 2, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

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