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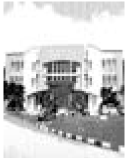

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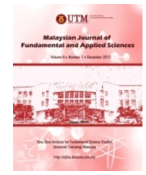

A Decision Making Method using Fuzzy Soft Sets

Samsiah Abdul Razak* and Daud Mohamad

Faculty of Computer and Mathematical Sciences, UiTM, 40450 Shah Alam, Selangor, Malaysia

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ABSTRACT

The introduction of soft set theory by Molodstov has gained attention by many as it is useful in dealing with uncertain data. It is advantageous to use due to its parameterization form of data. This concept has been used in solving many decision making problems and has been generalized in various aspects in particular to fuzzy soft set (FSS) theory. In decision making using FSS, the objective is to select an object from a set of objects with respect to a set of choice parameter using fuzzy values. Although FSS theory has been extensively used in many applications, the importance of weight of parameters has not been highlighted and thus is not incorporated in the calculation. As it depends on one's perception or opinion, the importance of the parameters may differ from one decision maker to another. Besides, existing methods in FSS only consider one or two decision makers to select the alternatives. In reality, group decision making normally involves more than two decision makers. In this paper we present a method for solving group decision making problems that involves more than two decision makers based on fuzzy soft set by taking into consideration the weight of parameters. The method of lambda – max which frequently utilize in fuzzy analytic hierarchy process (FAHP) has been applied to determine the weight of parameters and an algorithm for solving decision making problems is presented. Finally we illustrate the effectiveness of our method with a numerical example.

| Fuzzy soft sets | Uncertainty | Weight of parameters | Lambda-max method | Fuzzy analytic hierarchy process |

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1. INTRODUCTION

Soft set theory was first introduced by a Russian researcher Molodstov [1] with the intention to solve some complicated problems such as in economics, engineering and environment that are usually not successfully solved by classical methods due to the presence of uncertainties of various types. Soft set theory is preferred to other uncertainty concepts due to its ability to represent data in parametric form. At present, studies on the properties and applications of soft set theory is progressing rapidly and already used by researchers in many ways [2,3,4,5,6,7]. As a generalization of the standard concept of soft sets, Maji et al. [8] introduced the theory of fuzzy soft and applied it to decision making problems. Subsequently many researchers have extended and applied this theory in various decision making problems [9,10,11,12].

Cagman and Enginoglu [13] defined soft matrix, to make operations in theoretical studies in soft set more functional. Some properties of soft matrices and a soft max – min decision making (SMmDM) method are discussed. Futhermore, soft max – min decision function was used to solve a house selection problem involving two decision makers. In 2011, Yang and Ji [14], defined fuzzy soft matrix (FSM) which is very useful in representing and computing the data involving fuzzy soft sets. They also showed that the SMmDM method of [13], unable to solve decision making problems that involve more than two decision makers as it does not satisfy the commutative law.

Later Razak and Mohamad [15] extended the model given by Cagman and Enginoglu [16] where now the decision making problems involved three decision makers was catered using SMmDM method that satisfy the associative law and the researchers solved group decision making problems by incorporating the importance of weight of criteria using analytic hierarchy process (AHP). Cagman and Enginoglu [17] defined fuzzy soft matrix and constructed fuzzy soft max – min decision making method by using And – product in solving the problems.

In solving decision making problems using FSS, the considered parameters may have different importance due to diverse human perception that force us to give different weight to each of them. Besides, in many instances the decisions are made in group where more than one decision maker is needed. This is known as group decision making. Even though many approaches have been applied using soft set and fuzzy soft set theories, however these methods are limited to one decision maker. In this paper, we present FSMmDM incorporating the weight of criteria using Lambda – max method, an approach of criteria weight determination in Fuzzy Analytic Hierarchy Process (FAHP). FAHP was claimed to be better method compared to others [18]. The generalization of FSMmDM method given in [17] to group decision making problem of more than two decision makers utilizing the associative law in

*Corresponding author.
E-mail: amsha_abdrazak @ yahoo.com .my (SamsiahAbdul Razak)

[15] is presented in this paper. A numerical example is finally presented in this paper.

2. FUZZY SOFT MATRICES

A fuzzy set is simply a class of elements with continuum membership grade between 0 and 1 inclusively. A triangular fuzzy number, is a fuzzy set that is convex and normal (highest membership grade of 1), represented by 3 – tuple (*l*, *m*, *u*), is described by the membership function defined as

$$\mu(x)=\begin{cases}\dfrac{x-l}{m-l}, & l\le x\le m\\ \dfrac{u-x}{u-m}, & m\le x\le u\\ 0, & otherwise\end{cases}$$

Maji et al. [8] defined fuzzy soft set as a generalization of standard soft sets in following manner:

Definition 1: Let U be an initial universe set and E be a set of all parameters. Let *F* (*U*) be the set of all fuzzy sets in U. $(\tilde{F},A)$ is called a fuzzy soft set over U where $A\subseteq E$ and $\tilde{F}$ is a mapping given by

$$\tilde{F}:A\to F(U).$$

In general, for every $x\in A$, $\tilde{F}$ [x] is a fuzzy set in U and it is called fuzzy value set of parameter *x*. If for every $x\in A, \tilde{F}$ [x] is a crisp subset of U, then (F,A) is degenerated to be the standard soft set.

Cagman and Enginoglu [17] developed fuzzy soft decision making method by the following definitions.

Definition 2: Let $(\tilde{F},A)$ be a fuzzy soft set over U, where $U=\{u_1,u_2,\ldots,u_m\}$ be an initial universe set, $E=\{e_1,e_2,\ldots e_n\}$ be a set of parameters and $A\subseteq E$. For $\forall u_i\in U$ and $\forall e_j\in E$, there exists a membership degree $[a_{ij}]=f_{e_j}(u_i)$. The membership degrees will be presented in the following form as in Table 1.

Table 1. Evaluation of membership degrees fuzzy soft matrices $(FSM_{m\times n})$

	e_1	e_2	$\cdots$	e_n
u_1	a_{11}	a_{12}	$\cdots$	a_{1n}
u_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}

The matrix $A_{m\times n}=[a_{ij}]_{m\times n}=\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\vdots&\vdots&\ddots&\vdots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{bmatrix}$

is called fuzzy soft matrix of $(\tilde{F},A)$ over U.

Definition 3: Let $[a_{ij}],[b_{ik}]\in FSM_{m\times n}$. The *And* – product $\wedge$ between $[a_{ij}]$ and $[b_{ik}]$ is defined by

$\wedge: FSM_{m\times n}\times FSM_{m\times n}\to FSM_{m\times n^2}$, $[a_{ij}]\wedge[b_{ik}]=[t_{ip}]$

where $[c_{ip}]=\min\{a_{ij},b_{ik}\}$ such that $p=n(j-1)+k.$

3. METHODOLOGY

There are two procedures involved in the proposed method. The first procedure is the determination of criteria weight by using Lambda – max method in fuzzy AHP proposed in [19]. The second procedure is to solve the group decision making problems. This paper utilizes FSMmDM method in [17] and generalize it to n $(n>2)$ decision makers as in [15]. The details of both procedures are given below.

3.1 Criteria weight determination

The analytic hierarchy process (AHP) was first proposed by Saaty in 1971. In 1983, Laarhoven and Pedryz [20] proposed fuzzy AHP, which compared fuzzy ratios described in triangular fuzzy number. Several methods have been introduced to determine criteria weight in fuzzy AHP [18] but the Lambda – max method [19] is our focus in this study. The procedure of the Lambda – max method involves 4 steps as follows:

Step 1: Apply α – cut. To obtain the positive matrix of decision maker s, let $\alpha=1$, $\tilde{T}_m^s=[\tilde{r}_{ij}]_m^s$, and let $\alpha=0$ to obtain the lower bound and upper bound positive matrices of decision maker *s*, $\tilde{T}_l^s=[\tilde{r}_{ij}]_l^s$ and $\tilde{T}_u^s=[\tilde{r}_{ij}]_u^s$. Calculate the weight vector based on the weight calculation procedure in AHP, $W_m^s=(w_i)_m^s$, $W_l^s=(w_i)_l^s$, and $W_u^s=(w_i)_u^s$, $i=1,2,\ldots,n.$

Step 2: In order to minimize the fuzziness of the weight. Two constants, M_l^s and M_u^s are obtained as follows:

$$M_l^s=\min\left\{\frac{w_{im}^s}{w_{il}^s}\right\},\ 1\le i\le n$$

$$M_u^s=\min\left\{\frac{w_{im}^s}{w_{iu}^s}\right\},\ 1\le i\le n$$

and the upper and lower bounds of the weight are defined as:

$$W_{il}^{*s} = M_l^s w_{il}^s \;,\; W_{iu}^{*s} = M_u^s w_{iu}^s \;,$$

Hence the lower and upper bounds weight vectors are $(w_i^*)_l^s$ and $(w_i^*)_u^s$ respectively for $i = 1,2,\ldots,n.$

Step 3: By combining the upper bound, the middle bound and lower bound weight vectors, the fuzzy weight matrix for decision maker s can be obtained and is defined as $\widetilde{W}_i^s = (w_{il}^{*s}, w_{im}^{*s}, w_{iu}^{*s}), \quad i = 1,2,\ldots,n.$

Step 4: Calculate local fuzzy weights and global fuzzy weight with repetition from step 1 until step 3.

3.2 Fuzzy soft max – min decision making (FSMmDM) method

Cagman and Enginoglu [17] introduced a fuzzy soft (*fs*) max – min decision making method by using *And* – product and defined as follows:

Definition 4: Let $[c_{ip}] \in FSM_{m\times n^2}, I_k = \{p : \exists i, c_{ip} \neq 0, \ (k-1)n, < p \leq kn\}$ for all $k \in I = \{1,2,\ldots,n\}$. Then max – min decision function, denoted by Mm, is defined as:

$$Mm : FSM_{m\times n^2} \to FSM_{m\times 1}, \quad Mm[c_{ip}] = \left[\max_{k\in I}\{t_k\}\right]$$

where $t_k = \begin{cases} \min\limits_{p\in I_k}\{c_{ip}\}, & \text{if } I_k \neq \phi, \\ 0, & \text{if } I_k = \phi \end{cases}$

The one column soft matrix $Mm[c_{ip}]$ is called max – min decision fuzzy soft matrix.

Definition 5: Let $U = \{u_1, u_2, u_3, \ldots, u_m\}$ be an initial universe and $Mm[c_{ip}] = [d_{i1}]$. Then a subset of U can be obtained by using $[d_{i1}]$ as in the following expression

$$Opt[d_{i1}](U) = \{d_{il}/u_i : u_i \in U, d_{i1} \neq 0\},$$

which is called an optimum set of U.

Now using Definition 4 and 5, the FSMmDM method is as in the following algorithm.

Step 1: Choose the feasible subsets of the set of parameters.

Step 2: Use the matrix form to construct the *fs* – matrix for each set of parameters.

Step 3: Find the *And* – product for the *fs* – matrices.

Step 4: Find a max – min decision *fs* – matrix.

Step 5: Find an optimum set of U.

$$Opt_{Mm}(U) = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}^T.$$

3.3 Fuzzy soft max – min decision making (FSMmDM) method with criteria weight

Our proposed procedure for decision making is given as

Step 1: Evaluate the membership value of alternatives with respect to each criteria in decision making problem.

Step 2: Use the matrix form to construct the fuzzy soft matrices for each set of criteria.

$$[r_{ij}^k]_{m\times n} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix},$$

where $[r_{ij}]$ is a fuzzy soft matrix of decision maker k, m refers to the number of alternatives involved in problems and n refers to the parameters/criteria.

Step 3: Construct the matrix A_{ij} that combine the weight of criteria $\widetilde{W}_k = (w_1, w_2, \ldots, w_a)$ with the evaluation of alternatives by decision makers from Step 2.

$$A_{ij} = [r_{ij}]_{m\times n}\widetilde{W}_k = \begin{bmatrix} r_{11}\otimes w_1 & r_{12}\otimes w_2 & \cdots & r_{1n}\otimes w_a \\ r_{21}\otimes w_1 & r_{22}\otimes w_2 & \cdots & r_{2n}\otimes w_a \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}\otimes w_1 & r_{m2}\otimes w_2 & \cdots & r_{mn}\otimes w_a \end{bmatrix}$$

Step 4: Find the *And* – product of fuzzy soft matrices $\left(\text{e.g.}\left(DM_{n-1} \wedge DM_n = A\right)\right)$.

$$[A_{ij}] \wedge [B_{ik}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix} \wedge \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n} \\ B_{21} & B_{22} & \cdots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mn} \end{bmatrix}$$

The fuzzy soft matrix of size $(m\times n^2)$ is obtained. There are n blocks of $(m\times n)$ elements in the above matrix.

Step 5: Find the minimum of *And* – product between $[A_{ij}]$ and $[B_{ik}]$, for each n blocks of $(m\times n)$ elements above.

$$t_{ir} = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} & t_{m2} & \cdots & t_{mn} \end{bmatrix},$$

where $t_{ir} = \min\limits_{r=i,2,\ldots,n} [A_{ij} \wedge B_{ik}]$

Step 6: Find the *And* – product between $[t_{iO}]$ and $[C_{il}]$

$$[t_{ir}] \wedge [C_{il}] = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} & t_{m2} & \cdots & t_{mn} \end{bmatrix} \wedge \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix},$$

and repeat Step 5 to find the minimum of $[t_{ir}]$ *and* $[C_{il}]$ for each n blocks of $(m\times n)$ elements. Then the matrix

$$\left[v_{ip}\right]=\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix},$$

is obtained with $v_{ip} = \min_{p=1,2,\ldots,n}\left[t_{ir} \wedge u_{il}\right]$

Step 7: Find the max – min decision fuzzy soft matrix,

$$Mm(([A_{ij}] \wedge [B_{ik}]) \wedge [C_{il}]) = [u_1 \quad u_2 \quad \cdots \quad u_n]^T.$$

Step 8: Find an optimum set of

$$Opt_{Mm}(U) = [u_1, u_2, \ldots, u_n]^T$$

4. NUMERICAL APPLICATIONS: Manpower Recruitment Problem

As an illustration, a manpower recruitment problems by Chaudhuri et al. [20] is revisited for the purpose. A fuzzy soft (f_A, E) describes the manpower recruitment selection problem as a programmer. Three staffs in the Human Resources Department are involved as decision makers, denoted by *A*,*B* and *C* respectively. There are eight criteria considered as the parameters and seven programmers to be recruited by a Software Development Organization.

The decision is made based on these criteria. Weight of criteria will be calculated based on the Lambda – max method.The FSMmDM method is used to solve this problem.

4.1 Manpower recruitment problem

Let $U = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7\}$ be the universal set of seven programmers to be recruited by a Software Development Organization as a possible alternative. Let $E = \{e_1, e_2, e_3, e_3, e_4, e_5, e_6, e_7, e_8\}$ be the set of parameters (criteria for every programmer) , such that e_1, e_2, e_3, e_4, e_5, e_6, e_7, and e_8 represent the parameters “hardworking”, “discipline”, “honest”, “obedient”, “intelligence”, “innovative”, “entrepreneurial attitude”, and “aspirant” respectively. Hardworking and discipline describe the punctuality of the programmer. Honesty and obedient describe the truth in the behavior of the programmer meanwhile intelligence and innovative describe the innovative attitude of the programmer. Finally entrepreneurial attitude and aspirant describe the exploratory mindset of the programmer.

Table 2. Fuzzy comparison matrix of criteria with respect to overall goal by DM1

DM1	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	(1,1,1)	(1/7,1/5,1/3)	(1/6,1/4,1/2)	(1/9,1/7,1/5)	(1/5,1/3,1)	1/6,1/4,1/2)	(1/7,1/5,1/3)	(1/9,1/8,1/6)
C_2	(3,5,7)	(1,1,1)	(1/4,1/2,1)	(1/9,1/8,1/6)	(1/7,1/5,1/3)	(1/8,1/6,1/4)	(1/9,1/7,1/5)	(1/7,1/5,1/3)
C_3	(2,4,6)	(1,2,4)	(1,1,1)	(2,4,6)	(4,6,8)	(3,5,7)	(6,8,9)	(1/6,1/4,1/2)
C_4	(5,7,9)	(6,8,9)	(1/6,1/4,1/2)	(1,1,1)	(1/7,1/5,1/3)	(1/5,1/3,1)	(1/8,1/6,1/4)	(1/9,1/7,1/5)
C_5	(1,3,5)	(3,5,7)	(1/8,1/6,1/4)	(3,5,7)	(1,1,1)	(1/9,1/7,1/5)	(1/7,1/5,1/3)	(1/8,1/6,1/4)
C_6	(2,4,6)	(4,6,8)	(1/7,1/5,1/3)	(1,3,5)	(5,7,9)	(1,1,1)	(1/8,1/6,1/4)	(1/5,1/3,1)
C_7	(3,5,7)	(5,7,9)	(1/9,1/8,1/6)	(4,6,8)	(3,5,7)	(4,6,8)	(1,1,1)	(1/5,1/3,1)
C_8	(6,8,9)	(3,5,7)	(2,4,6)	(5,7,9)	(4,6,8)	(1,3,5)	(1,3,5)	(1,1,1)

Table 3. Fuzzy comparison matrix of criteria with respect to overall goal by DM2

DM2	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	(1,1,1)	(1/8,1/6,1/4)	(1/9,1/7,1/5)	(1/7,1/5,1/3)	(1/9,1/8,1/6)	(1/6,1/4,1/2)	(1/5,1/3,1)	(1/5,1/3,1)
C_2	4,6,8)	(1,1,1)	(3,5,7)	(5,7,9)	(1,3,5)	(2,4,6)	(1,2,4)	(1/7,1/5,1/3)
C_3	(5,7,9)	(1/7,1/5,1/3)	(1,1,1)	(3,5,7)	(4,6,8)	(1,3,5)	(5,7,9)	(1/6,1/4,1/2)
C_4	(3,5,7)	(1/9,1/7,1/5)	(1/7,1/5,1/3)	(1,1,1)	(1/6,1/4,1/2)	(1/7,1/5,1/3)	(1/5,1/3,1)	(1/9,1/7,1/5)
C_5	(6,8,9)	(1/5,1/3,1)	(1,/8,1/6,1/4)	(2,4,6)	(1,1,1)	(3,5,7)	(1,3,5)	(1/5,1/3,1)
C_6	(2,4,6)	(1/6,1/4,1/2)	(1/5,1/3,1)	(3,5,7)	(1/7,1/5,1/3)	(1,1,1)	(2,4,6)	(1/6,1/4,1/2)
C_7	(1,3,5)	(1/4,1/2,1)	(1/9,1/7,1/5)	(1,3,5)	(1/5,1/3,1)	(1/6,1/4,1/2)	(1,1,1)	(1/7,1/5,1/3)
C_8	(1,3,5)	(3,5,7)	(2,4,6)	(5,7,9)	(1,3,5)	(2,4,6)	(3,5,7)	(1,1,1)

Table 4. Fuzzy comparison matrix of criteria with respect to overall goal by DM3

DM3	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	(1,1,1)	(3,5,7)	(5,7,9)	(4,6,8)	(2,4,6)	(6,8,9)	(3,5,7)	(3,5,7)
C_2	1/7,1/5,1/3	(1,1,1)	(3,5,7)	(6,8,9)	(1,3,5)	(2,4,6)	(1,2,4)	(4,6,8)
C_3	(1/9,1/7,1/5)	1/7,1/5,1/3	(1,1,1)	(1/6,1/4,1/2)	(1/7,1/5,1/3)	(1/8,1/6,1/4)	(1/5,1/3,1)	(,1/4,1/2,1)
C_4	(1/8,1/6,1/4)	1/9,1/8,1/6	(2,4,6)	(1,1,1)	(1/8,1/6,1/4)	(1/7,1/5,1/3)	(1/5,1/3,1)	(1/6,1/4,1/2)
C_5	(1/6,1/4,1/2)	1/5,1/3,1	(3,5,7)	(4,6,8)	(1,1,1)	(1/6,1/4,1/2)	(1/4,1/2,1)	(1/7,1/5,1/3)
C_6	(1/9,1/8,1/6)	1/6,1/4,1/2	(4,6,8)	(3,5,7)	(2,4,6)	(1,1,1)	(1,3,5)	(1,3,5)
C_7	(1/7,1/5,1/3)	1/4,1/2,1	(1,3,5)	(1,3,5)	(1,2,4)	(1/5,1/3,1)	(1,1,1)	(1/6,1/4,1/2)
C_8	(1/7,1/5,1/3)	(1/8,1/6,1/4)	(1,2,4)	(2,4,6)	(3,5,7)	(1/5,1/3,1)	(2,4,6)	(1,1,1)

4.1.1 Constructing the comparison matrices in FAHP

The pair-wise comparison matrix, is constructed from the evaluation by each decision maker according to the nine point scale commonly used in FAHP [18]. The fuzzy evaluation matrix are presented in triangular fuzzy numbers for three decision maker are shown in Table 2 – 4.

A. Criteria weight for each decision maker

Evaluating all the inputs using Lambda – max method, we obtained the weight for each decision maker as in Table 5.

Table 5. Criteria weight by every decision makers

Criteria	(W_A)	(W_B)	(W_C)
$\mathbf{C_1}$	0.024	0.025	0.389
$\mathbf{C_2}$	0.031	0.208	0.203
$\mathbf{C_3}$	0.220	0.175	0.025
$\mathbf{C_4}$	0.057	0.036	0.030
$\mathbf{C_5}$	0.061	0.122	0.063
$\mathbf{C_6}$	0.109	0.082	0.128
$\mathbf{C_7}$	0.177	0.053	0.068
$\mathbf{C_8}$	0.322	0.299	0.093

B. Calculation of fuzzy soft max – min decision making (FSMmDM) method

Step 1: Evaluation of membership degree by each decision makers.

Step 2: Construct fuzzy soft evaluation in Step 1 into matrix form.

$$A_{ij} = \begin{bmatrix} 0.78 & 0.50 & 0.76 & 0.50 & 0.70 & 0.70 & 0.90 & 0.60 \\ 0.70 & 1.00 & 0.86 & 0.80 & 0.50 & 0.80 & 0.78 & 0.50 \\ 0.50 & 0.70 & 0.54 & 0.45 & 0.60 & 0.65 & 0.52 & 1.00 \\ 0.59 & 0.20 & 0.20 & 0.60 & 0.74 & 0.73 & 0.60 & 0.87 \\ 1.00 & 0.70 & 0.89 & 0.70 & 0.80 & 0.82 & 1.00 & 1.00 \\ 0.60 & 1.00 & 0.90 & 1.00 & 0.90 & 0.68 & 0.89 & 0.66 \\ 1.00 & 0.80 & 0.50 & 0.52 & 1.00 & 0.56 & 0.83 & 0.70 \end{bmatrix}$$

$$B_{ik} = \begin{bmatrix} 0.68 & 0.89 & 0.63 & 0.50 & 1.00 & 0.80 & 0.80 & 0.85 \\ 0.85 & 0.65 & 0.81 & 0.44 & 1.00 & 0.90 & 0.74 & 0.96 \\ 0.55 & 0.75 & 0.65 & 0.90 & 0.68 & 1.00 & 0.68 & 1.00 \\ 0.65 & 0.79 & 0.80 & 1.00 & 0.98 & 0.89 & 0.82 & 1.00 \\ 0.82 & 0.89 & 0.80 & 0.75 & 1.00 & 0.98 & 0.90 & 0.89 \\ 0.95 & 1.00 & 0.90 & 0.65 & 0.69 & 1.00 & 1.00 & 0.68 \\ 0.56 & 1.00 & 0.75 & 0.63 & 0.86 & 0.76 & 0.57 & 1.00 \end{bmatrix}$$

$$C_{il} = \begin{bmatrix} 0.60 & 0.70 & 0.59 & 0.80 & 0.70 & 1.00 & 0.70 & 1.00 \\ 0.50 & 0.56 & 0.67 & 0.60 & 0.56 & 0.86 & 0.64 & 0.50 \\ 0.75 & 0.90 & 0.65 & 0.64 & 0.82 & 0.56 & 0.73 & 0.60 \\ 0.55 & 0.85 & 0.75 & 0.50 & 0.67 & 0.64 & 0.87 & 0.70 \\ 0.95 & 0.54 & 1.00 & 1.00 & 0.86 & 1.00 & 0.68 & 0.50 \\ 0.50 & 0.60 & 0.90 & 0.91 & 0.70 & 1.00 & 0.80 & 0.60 \\ 0.60 & 0.73 & 0.60 & 1.00 & 0.90 & 1.00 & 1.00 & 0.83 \end{bmatrix}$$

Step 3: Incorporate the criteria weight for each decision maker in the fuzzy soft matrix. We obtain:

$$[A_{ij} \times W_A] = [R_{ij}] \begin{bmatrix} 0.024 & 0.016 & 0.167 & 0.029 & 0.043 & 0.076 & 0.159 & 0.193 \\ 0.017 & 0.031 & 0.189 & 0.046 & 0.031 & 0.087 & 0.138 & 0.161 \\ 0.012 & 0.022 & 0.119 & 0.026 & 0.037 & 0.071 & 0.092 & 0.322 \\ 0.014 & 0.006 & 0.044 & 0.034 & 0.045 & 0.080 & 0.106 & 0.280 \\ 0.024 & 0.022 & 0.196 & 0.040 & 0.049 & 0.089 & 0.177 & 0.322 \\ 0.014 & 0.031 & 0.198 & 0.057 & 0.055 & 0.074 & 0.158 & 0.213 \\ 0.24 & 0.025 & 0.110 & 0.030 & 0.061 & 0.061 & 0.147 & 0.225 \end{bmatrix}$$

$$[B_{ij} \times W_B] = [S_{ij}] = \begin{bmatrix} 0.017 & 0.185 & 0.110 & 0.018 & 0.122 & 0.066 & 0.042 & 0.254 \\ 0.021 & 0.135 & 0.142 & 0.016 & 0.122 & 0.074 & 0.039 & 0.287 \\ 0.014 & 0.156 & 0.114 & 0.032 & 0.083 & 0.082 & 0.036 & 0.299 \\ 0.016 & 0.164 & 0.140 & 0.036 & 0.120 & 0.073 & 0.043 & 0.299 \\ 0.021 & 0.185 & 0.140 & 0.027 & 0.122 & 0.080 & 0.048 & 0.266 \\ 0.024 & 0.208 & 0.158 & 0.023 & 0.084 & 0.082 & 0.053 & 0.203 \\ 0.014 & 0.208 & 0.131 & 0.023 & 0.105 & 0.062 & 0.030 & 0.299 \end{bmatrix}$$

$$[C_{ij} \times W_c] = [T_{ij}] = \begin{bmatrix} 0.233 & 0.142 & 0.015 & 0.024 & 0.044 & 0.128 & 0.048 & 0.093 \\ 0.195 & 0.114 & 0.017 & 0.018 & 0.035 & 0.110 & 0.044 & 0.048 \\ 0.292 & 0.183 & 0.016 & 0.019 & 0.052 & 0.072 & 0.050 & 0.056 \\ 0.214 & 0.173 & 0.019 & 0.015 & 0.042 & 0.082 & 0.059 & 0.065 \\ 0.370 & 0.110 & 0.025 & 0.030 & 0.054 & 0.128 & 0.046 & 0.047 \\ 0.195 & 0.122 & 0.023 & 0.027 & 0.044 & 0.128 & 0.054 & 0.056 \\ 0.233 & 0.148 & 0.015 & 0.030 & 0.057 & 0.128 & 0.068 & 0.077 \end{bmatrix}$$

Step 4: Using *And* – product, the product of fuzzy soft matrices between $[R_{ij}]$ and $[T_{il}]$ is obtained as follows:

$$\left[\begin{matrix} 0.017 & 0.024 & 0.024 & 0.024 & 0.024 & 0.024 & 0.024 & 0.024 \\ 0.017 & 0.017 & 0.017 & 0.017 & 0.017 & 0.017 & 0.017 & 0.017 \\ 0.012 & 0.012 & 0.012 & 0.012 & 0.012 & 0.012 & 0.012 & 0.012 \\ 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 \\ 0.021 & 0.024 & 0.0204 & 0.024 & 0.024 & 0.024 & 0.024 & 0.024 \\ 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 \\ 0.014 & 0.024 & 0.023 & 0.024 & 0.024 & 0.024 & 0.024 & 0.024 \end{matrix}\right. \quad \begin{matrix} 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.017 \\ 0.021 & 0.031 & 0.031 & 0.016 & 0.031 & 0.031 & 0.031 & 0.0201 \\ 0.014 & 0.022 & 0.022 & 0.022 & 0.022 & 0.022 & 0.022 & 0.014 \\ 0.006 & 0.006 & 0.006 & 0.006 & 0.006 & 0.006 & 0.006 & 0.016 \\ 0.021 & 0.022 & 0.022 & 0.022 & 0.022 & 0.022 & 0.022 & 0.021 \\ 0.024 & 0.031 & 0.031 & 0.023 & 0.031 & 0.031 & 0.031 & 0.024 \\ 0.014 & 0.025 & 0.025 & 0.023 & 0.025 & 0.025 & 0.025 & 0.014 \end{matrix} \quad \begin{matrix} 0.167 & 0.110 & 0.018 & 0.122 & 0.066 & 0.066 & 0.042 & 0.167 \\ 0.135 & 0.142 & 0.016 & 0.122 & 0.074 & 0.074 & 0.039 & 0.189 \\ 0.119 & 0.114 & 0.032 & 0.083 & 0.083 & 0.082 & 0.036 & 0.119 \\ 0.044 & 0.044 & 0.036 & 0.044 & 0.044 & 0.044 & 0.043 & 0.044 \\ 0.185 & 0.140 & 0.027 & 0.122 & 0.122 & 0.080 & 0.048 & 0.196 \\ 0.198 & 0.158 & 0.023 & 0.084 & 0.084 & 0.082 & 0.053 & 0.198 \\ 0.110 & 0.110 & 0.023 & 0.105 & 0.105 & 0.062 & 0.030 & 0.110 \end{matrix}$$

$$\begin{matrix} 0.017 & 0.029 & 0.029 & 0.018 & 0.029 & 0.029 & 0.029 & 0.029 \\ 0.021 & 0.046 & 0.046 & 0.016 & 0.046 & 0.046 & 0.039 & 0.046 \\ 0.014 & 0.026 & 0.026 & 0.026 & 0.026 & 0.026 & 0.026 & 0.026 \\ 0.016 & 0.034 & 0.034 & 0.034 & 0.034 & 0.034 & 0.034 & 0.034 \\ 0.021 & 0.040 & 0.040 & 0.027 & 0.040 & 0.040 & 0.040 & 0.040 \\ 0.024 & 0.057 & 0.057 & 0.023 & 0.057 & 0.057 & 0.053 & 0.057 \\ 0.014 & 0.030 & 0.030 & 0.023 & 0.030 & 0.030 & 0.030 & 0.030 \end{matrix} \quad \begin{matrix} 0.017 & 0.043 & 0.043 & 0.018 & 0.043 & 0.043 & 0.042 & 0.043 \\ 0.021 & 0.031 & 0.031 & 0.016 & 0.031 & 0.031 & 0.031 & 0.031 \\ 0.014 & 0.037 & 0.037 & 0.032 & 0.037 & 0.037 & 0.036 & 0.037 \\ 0.016 & 0.045 & 0.045 & 0.036 & 0.045 & 0.045 & 0.043 & 0.045 \\ 0.021 & 0.049 & 0.049 & 0.027 & 0.049 & 0.049 & 0.048 & 0.049 \\ 0.024 & 0.055 & 0.055 & 0.023 & 0.055 & 0.055 & 0.053 & 0.055 \\ 0.014 & 0.061 & 0.061 & 0.023 & 0.061 & 0.061 & 0.030 & 0.061 \end{matrix} \quad \begin{matrix} 0.017 & 0.076 & 0.076 & 0.018 & 0.076 & 0.066 & 0.042 & 0.076 \\ 0.021 & 0.087 & 0.087 & 0.016 & 0.087 & 0.074 & 0.039 & 0.087 \\ 0.014 & 0.071 & 0.070 & 0.032 & 0.071 & 0.071 & 0.036 & 0.071 \\ 0.016 & 0.080 & 0.080 & 0.026 & 0.080 & 0.073 & 0.043 & 0.080 \\ 0.021 & 0.089 & 0.089 & 0.027 & 0.089 & 0.080 & 0.048 & 0.089 \\ 0.024 & 0.074 & 0.074 & 0.023 & 0.074 & 0.074 & 0.053 & 0.074 \\ 0.014 & 0.061 & 0.061 & 0.023 & 0.061 & 0.061 & 0.030 & 0.061 \end{matrix}$$

$$\begin{matrix} 0.017 & 0.159 & 0.110 & 0.018 & 0.122 & 0.066 & 0.042 & 0.159 \\ 0.021 & 0.135 & 0.138 & 0.016 & 0.122 & 0.074 & 0.039 & 0.138 \\ 0.014 & 0.092 & 0.092 & 0.032 & 0.083 & 0.082 & 0.036 & 0.092 \\ 0.016 & 0.106 & 0.106 & 0.036 & 0.106 & 0.073 & 0.043 & 0.106 \\ 0.021 & 0.177 & 0.140 & 0.027 & 0.122 & 0.080 & 0.048 & 0.177 \\ 0.024 & 0.158 & 0.158 & 0.023 & 0.084 & 0.082 & 0.053 & 0.158 \\ 0.014 & 0.147 & 0.0131 & 0.023 & 0.105 & 0.062 & 0.030 & 0.147 \end{matrix} \quad \left.\begin{matrix} 0.017 & 0.185 & 0.110 & 0.018 & 0.122 & 0.066 & 0.042 & 0.193 \\ 0.021 & 0.135 & 0.142 & 0.016 & 0.122 & 0.074 & 0.039 & 0.161 \\ 0.014 & 0.156 & 0.114 & 0.032 & 0.083 & 0.082 & 0.036 & 0.299 \\ 0.016 & 0.164 & 0.140 & 0.036 & 0.120 & 0.073 & 0.043 & 0.280 \\ 0.021 & 0.185 & 0.140 & 0.027 & 0.122 & 0.080 & 0.048 & 0.266 \\ 0.024 & 0.208 & 0.158 & 0.023 & 0.084 & 0.082 & 0.053 & 0.204 \\ 0.014 & 0.208 & 0.131 & 0.023 & 0.105 & 0.062 & 0.030 & 0.225 \end{matrix}\right]$$

which is 7x64 matrix

Step 5: Observe that there are 8 blocks of 7x8 elements in the above matrix. Determine the minimum value of each row in each block, we then obtain $[X_{ij}]$:

$$[X_{ij}]=\begin{bmatrix} 0.017 & 0.016 & 0.017 & 0.017 & 0.017 & 0.017 & 0.017 & 0.017 \\ 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 \\ 0.012 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 \\ 0.014 & 0.006 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 \\ 0.021 & 0.021 & 0.021 & 0.021 & 0.021 & 0.021 & 0.021 & 0.021 \\ 0.014 & 0.023 & 0.023 & 0.023 & 0.023 & 0.023 & 0.023 & 0.023 \\ 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 \end{bmatrix}$$

Step 6: Find *And* – product between $[X_{ij}]$ and $[S_{ij}]$ similar to Step 4. Observe that there are 8 blocks of 7x8 elements in fuzzy matrix. Then repeat Step 5 for *And* – product between $[X_{ij}]$ and $[S_{ij}]$. For each block, we choose the minimum value of each row.

Hence the min for the *And* – product obtain as follows:

$$\min([X_{ij}]\wedge[S_{ij}])=[Y_{ij}]=$$

$$\begin{bmatrix} 0.017 & 0.016 & 0.017 & 0.017 & 0.017 & 0.017 & 0.017 & 0.017 \\ 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 \\ 0.012 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 \\ 0.014 & 0.006 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 \\ 0.021 & 0.021 & 0.021 & 0.021 & 0.021 & 0.021 & 0.021 & 0.021 \\ 0.014 & 0.023 & 0.023 & 0.023 & 0.023 & 0.023 & 0.023 & 0.023 \\ 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 & 0.014 \end{bmatrix}$$

Step 7: Obtain the maximum value for each row in fuzzy soft matrix in Step 7 as:

$$Mm(([R_{ij}]\wedge[S_{ij}])\wedge[T_{ij}])=Mm[Y_{ij}]=\begin{bmatrix} 0.017 \\ 0.016 \\ 0.014 \\ 0.016 \\ 0.021 \\ 0.023 \\ 0.014 \end{bmatrix}$$

Step 8: Finally, we find the optimum fuzzy soft set as in Definition 5 which is the maximum value among the elements in Step 7, that is

$$Opt_{Mm(([R_{ij}]\wedge[S_{ij}])\wedge[T_{ij}])}(U)=\{M_6\}.$$

Hence the human resources department will select programmer 6 as the preferred programmer in Software Development Organization.

5. CONCLUSION

Fuzzy soft set theory has been applied in many fields especially in solving decision making problems. In this paper we presented the FSMmDM method incorporating together with important weight of each criteria involved, obtained by using Lambda – max method [19]. We also generalized the FSMmDM method proposed by [17] as a group decision making method as in [15].

We gave a numerical example of group decision making problem in manpower recruitment that demonstrated the generalization method incorporating the weight of parameters can be effectively applied for such problem.

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