RESEARCH ARTICLE

Simulation of AH flows and deformation of DMD in a 3D AC

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Abstract

This paper presents the interaction between the aqueous humour (AH) flows and the deformation of Descemet membrane detachment (DMD) in a 3D anterior chamber (AC). Arbitrary Lagrangian Eulerian (ALE) method is used to model the problem. Finite element method using COMSOL Multiphysics software is adopted to solve the governing equations for the AH flows and the deformation of DMD. The fluid flow behaviour and the deformation of the detached Descemet membrane are analysed in order to comprehend the progression of the DMD in the AC due to the AH flows and vice versa. The re-attachment or re-detachment of the DMD is significantly affected by the AH flows. Advance treatment for the DMD can be developed based on a better understanding of the interaction between the AH flows and the DMD.

Keywords: Aqueous humour, descemet membrane detachment, arbitrary lagrangian eulerian

INTRODUCTION

Descement membrane detachment (DMD) is a human eye disease and it causes blindness. DMD happens in the region called as anterior chamber (AC). AC is bounded by cornea, iris and pupil and filled by aqueous humour (AH) which has properties similar to water. Cornea is the clear dome like surface that exists at the front of a human eye and it covers the iris and the pupil, as shown in Figure 1. The cornea is made up by three main layers and two auxiliary layers, those are epithelium, stroma, endothelium, Browman layer and Descemet membrane (DM). Epithelium is the outermost layer and endothelium is the innermost layer of the cornea. Stroma is the intermedium layer. Moreover, Browman layer is the layer lies between the stroma and epithelium and DM is the layer lies between the stroma and endothelium. DMD happens when there is a tear or break on the DM which allows the AH flows into the subspaces between the stroma and DM, and consequently separate the DM away from the stroma.
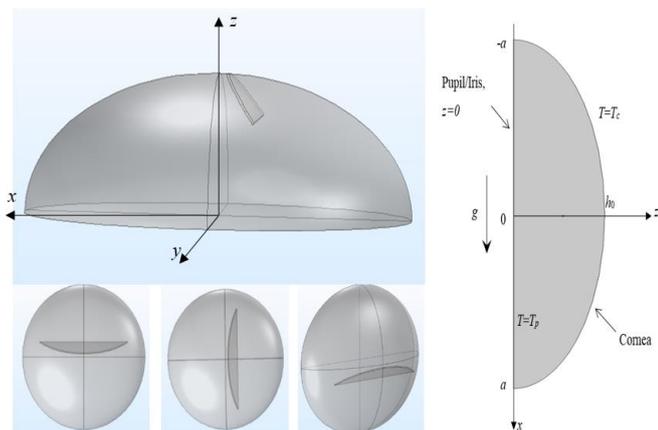
Fig. 1 Schematic diagram of the AC of human eye.

Mulhern *et al.* (1996), Potter and Zalatimo (2005) and Ünlü and Aksünger (2000) have reported that the DMD is induced by cataract surgery, iridectomy, trabeculectomy, corneal transplantation, deep lamellar keratoplasty, holmium laser sclerostomy, alkali burn and viscocanalostomy. The technique of curing of DMD caused by cataract surgery with sulphur hexafluoride injection is discussed by Sevillano *et al.* (2008). Further, 2005 studied the case of treating the DMD by injecting intracameral perfluoropropane (C_3F_8) into the AC. Recently, Couch and Baratz (2009) investigated two cases of delayed bilateral DM, one eye was fixed surgically and the other eye improved spontaneously. They estimated that the spontaneous reattachment happen because of the AH flow in the AC driven by the buoyancy effects. Motivated by Couch and Baratz (2009), Ismail *et al.* (2013) developed a new mathematical model of the AH flow in the AC with DMD based on the work done by Canning *et al.* (2002), to explain the spontaneous reattachment phenomena. The authors concluded that the AH flow driven by the temperature difference across the eye and the orientation of the patient may control the clinical outcomes for the DMD.

Ismail *et al.* (2013) assumed that the AH flow in the AC is only driven by the gravity and buoyancy and hence, they considered the flow to be essentially two dimensional as explained in Canning *et al.* (2002). Therefore, they studied a two dimensional 'sectionally thin-layer' flow when a DMD is present in the AC. The surface of the cornea and the iris are assumed to be stationary tissue. The iris or pupil is assumed to have a fix temperature $T_p = 302K$. The cornea surface is imposed with a constant heat flux. The lubrication theory limit of the Navier-Stokes equations were employed by Ismail *et al.* (2013), in order to study the problem analytically. They concluded that the spontaneous reattachment of DMD is affected by the AH flow in AC.

However, the exact mechanisms of the spontaneous reattachment appears not to be known. This is because the interaction between the AH flows and the deformation of DMD is not considered in the previous studies. DMD in the flow stream of AH is subjected to fluid

forces that will cause it to deform. The fluid forces are dependent on the geometry of the DMD and also the deformation, since the AH flows. Thus, the governing equations for the DMD as well as for the fluid need to be solved simultaneously. Lagrangian description is usually used to formulate the DMD, and Eulerian description is most often used to express the fluid for a convenient description of its convective nature. It is inconvenient to descript the fluid flow by using Lagrangian description which employs a referential frame moving with the flow to track the fluid motion. This is because a frequent remeshing is required due to large deformations due to the fluid motion, consequently increasing the required computational resources. On the other hand, if the Eulerian description is applied to descript the motion of DMD, the deformation of the DMD due to the fluid forces is difficult to track.

Therefore, the arbitrary Lagrangian Eulerian (ALE) technique is introduced which combine the advantage of both the methods and the obstacles of a purely Eulerian or purely Lagrangian implementation are avoided (Kjellgren and Hyva Èrinen, 1998; Kuhl *et al.*, 2003; Lo and Young, 2004; Braescu and George, 2007; Anjos *et al.*, 2014; Jafari and Okutucu-Özyurt, 2016). The technique has been used by few researchers to investigate the human eye problem. The effect of intraocular pressure (IOP) variations (10, 20, and 30 mmHg) in the stresses and deformations of the human eye components is analyzed by Karimi *et al.* (2016a) with the aid of a computational Lagrangian-Eulerian coupling model. Karimi *et al.* (2016b) studied the stresses and deformations of all the human eye components due to the high explosive detonation by using fluid-structure interaction (FSI) model. Further, Karimi *et al.* (2016c) investigated the stresses and deformations of all the human eye components due to the tennis ball impact using a 3D computational dynamic fluid-structure interaction model.

The present study is aimed to deeply investigate the role of the AH flows in the deformation of the DMD using a 3D computational ALE model. The AH flows is modelled by using the fluid mechanical theory which developed by Canning *et al.* (2002) with the presence of detached DM. COMSOL Multiphysics which compute the results based on finite element method is applied to solve the model. Finite element methods is easily applied to objects with irregular geometry like the shape of the cornea. Moreover, it is convenient to solve problem with mixed boundary condition and problem with the objects composed of several different materials in its medium (for example, see Nouri *et al.*, 2002; Marcon *et al.*, 2002; Lewis *et al.*, 2008; Reddy and Gartling, 2010; Zienkiewicz *et al.*, 2013).

MATHEMATICAL FORMULATION

Model Construction

A three-dimensional AH flow driven by buoyancy effects in the AC with the presence of DMD has been considered. The temperature gradient is induced by the different temperature at the back of the AC, which is estimated to core body temperature (37^0C) to the outside of the cornea (say 24^0C). Buoyant convection is happen within the AC due to the temperature gradient that occur across the AC of the eye. $z = 0$ represent the plane formed by pupil aperture and the iris and the anterior surface of the cornea is represented by $z = h(x, y) = h_0(1 - (x/a)2 - (y/a)2)1/2$, which the AH flow is introduced in a Cartesian coordinate (x, y, z). At the iris, the temperature is fixed at T_p which is close to the human body temperature, 310.15K, and the temperature at the cornea is assumed to be T_c , around 308.15 K. The different temperatures between the back of the AC and the cornea induce the temperature gradient inside the AC. The patient is assumed to be in an upright position thus the gravity, *g* is acting downward as shown in Figure 1. Following Canning *et al.* (2002) and Ismail *et al.* (2013), a set of typical values for the AH in the AC of the human eye is used as shown in Table 1.

Table 1: Properties of the AH used in the model.

Parameter	Description	Unit
a	Radius of the AC	5.5×10⁻³ m
α	Coefficient of thermal expansion	3×10^{-4} K⁻¹
c_P	Specific heat of the AH	4200 Jkg⁻¹K⁻¹
g	Gravity	9.8 ms⁻²
h_0	Maximum height of the cornea	2.75×10⁻³ m
k	Thermal conductivity of the AH	0.57 Wm⁻¹K⁻¹
μ	Viscosity of the AH	0.9×10^{-3} kgm⁻¹s⁻¹
ρ	Density of the AH	1000 kgm⁻³
T_c	Temperature at the cornea	308.15 K
T_p	Temperature at the iris/pupil	310.15 K
U	Typical buoyancy-driven flow speed in the AC	10^{-4} ms⁻¹

The AH is assumed to be Newtonian, incompressible and driven by the temperature gradient. Therefore, the Navier-Stokes equations and energy equations which are derived based on the principle of conservation of energy are used to govern the fluid flow. The derivation of energy equations can be found in Batchelor (2000) and Kundu *et al.* (2012). Variation of density is happen due to the difference in temperature. The variation of the density cannot be neglected as it gives rise to the flow. This adds to the complexity in the analysis of the flow. Boussinesq approximation is applied to allow a more convenient procedure to obtain the solution. Boussinesq approximation Jaluria (1980) states that the density difference due to the interaction between the gravity is considered to vary with temperature where the density elsewhere is considered constant. Thus, the Boussinesq approximation governing equations of the AH flow driven by the temperature difference across the AC are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$-\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2}\right) - \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right)$$
$$+ g\left(1 - \alpha\left(T - T_c\right)\right) = 0,$$

$$-\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2}\right) - \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z}\right) = 0, \quad (1)$$

$$-\frac{\partial p}{\partial z} + \upsilon\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2}\right) - \rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial x}\right) = 0,$$

$$\frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = 0,$$

where υ is the kinematic viscosity of the fluid. The boundary conditions for the velocity are:

$$u(x, y, 0) = v(x, y, 0) = w(x, y, 0) = 0,$$
$$u(x, y, h(x, y)) = v(x, y, h(x, y)) = w(x, y, h(x, y)) = 0. \quad (2)$$

The boundary conditions for temperature are as follows:

$$T = T_p \text{ at } z = 0, \ T = T_c \text{ at } z = h(x, y). \quad (3)$$

where h_0 is a typical depth of the AC, T_c and T_p denote the temperature at the cornea and the plane formed by the pupil and the iris respectively. By assuming that the fluxes and the pressures at each point *x* are continuous, the pressure is known and is equal to the

constant pressure $p = p_a$ at $x = a$, where p_a is the normal atmospheric pressure and equal to 101325 Pa. Finite element method is applied to solve equation (1) by subjected to equations (2) and (3).

Further, as a preliminary to the study, the detached DM is assumed to be a rigid, thin and small flap attached onto the anterior surface of the cornea and has the same temperature as the cornea, since originally it is a part of the cornea. Generally, DMD does not have a fix shape. For simplicity, in the present study, DMD is assumed to be a flat plate as shown in Figure 2. The Descemet's membrane is approximately 0.01 mm thick. The length of the DMD is taken to be 2 mm. The cornea and the DMD are modelled as stationary tissues, so the posterior cornea surface and DMD surface are set to non-slip condition. The temperature of these surfaces is set to 308.15 K. Also, the iris/pupil is modeled as stationary and the velocity is set to zero along the surface and temperature is set to 310.15 K. Additionally, the problem domain and boundary conditions of the DMD are shown in Figure 2.
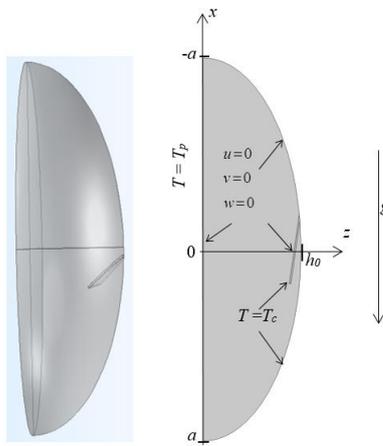
Fig. 2 Problem domain and boundary conditions of the DMD problem.

The computed approximation solutions of the fluid forces are then used to determine the deformation of the detach membrane. The DMD is assumed as a thin flexible elastic plate or beam. Krichhoff theory for small deflection of thin plate (see Ventsel and Krauthammer, 2001 and Ismail *et al.*, 2013 for detail.) is adopted to estimate the deformation of the DMD under the influence of the fluid flows. The detachment, $D(x, y, z)$ satisfies the following equation:

$$D_m\left(\frac{d^4 D}{dx^4} + \frac{d^4 D}{dy^4} + \frac{d^4 D}{dz^4}\right) = \nabla \cdot \sigma_f, \qquad (4)$$

where D_m is the flexural rigidity of the DMD and is defined as $D_m = E_m d_m^3 / 12\left(1 - \upsilon_D^2\right)$. E_m is the Young's modulus, d_m is the thickness of the DMD and υ_D is the Poisson ration of the DM material. σ_f is the fluid stress tensor which obtained from the previous calculation. Following Ismail et al. (2013), we set $E_m = 50000 \text{ Pa}^d$, $d_m = 10\mu\text{mc}$ and $\upsilon_D = 0.45$.

Computational Mesh and Numerical Method

COMSOL Multiphysics 5.2 was used to compute the numerical results. A personal computer with a processor speed of 2.30 GHz and a RAM of 8 GB was used to perform the computation in present study. The governing equations (1) subjected to the boundary conditions (2) and (3) were first be solved by using finite element method. Then, the computed fluid forces were used to solve equation (4).

The three dimensional model is meshed using tetrahedral elements. The total number of elements in the mesh of the DMD

problem (see Figure 3) is 33257. In order to show that the results do not affected by the number of elements that used to mesh the geometry, a mesh test was conducted as shown in Figure 3b. The results did not depend on the number of element that use to mesh the geometry, thus, we hold the view that the results is independent from the number of elements. The desired variables such as velocity, pressure and temperature are approximated by using Lagrange cubic polynomial. The displacement of the DMD is also approximated by applying Lagrange cubic polynomial.
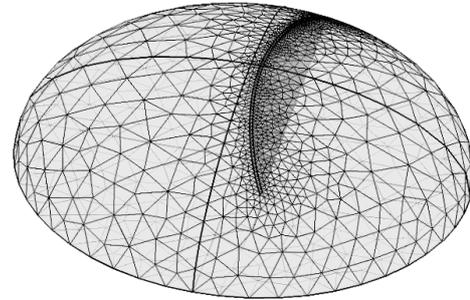
Fig. 3 Mesh plot.
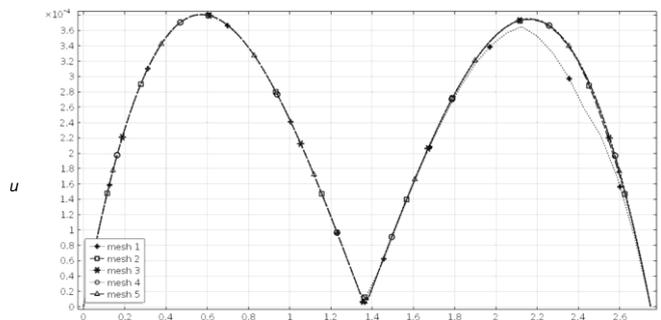
Fig. 4 Magnitude of the velocity along the line (0,0,0) to (0,0,0.0275) in the AC without DMD with different mesh.

RESULTS AND DISCUSSIONS

Figure 4 shows the streamline plot for the fluid flow in the AC driven by the buoyancy convection without the DMD. The streamline is concur to the streamline plotted by Ismail et al. (2013) (see Figure 3 in Ismail *et al.*, 2013). Moreover, the maximum flow speed observed in present communication is 3.680×10^{-4} m/s and it is located at position $(0, 5.441 \times 10^{-4})$. It is great agreement with the analytical determined maximum flow speed by Ismail *et al.* (2013) which is 3.962×10^{-4} m/s and it is located at $(0, 5.811 \times 10^{-4})$. This has validated the method applied in present study to compute the numerical solutions, as well as enhances our confidence to the results obtained in the research.
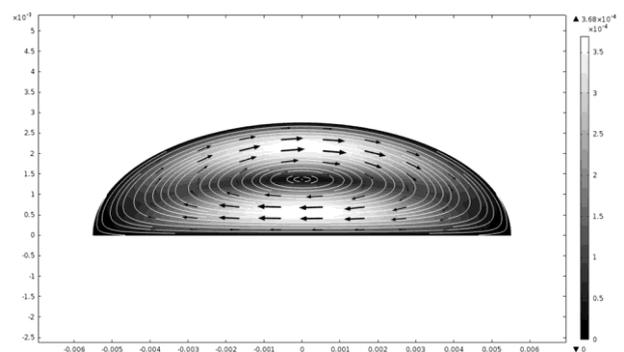
Fig. 5 Streamline plot at plane $y = 0$ for case without DMD.

The AH circulates in the AC, as illustrate in Figures 5 for gravity acting upward in *x*-direction. As a result of the temperature gradient between the cornea and the iris/pupil, the AH adjacent to the iris/pupil is heated and eventually rises as buoyant flow. Then, the AH adjacent to the cornea becomes colder and hence becomes heavier than ambient fluid (see Figure 5). The existence of DMD in the AC yield an opposing effect on the buoyancy. The DMD increases the resistance to the AH flow, which is adjacent to the cornea, then reduces the velocity of the downward AH flow if compared with the case without DMD. In the subspace between the detached DMD and the stroma, the AH flow is able to turn around the corner and the flow is slow in this region and Moffatt vortices are detected Ismail *et al.* (2013) as illustrated in Figure 5. An opposite phenomena is observed when the gravity is assumed to act downward in the *x*-direction as shown in Figure 6.
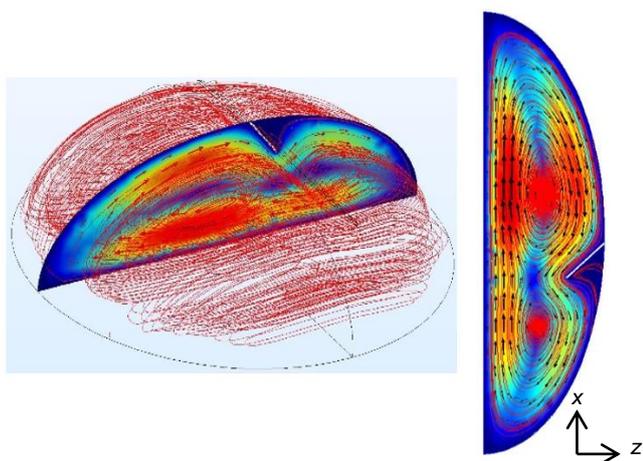
Fig. 6 Streamline in AC when the gravity is acting upward in *x*-direction.
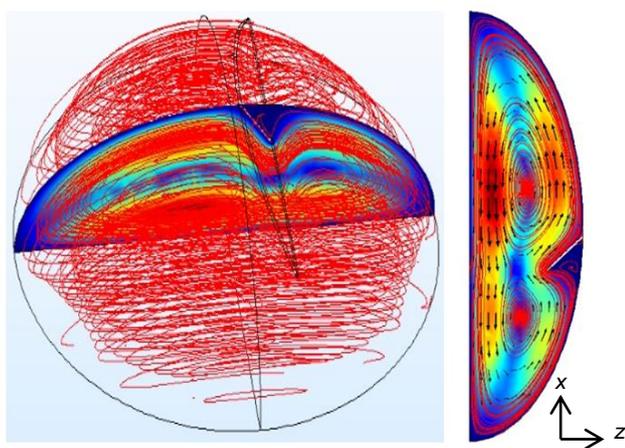
Fig. 7 Streamline in AC when the gravity is acting downward in *x*-direction.

The reattachment or redetachment of the DMD is contributed by the movement of the fluid in the subspace between the detached membrane and the stroma. As observed in Figure 7, the detached membrane is pushed away from its original position and toward to the stroma. This is because the pressure acting surrounding the DMD is larger than the pressure acting in the subspace between the detached membrane and the stroma, see Figure 8. This imply that the movement of the fluid surrounding the DMD is significant in process of reattachment or redetachment of the DMD. However,when the direction of the gravity is changed and in downward *x*-direction, the detached membrane is moved away from the stroma as shown in Figure 9. This is because the changing of the direction of the gravity has altered the AH flow in the AC, thus the pressure distribution in the

AC is also altered as illustrated in Figure 10. The pressure in the subspace between the detached membrane and the stroma is higher than the pressure acting surrounding the DMD. Moreover, the flow of the AH in the AC also causes the DMD become more severe (see Figure 10).
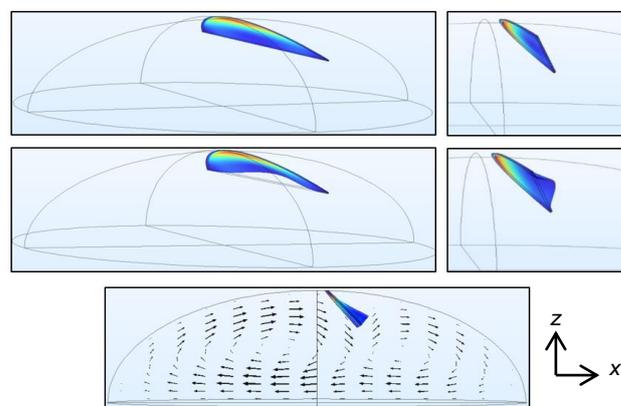
Fig. 8 Deformation of the DMD in AC when the gravity is acting upward in *x*-direction.
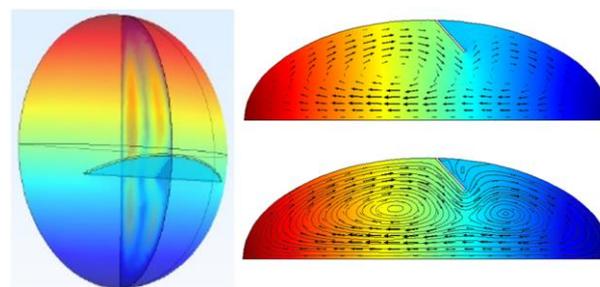
Fig. 9 .Pressure distribution in AC when the gravity is acting upward in *x*-direction.
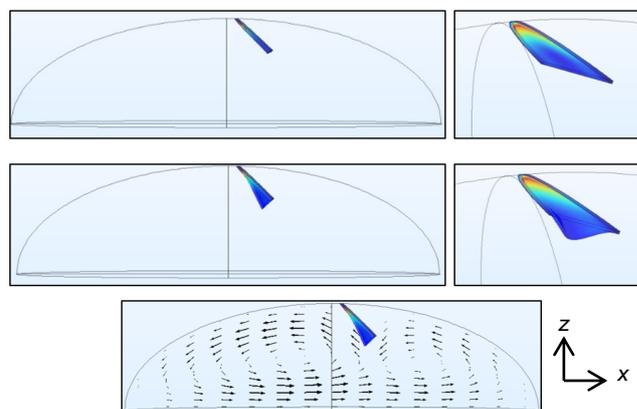
Fig. 10 Deformation of the DMD in AC when the gravity is acting downward in *x*-direction.
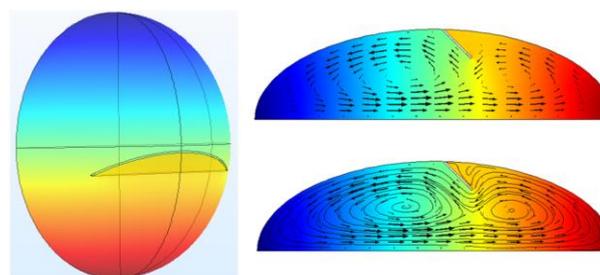
Fig. 11 Pressure distribution in AC when the gravity is acting downward in *x*-direction.

Therefore, we can conclude that the reattachment or redetachemnt of the DMD is depended on the AH flow, which driven by the temperature gradient, in the AC. We speculate that the reattachment of the DMD phenomena may only happen under certain conditions.

CONCLUSIONS

The behavior of the AH convection flow in a three dimensional AC in the presence of DMD have been numerically studied. The velocity streamline, pressure distribution and deformation of the DMD due to the fluid flow were obtained and illustrated graphically. Some interesting finding of the study can be summarized as follows:

1. The reattachment or redetachement of the DMD is depended on the AH flow which is driven by the temperature gradient between the iris/pupil and the cornea, in the AC .
2. The direction of the gravity has a significant effect to the AH flow in the AC. Therefore, the effect of the gravity need to be take into account in order to induce reattachment of the DMD.
3. The method proposed in present communication is appropriate and efficient in simulating the fluid flow in human eye as well as when different conditions are considered.

In our observation, to fully realize the behavior of the AC flow under the effect of the DMD, more research have to be done. For example, the fluid flow in the AC may be changed due to the blinking of the eye. This is because the temperature gradient in the AC is altered when the eye is blinking. In the future research, the blinking process of the human eye have to be considered in order to fully understand the dynamic of the AH flow in the AC with DMD. Further, the mechanisms of the reattachment or redetachment of the DMD is not completely developed. Many factors are not considered in present study such as the surface tension of the detached membrane, the rotation effect of the eye and so on.

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