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RESEARCH ARTICLE

# Mathematical modeling for controlled released fertilizer

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**Abstract**

Controlled release fertilizers (CRFs) are essential for sustainable agriculture system. CRFs are designed to maintain the constant optimum release rate of nutrients from the coated granule. This increase the plant uptake of nutrients hence reduces the soil pollution and decreases the crop expenditure. In the literature, the maximum studies have been done by considering the molecular diffusion as the only phenomenon responsible for nutrient release from CRFs. The molecular diffusion model is solved mostly by using the variable separable methods and Laplace transform as well as finite difference methods by different researchers. The release of NPK (nutrient) depends on both molecular diffusions which are expressed by Fick's second law of diffusion and ionic diffusion, due to the electrolytic behavior of NPK in the soil. In this work, an analytical solution is presented. The obtained solution helps to find the effect of granule coating thickness, nutrient release rate, pH of the soil and temperature of the soil on the nutrient release profiles.

***Keywords***: Adomian decomposition method; nutrient release; NPK; ionic diffusion; molecular diffusion

## INTRODUCTION

The fertilizers enhance the production efficiency of the crops and maintain good health of the plant [1]. Uncoated fertilizers are applied to soil due to an instantaneous release of nutrients up to 30-50 % of nutrient is utilized by plant the remaining unutilized nutrients mixed with water and soil causes environmental degradation [2]. The fertilizers are coated with polymers and biopolymers to maintain the optimal release of nutrients and helps the plants to uptake more nutrients by the crop and plants [3]. The polymers and biopolymers are in divided into two categories hydrophobic and hydrophilic and release of nutrient takes place due to diffusion and degradation of biopolymer coating of CRF. The release from coated CRF granule takes place mainly due to diffusion (up to 80%) of the nutrient. Hence it is important to consider the phenomenon of diffusion for modelling the release of nutrient from CRFs.

The modelling of the CRFs in the literature deal with the diffusion phenomenon. For example, Al-Zahrani [4] developed an analytical solution using Laplace transformation for the diffusion model to predict the concentration of nutrient inside the granule. Later, Shaviv et al [5] developed a diffusion based model by considering the nutrient release as non-fickian in nature. The model successfully predicted the nutrient release from the granule, the release profiles has been divided into three stages, lag period, constant release, and decay release. Du et al [6] considered that nutrient from the granule follows the Fick's second law diffusion and developed an analytical solution using the variable separable method. The concept ionic diffusion of nutrient release from CRF was first considered by Basu et al [7]. The Ionic diffusion model was solved using a two-level finite explicit method.

The literature studies mostly based on the assumption that the diffusion and nutrient release is non-electrolytic or molecular it is only applicable for urea as a nutrient inside the coated granule due to its non-electrolytic behavior [8]. In case of potassium sulfate, and diammonium phosphate as the nutrient ionic diffusion plays a significant role as both the nutrients are electrolyte in nature. Potassium sulfate is a strong electrolyte, diammonium phosphate is a feeble electrolyte [8]. If the nutrient is electrolytic in nature, there will be significant interactions between the nutrients inside the coated granule. These interactions influence the release of nutrients from the granule. Sometimes, these impacts deliver bizarre midpoints of dissemination coefficients of various solutes; in others, they propose a solid reliance of dispersion on fixation; in still others, they result in dispersion that is much slower than anticipated. We consider combined NPK as the nutrient (potassium sulfate, urea, diammonium phosphate) stuffed in spherically shaped biopolymer coated CRF granule. After immersion, solid electrolytes totally ionize, though powerless electrolytes create an arrangement of particles and atoms in remaining equilibrium with each other.

The release of NPK nutrient from CRF granule is considered due to surface erosion. The diffusion coefficient is dependent of the pH of the soil, temperature of the soil, soil moisture. Along with diffusion coefficient the effect of the radii of the granule is considered on the nutrient release phenomenon.

## MATHEMATICAL MODEL

Let us consider the spherical shaped granule with center '*o*' and radii of the granule a, the coating thickness of the granule is '*l*'. By using the Fick's law of diffusion for granule the diffusion equation is given by Equation (1) [8].

$$\frac{\partial C}{\partial t}=\frac{\partial}{\partial x}\left(D_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_z\frac{\partial C}{\partial z}\right) \tag{1}$$

where *c* is the concentration of nutrient inside the granule, *t* is the current time and *x*, *y*, and *z* are the axis with respect to the coordinates and $D_x$, $D_y$, and $D_z$ are diffusion coefficient respectively.

For spherical shaped granule, the Fick's law of diffusion in one dimension is given by Equation (2).

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r}\right) \tag{2}$$

where $r$ is the radial distance $D$ is the diffusion coefficent. Equation (2) can be solved using the initial and boundary conditions given from Equations (3) to (5).

The initial condition at time *t=0* is given below:

$$C(r,0) = C_0 \tag{3}$$

The boundary conditions at *r=0* and *r=l* is given below:

$$C(0,t) = C_0 \tag{4}$$

$$C(l,t) = q \tag{5}$$

The ionic diffusion phenomenon represented and derived by Basu et al [8] is given below:

$$\frac{\partial C_s}{\partial t} = D_s\left(\frac{\partial^2 C_s}{\partial r^2} + \frac{2}{r}\frac{\partial C_s}{\partial r}\right) \tag{6}$$

where $C_s$ is the concentration of strong electrolyte. $D_s$ is the diffusion coefficient of strong electrolyte which consist of cations and anions which is given below.

$$D_s = \frac{2}{\dfrac{1}{D_c} + \dfrac{1}{D_a}} \tag{7}$$

To study the effect of pH on the nutrient release profile of the CRF the diffusion coefficient of the cations and anions is considered to be dependent of the pH. Depending on the pH diffusion of water can be alkaline, neutral and acidic [10].

$$D_c = D_c + \alpha(pH - 1.0)(6.5 - pH)/(2.75)^2 \tag{8}$$

$$D_a = D_a + \beta(pH - 7.5)(14 - pH)/(3.25)^2 \tag{9}$$

where $D_c$ and $D_a$ are the diffusion coefficent of cations and anions respectively, $\alpha$ and $\beta$ are two constant parameters.

The effect of temperature is given in the Equation (10) is from Lu et al [11], from Equation (10) it infer that diffusion coefficient is dependent on temperature.

$$D = \lambda_1 T^{1.5} + \lambda_2 \tag{10}$$

where $T$ is temperature and $\lambda_1$ and $\lambda_2$ are the constants which depends on the chemical structure of the soil and nutrients inside the granule. Equation (6) is solved with the help of modified Adomian decomposition method [12], [13]. The results obtained from model is presented in the next section.

## RESULTS AND DISCUSSION

The effect of the coating thickness of the granule on the nutrient release profile is given in Fig. 1. It shows that as the coating thickness increase the nutrient release rate decreases and take longer time for 100% release. This happens due to obstacle gave by the coating material and reducing the diffusion of water and nutrient. The effect of pH on the diffusion coefficient of cations and anions is shown in Fig. 2 and Fig. 3 respectively. Fig.2 shows that effect of pH is even and as pH increases the diffusion coefficient of cations increases and increase in the nutrient release rate, in the case of anions the pattern was not even, as the pH increases the diffusion coefficient of anions increases until pH reaches 10 after that the diffusion coefficient reduces.

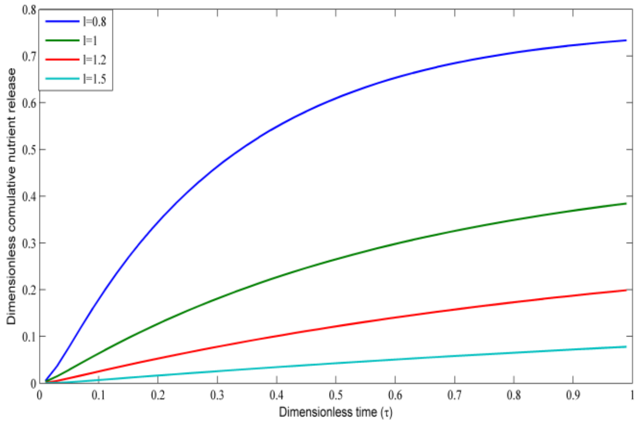

**Fig. 1** Nutrient release v/s time for different coating thickness of the granule.

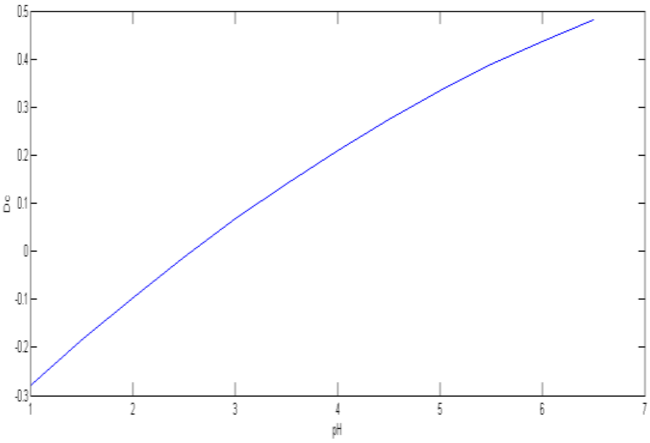

**Fig. 2** Effect of pH on cations ($D_c$).

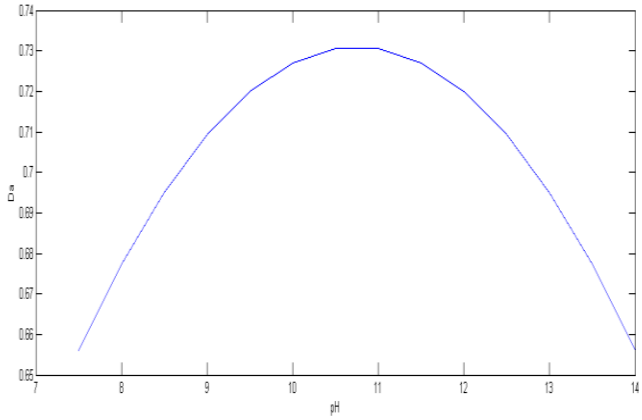

**Fig. 3** Effect of pH on anions ($D_a$)

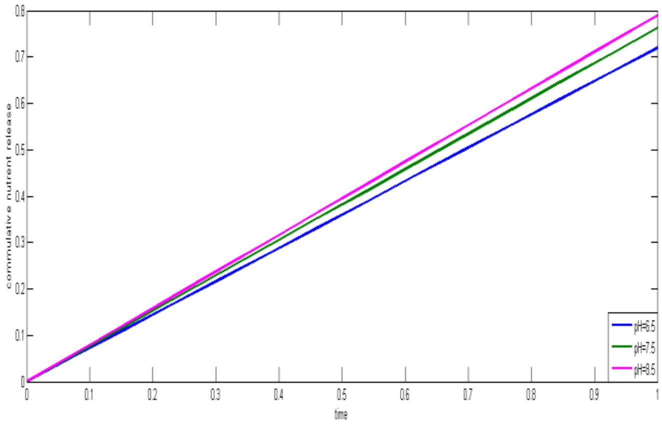

**Fig. 4** Effect of pH on nutrient release profile with time.

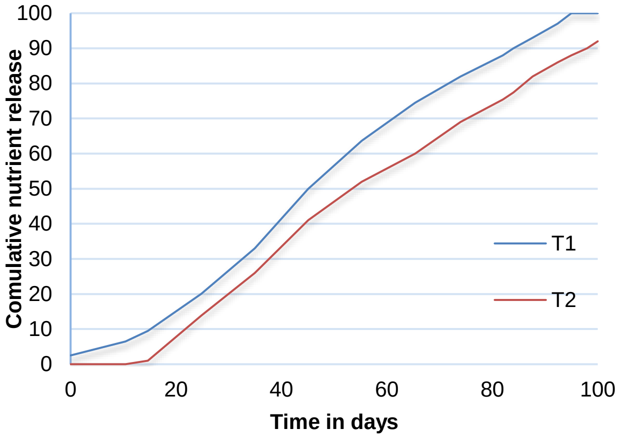

**Fig. 5** Effect of temperature on nutrient release from CRF.

As the pH increases, the nutrient release becomes faster which is shown in Fig. 4. The range of pH selected in this case is in between 6.5 to 8.5 in this range diffusion coefficient of both anions and cations increases as shown in Fig. 2 and Fig. 3 respectively, and hence increase the nutrient release rate. The effect of temperature on the nutrient release profile is shown in Fig.5. From Fig. 5 it can infer that as the temperature increases the nutrient release rate will be higher due to diffusion coefficient increases in an increase in the temperature of the surround. The diffusion coefficient is directly related to the nutrient release. Above obtained results helps in the better design and development of CRF as it gives an in-depth understanding of the parameters which affect the nutrient release profiles of the CRF.

## CONCLUSION

The ionic diffusion of nutrient from biopolymer controlled release ferilizer is considered in the form of a Fick's law of diffusion and the solution of the model is obtained using modified adomian decomposition method. The analytical solutions help to determine the effect of coating thcikness, pH, and temperature of the soil on cumulative nutrient release. Understanding of this physical parameters helps in the better design and devolpment modern CRF techinques.

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