RESEARCH ARTICLE

Mixed convection boundary layer flow past a horizontal circular cylinder in viscoelastic nanofluid with constant wall temperature

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Abstract

The steady of two-dimensional convection boundary layer flow of viscoelastic nanofluid over a circular cylinder with constant wall temperature is investigated in this paper. Carboxymethyl cellulose solution-water (CMC-water) is chosen as the base fluid and copper as a nanoparticle with the Prandtl number Pr = 6.2. The governing boundary layer partial differential equations are transformed into dimensionless forms. Then the obtained equations are solved numerically by using the Keller-Box method. This paper focus on the effect of selected parameter on the flow and heat transfer characteristics and be presented in graphs. The results show that, the velocity profiles are increased while the temperature profiles are decreased by increasing the values of nanoparticles volume fraction and viscoelastic parameter, respectively. Also, the values of reduced skin friction are increased by increasing mixed convection parameter, but the values of heat transfer coefficient produce an opposite behavior with an increasing in mixed convection parameter.

Keywords: Viscoelastic, nanofluid, mixed convection, horizontal circular cylinder, constant wall temperature

INTRODUCTION

The concept of nanofluid was first manifested by series of research at Argonne National Laboratory and Choi [1] was the first to call the fluids with particles of nanometer dimension suspended in them as "Nano-fluids" which has gained popularity. Nanoparticles used in nanofluids can be classified by materials. The nanoparticles are consisting of nano sized metals, oxides and carbon nanotubes. Recently, study of nanofluids has become most popular among researchers' due to its various applications in many industries, engineering and medical sciences as well. Basically, this kind of fluid have extremely high thermal conductivities compared to the conventional liquids, then nanofluids have been proposed as a route for surpassing the performance of heat transfer liquids. Many researchers had involved in the study of boundary layer flow problem in nanofluids such as, Nield and Kuznetsov [2], Khan and Pop [3], Ahmad and Pop[4], Chamkha et al. [5] ,Qasim et al. [6], Zaib et al. [7] and Noghrehabadi et al. [8]. Recently, Sabir et al. [9] investigated mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink. In reality, most fluids are non-Newtonian. Non-Newtonian fluid means that their viscosity is dependent on shear rate (thickening) or the deformation history. Nowadays, the non-Newtonian nanofluids have received much considerable interest and concern by the researchers' due to the potential of nanofluids applications in many types of industry. Rawi et al. [10], Mabood et al. [11], Sarkar et al. [12], and Bouchoucha and Bessaih [13] have been considered some interesting studies involving the non-Newtonian nanofluids with various conditions and geometries have been considered. Motivated by the above-mentioned studies, this paper considers the steady of two-dimensional mixed convection boundary layer flow past a horizontal circular cylinder in viscoelastic nanofluid with constant wall temperature.The Tiwari and Das model

[14] was used in this study. To the best of our knowledge, this problem has never been considered before and the results reported here are new. The influence of relevant parameters on the dimensionless fluid velocity, the temperature, the nanoparticle volume fraction, the rate of heat transfer, and the rate of skin friction coefficients are investigated and shown graphically and discussed.
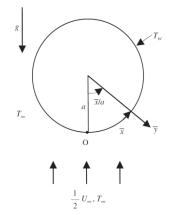
MATHEMATICAL FORMULATION

Fig.1 Physical model and coordinate system.

In this paper, the steady mixed convection boundary layer flow past an isothermal horizontal circular cylinder of radius a placed in a viscoelastic nanofluid is studied. Fig.1 illustrates the geometry of the problem and the corresponding coordinate system. It is assumed that

the constant temperature of the surface of the cylinder is T_w, and that of the ambient fluid is T_∞, where $T_w > T_\infty$ corresponds to a heated cylinder (assisting flow) and $T_w < T_\infty$ corresponds to a cooled cylinder (opposing flow), respectively. It is also assumed that the viscous dissipation is neglected. Further, following Merkin [15], we assume that the velocity of the freestream is $(1/2)U_\infty$. Under these assumptions along with the Boussinesq approximation, the boundary layer equations can be written as follows, Merkin [15], Nazar [16];

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \rho_{nf}\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}} + \mu_{nf}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$
$$+ k_0\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right)\right] + g(\rho\beta)_{nf}(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$(\rho C_p)_{nf}\left[\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right] = k_{nf}\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

subjected to the boundary conditions:

$$\bar{u} = 0, \ \bar{v} = 0, \ T = T_w \qquad \text{at} \qquad \bar{y} = 0, \ \bar{x} \ge 0,$$

$$\bar{u} = \bar{u}_e(\bar{x}), \ \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \ T = T_\infty \qquad \text{as} \qquad \bar{y} \to \infty, \ \bar{x} \ge 0, \tag{4}$$

where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates along the surface of the cylinder. The value is starting from the lower stagnation point of the cylinder. While $\bar{y}$ is the coordinate measured normal to the surface of the cylinder, $\bar{u}$ and $\bar{v}$ are the velocity components, $\bar{u}_e(\bar{x})$ is the velocity outside the boundary layer, T is the temperature of selected fluid, $k_0(\ge 0)$ is the constant of the viscoelastic material with $k_0 = 0$, ρ_{nf} and μ_{nf} are the density and dynamic viscosity of nanofluid, $(\beta)_{nf}$ is the thermal expansion of nanofluid, k_{nf} is the effective thermal conductivity of the nanofluid and $(\rho C_p)_{nf}$ is the heat capacitance of nanofluid. These nanofluid constants are defined by,

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s,$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s,$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \ \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,$$

$$k_{nf} = k_f\frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}. \tag{5}$$

where ϕ is the nanoparticle volume fraction of the nanofluid. The thermophysical properties of nanoparticles and base fluid is given in Table 1 [17].

Table 1 Thermophysical properties of nanoparticles and base fluid.

Physical Properties	CMC-water	Cu
$\rho(\text{kg }m^{-3})$	997.1	8933
$C_p(\text{J kg}^{-1}\text{K}^{-1})$	4179	385
$k(\text{Wm}^{-1}\text{K}^{-1})$	0.613	401
$\beta \times 10^5(\text{K}^{-1})$	21	1.67

By introducing the following non-dimensional variables;

$$x = \bar{x}/a, \quad y = \text{Re}^{1/2}(\bar{y}/a), \quad u = \bar{u}/U_\infty,$$

$$v = \text{Re}^{1/2}(\bar{v}/U_\infty), \ u_e(x) = \bar{u}_e(\bar{x})/U_\infty, \ \theta = (T - T_\infty)/(T_w - T_\infty) \tag{6}$$

where $\text{Re} = U_\infty a/v$ is the Reynolds number. By substitution equation (6) into equations (1)-(3), the dimensionless equations becomes,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{7}$$

$$\left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right]\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right]u_e\frac{\partial u_e}{\partial x} + \frac{1}{(1+\phi)^{2.5}}\frac{\partial^2 u}{\partial y^2}$$
$$+ K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right] + \left[(1-\phi) + \phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\sin(x), \tag{8}$$

$$\left[(1-\phi) + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right] = \frac{k_{nf}}{k_f}\frac{1}{\text{Pr}}\frac{\partial^2 \theta}{\partial y^2}, \tag{9}$$

with the boundary conditions,

$$u = 0, \quad v = 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \ x \ge 0,$$

$$u = u_e(x), \frac{\partial u}{\partial y} = 0, \ \theta = 0 \quad \text{as} \quad y \to \infty, \ x \ge 0, \tag{10}$$

where Pr is the Prandtl number, K known as dimensionless viscoelastic parameter and is known as constant mixed convection parameter, that are defined by,

$$K = \frac{k_0 U_\infty}{a\rho v}, \qquad \lambda = \frac{Gr}{\text{Re}^2}, \tag{11}$$

where $Gr = g\beta(T_w - T_\infty)a^3/v^2$ is also known as a Grashof number. $\lambda > 0$ is correlate to an auxiliary flow while $\lambda < 0$ is correlate to the opposing flow. But $\lambda = 0$ is correlate to the forced convection flow, respectively. In this problem, if $K = 0$ it is referring to the case of viscous (Newtonian) fluids.

Further, we introduce now the stream function defined as usual as;

$$\psi = xF(x,y), \qquad \theta = \theta(x,y), \tag{12}$$

where ψ is the stream function that define as,

$$u = \frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x}, \tag{13}$$

Using (12) and (13) into equations (8) and (9) to obtain,

$$\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[\left(\frac{\partial F}{\partial y}\right)^2+x\frac{\partial F}{\partial y}\left(\frac{\partial^2 F}{\partial x\partial y}\right)-x\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial y^2}-F\frac{\partial^2 F}{\partial y^2}\right]$$

$$=\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\frac{\sin x\cos x}{x}+\frac{1}{(1+\phi)^{2.5}}\frac{\partial^3 F}{\partial y^3}$$

$$+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\frac{\sin x}{x}$$

$$+K\left[\begin{array}{l}2\dfrac{\partial F}{\partial y}\dfrac{\partial^3 F}{\partial y^3}-F\dfrac{\partial^4 F}{\partial y^4}-\left(\dfrac{\partial^2 F}{\partial y^2}\right)^2\\[2mm]+x\left(\dfrac{\partial^2 F}{\partial x\partial y}\dfrac{\partial^3 F}{\partial y^3}-\dfrac{\partial F}{\partial x}\dfrac{\partial^4 F}{\partial y^4}+\dfrac{\partial F}{\partial y}\dfrac{\partial^4 F}{\partial x\partial y^3}-\dfrac{\partial^2 F}{\partial y^2}\dfrac{\partial^3 F}{\partial x\partial y^2}\right)\end{array}\right],\qquad(14)$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[x\frac{\partial F}{\partial y}\frac{\partial\theta}{\partial x}-x\frac{\partial F}{\partial x}\frac{\partial\theta}{\partial y}-F\frac{\partial\theta}{\partial y}\right]=\frac{k_{nf}}{k_f}\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2},$$

$$(15)$$

which are subject to the following boundary conditions;

$$F=0,\qquad\frac{\partial F}{\partial y}=0,\ \theta=1,\qquad\qquad\text{at }y=0,\ x\geq0,$$

$$\frac{\partial F}{\partial y}=\frac{\sin x}{x},\ \frac{\partial^2 F}{\partial y^2}=0,\ \theta=0,\qquad\text{as }y\to\infty,\ x\geq0.\qquad(16)$$

The physical quantities interest of this problem is the wall temperature distribution $\theta_w(x)$ and the skin friction coefficient C_f, that are defined as,

$$C_f=\text{Re}^{1/2}\frac{\tau_w}{\rho U_\infty^2},\qquad\qquad\theta_w=\text{Re}^{-1/2}\frac{aq_w}{k(T_w-T_\infty)},\qquad(17)$$

where τ_w is the shear stress or skin friction while q_w is the heat flux from the surface of cylinder [1], that are given by

$$\tau_w=\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0}+k_0\left(\bar{u}\frac{\partial^2\bar{u}}{\partial\bar{x}\partial\bar{y}}+\bar{v}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+2\frac{\partial\bar{u}}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0},$$

$$q_w=-k\left(\frac{\partial T}{\partial\bar{y}}\right)_{y=0}.\qquad(18)$$

and it is known that μ is for dynamic viscosity while k is for thermal conductivity. By substitution of (6), (17) and (18), obtained

$$C_f=x\left(\frac{\partial^2 F}{\partial y^2}\right)_{y=0},\qquad\qquad\theta(x,0)=\theta_w(x).\qquad(19)$$

When $x\approx0$, in the case of lower stagnation point, introduced the similarity variable $xf(\eta)$, $\theta=\theta(\eta)$ and η as per below. Thus, it can be shown that equations (7)-(9) diminish to the ordinary differential equations as:

$$\frac{1}{(1+\phi)^{2.5}}f'''-\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[f'^2-ff''\right]$$

$$+K\left(2ff'''-ff^{iv}-f'^2\right)+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta=0,\qquad(20)$$

$$\frac{k_{nf}}{k_f}\frac{1}{\Pr}\theta'+\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]f\theta'=0,\qquad(21)$$

and the boundary condition (9) as

$$f(0)=0,\qquad f'(0)=0,\qquad\theta(0)=1,$$

$$f'(\infty)=1,\qquad f''(\infty)=0,\qquad\theta(\infty)=0,\qquad(22)$$

In this case, obtained
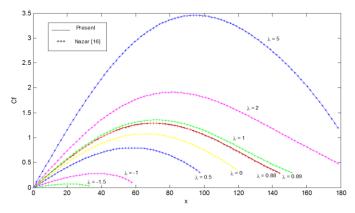
$$C_f=xf''(0),\qquad\theta_w(0)=\theta(0).\qquad(23)$$

The present study in equations (20)-(21) reduces to that of Nazar *et al.* [16] and Anwar *et al.* [18] for a viscoelastic fluid when $\phi=0$ (regular fluid).

RESULTS AND DISCUSSION

Both equations (14-15) with the boundary conditions (16), and the ordinary differential equations (20-21) with the boundary conditions (22) that were solved numerically using the Keller-box method. The effect of different values of the previous parameters on the velocity profiles, temperature profiles, reduced skin friction and heat transfer coefficients are reviewed in particularly.
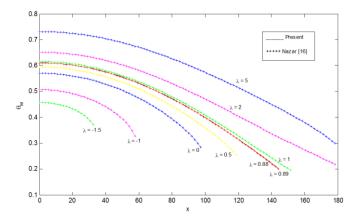
Figures 2 and 3 show the comparison of the present results for the skin friction coefficient C_f and the heat transfer from the cylinder θ_w with Nazar et al. [16] for a Newtonian fluid (K=0) with no nanofluid effects and Pr=1. It shows that the numerical solutions obtained by the present authors are in good agreement with Nazar et al. [16]. Thus, we confident that the present results are very accurate.

Figures 4 and 5 show the local skin friction C_f and local heat transfer θ_w for K=1, Pr=6.2 and various values of λ with the presence of nanoparticle volume fraction when $\phi=0.05$. From the graph, it shows that by the presence of nanoparticle volume fraction ϕ, the value of local skin friction is increased by increasing mixed convection parameter. The results show the same behavior for heat transfer coefficient where the value of θ_w is increased by increasing the values of λ.
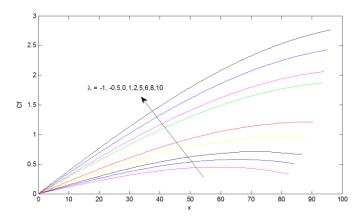

Fig. 2 Comparison of the local skin friction C_f for K=0 (Newtonian fluid), Pr=1, $\phi=0$ and various of λ
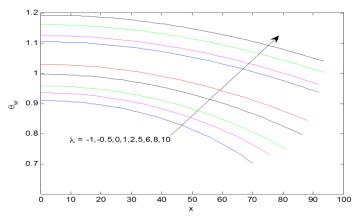
Fig.3 Comparison of the local heat transfer θ_w for K=0 (Newtonian fluid), Pr=1, $\phi = 0$ and various of λ
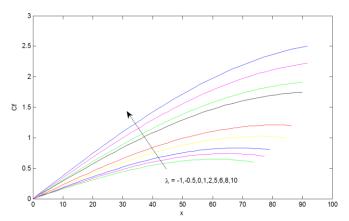

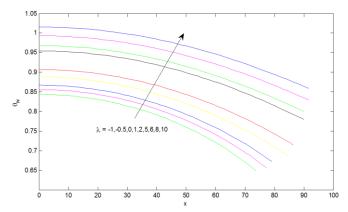**Fig.4** Local skin friction C_f for K=1, Pr=6.2, $\phi = 0.05$ and various of λ

Fig.5 Local heat transfer θ_w for K=1, Pr=6.2, $\phi = 0.05$ and various of λ

Fig.6 Local skin friction C_f for K=1, Pr=6.2, $\phi = 0.2$ and various of λ

Fig.7 Local heat transfer θ_w for K=1, Pr=6.2, $\phi = 0.2$ and various of λ

Fig.8 Velocity profiles for variety values of nanoparticle volume fraction ϕ for K=1, Pr=6.2, $\lambda = -1.5$ (opposing flow)

Fig.9 Temperature profiles for variety values of nanoparticle volume fraction ϕ for K=1, Pr=6.2, $\lambda = -1.5$ (opposing flow)
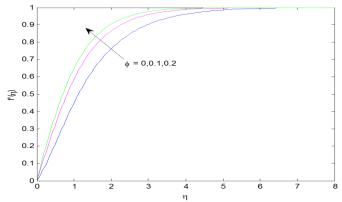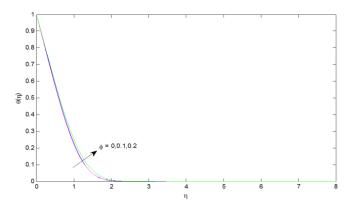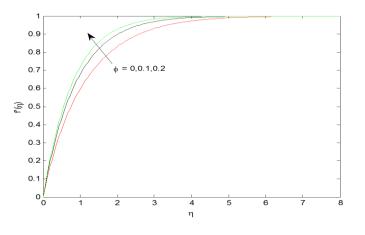
Further, Figures 6 and 7 show the local skin friction local skin friction C_f and local heat transfer θ_w with the same value of parameter but different value of nanoparticle volume fraction, which is $\phi = 0.2$. The same trend behavior is obverse as in Figures 4 and 5. Finally, Figures 8 to 11 illustrate the velocity and temperature profiles of the cylinder against *y* for the values of mixed convection parameter $\lambda = -1.5$ (opposing flow) and $\lambda = 1$ (assisting flow) when $K = 1$, $Pr = 6.2$ but with different values of nanoparticle volume fraction ϕ, which are varies from $0 \leq \phi \leq 0.2$. These figures show how the nanoparticle volume fraction ϕ affects the fluid velocity and temperature profiles. Thus, from the graph, both velocity and temperature profiles are increased by increasing the values of nanoparticles volume fraction ϕ and mixed convection parameter λ.
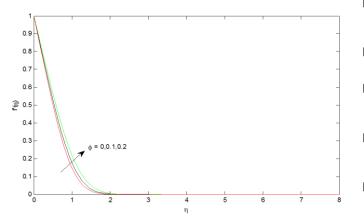
Fig.10 Velocity profiles for variety values of nanoparticle volume fraction ϕ for K=1, Pr=6.2, $\lambda=1$ (assisting flow)

Fig.11 Temperature profiles for variety values of nanoparticle volume fraction ϕ for K=1, Pr=6.2, $\lambda=1$ (assisting flow)

CONCLUSION

Steady of mixed convection boundary layer flow of viscoelastic nanofluids past a horizontal circular cylinder with a case of constant wall temperature problem has been solved numerically using Keller-box method. The numerical results for the skin friction, heat transfer coefficient, velocity and temperature profiles are presented for variety values of nanoparticles volume fraction, mixed convection parameter and viscoelastic parameter. The effect of velocity and temperature profiles due to the influence of nanoparticles volume fraction and mixed convection parameters are also analysed. The comparison of the results show that the present results agree very well with the previous published results reported by Nazar *et al.* [16] and Anwar *et al.* [18].

ACKNOWLEDGEMENT

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