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RESEARCH ARTICLE

New fuzzy generalized bi $\Gamma-$ideals of the type $(\in,\in\vee q_k)$ in ordered $\Gamma-$semigroups

Ibrahim Gambo [a], Nor Haniza Sarmin [a,*], Hidayat Ullah Khan [b], Faiz Muhammad Khan [c,d]

[a]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor, Malaysia.
[b]Department of Mathematics, University of Malakand, Khyber Pukhtunkhwa, 18800 Pakistan.
[c]Department of Applied Mathematics, School of Applied and Natural Sciences, Northwestern Polytechnical University Xi'an 710072, P.R. China.
[d]Department of Mathematics and Statistics, University of Swat, Khyber Pakhtunkhwa, 19130 Pakistan.

* Corresponding author: nhs@utm.my

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Abstract

The set defined on X represented by $A=\{(x,\lambda_A(x)), \text{ where } x\in X\}$. is called a fuzzy subset A of X. It is not always possible for membership functions of type $\lambda_A : X\to[0,1]$ to associate with each point x in a set X a real number in $[0,1]$ without the loss of some useful information. The importance of the ideas of "belongs to" $(\in)$ and "quasi coincident with" (q) relations between a fuzzy set and fuzzy point is evident from the research conducted during the past two decades. Ordered Γ-semigroup is a generalization of ordered semigroups and plays a vital role in the broad study of ordered semigroups. In this paper, we provide an extension of fuzzy generalized bi $\Gamma-$ideals and introduce $(\in,\in\vee q_k)-$fuzzy generalized bi $\Gamma-$ideals of ordered $\Gamma-$semigroup. The purpose of this paper is to link this new generalization with generalized bi $\Gamma-$ideals by using level subset and characteristic function.

Keywords: Generalized bi $\Gamma-$ideal, ordered $\Gamma-$semigroup, fuzzy Point; $(\in,\in\vee q_k)-$fuzzy generalized bi $\Gamma-$Ideal.

INTRODUCTION

Fuzzy algebraic structures of groups have begun in the spearheading paper of Rosenfeld [1] in 1971. He studied the concept of fuzzy subgroups and showed that numerous outcomes in groups can be extended and study in an elementary manner to develop the theory of fuzzy subgroups after a pioneered work on fuzzy set theory by Zadeh [2] in 1965. Thereafter, many researchers worked on the fuzzification of various algebraic structures. Sen [3] were the first to introduce the concept of a $\Gamma-$semigroup which is a generalization of both semigroup and ternary semigroup. Furthermore, Kwon and Lee [4], further studied po-$\Gamma-$semigroup and introduced the concept of weakly prime ideals and provided useful characterizations of weakly prime ideals. The concepts of fuzzy ideals, fuzzy bi-ideals and fuzzy quasi ideals in Γ-semigroups are discussed in [5, 6]. Furthermore, Khan *et al.* [7] introduced the concept of generalized bi $\Gamma-$ideals of type (λ,θ) in ordered semigroups. Furthermore, the fuzzification of $\Gamma-$structures by Dutta and Chanda can refer to [8, 9] they obtained a one to one correspondence between the set of all fuzzy prime ideals of the operator rings of the $\Gamma-$ring and the set of all fuzzy prime ideals of a $\Gamma-$ring. Jun and Lee [10] introduced the notion of fuzzy ideal in $\Gamma-$ring. The idea of fuzzy ideals of rings were introduced by Liu [11] and they also prove some fundamental properties of fuzzy ideals. Later Jun *et al.* [12] introduce the notion of fuzzy left (resp. right) ideals of $\Gamma-$near-rings, and studied their properties in that regards.

The importance of the ideas of "belongs to" $(\in)$ and "quasi coincident with" (q) relations between a fuzzy set and fuzzy point [13] is one of the evident from the research conducted during the past two decades. Jun [14], further generalized the concept of quasi coincident with relations between a fuzzy set and fuzzy point $(x_t q_k A)$ and defined $x_t q_k A$, if $\lambda_A(x)+t+k>1$, where $k\in[0,1)$. In this paper, we studied and provided the extension of the generalized form of fuzzy bi $\Gamma-$ideals in ordered $\Gamma-$semigroups and introduced the concept of $(\in,\in\vee q_k)-$fuzzy generalized bi $\Gamma-$ideals in ordered semigroups.

PRELIMINARIES

Some fundamental concepts and previous results are provided in this section that will be used throughout this paper and $A=\lambda_A$ is used for fuzzy set throughout this paper.

Given two nonempty sets G and Γ. Then the set G is called a $\Gamma-$semigroup if G satisfies the condition $(a\alpha b)\beta c = a\alpha(b\beta c)$ $\forall a,b,c\in G$ and $\alpha,\beta\in\Gamma$. Similarly, a nonempty subset S of a $\Gamma-$semigroup G is called a sub $\Gamma-$semigroup of G if $a\alpha b\in S$ $\forall a,b,c\in S$ and $\alpha\in\Gamma$. Given any nonempty subsets A and B of

G, $A\Gamma B=\{a\alpha b : a\in A,\ with\ b\in B\ and\ \alpha\in\Gamma\}$ [3, 15]. Since the invention of the definitions of $\Gamma-$semigroups then many researches are carried out in this direction of generalizations.

Example 2.1

Let $G=\{a,b,c\}$ and defined $\Gamma=\{\alpha\}$ with a mapping defined by $G\times\Gamma\times G\to G$ with an operation defined in the cayley table 1:

Table 1

α	a	b	c
a	a	a	a
b	a	b	a
c	a	a	c

Then G is a $\Gamma-$semigroup.

Definition 2.2 [7]

If G and Γ are non-empty sets, then a structure $(G,\Gamma,\le)$ is called an ordered Γ-semigroup if:

(b$_1$) $(a\alpha b)\beta c=a\alpha(b\beta c)$ for all $a,b,c\in G$ and $\alpha,\beta\in\Gamma$,

(b$_2$) $a\le b\to a\alpha x\le b\alpha x$ and $x\beta a\le x\beta b$ for all $a,b,x\in G$ and $\alpha,\beta\in\Gamma$.

Definition 2.3 [7]

A non-empty subset A of G is called a generalized bi Γ-ideal of G if the following conditions hold for all $a,b\in G$:

(b$_3$) $a\le b\in A\to a\in A$,

(b$_4$) $A\Gamma G\Gamma A\subseteq A$.

Definition 2.4 [7]

The set defined on X represented by $A=\{(x,\lambda_A(x)),\ \text{where } x\in X\}$ is called a fuzzy subset A of X.

Definition 2.5 [7]

Given a fuzzy subset A of G then A is called a fuzzy generalized bi Γ-ideal of G if the following conditions are satisfied, for all $x,y,z\in G$ and $\alpha,\beta\in\Gamma$:

(b$_5$) $x\le y\to\lambda_A(x)\ge\lambda_A(y)$,

(b$_6$) $\lambda_A(x\alpha y\beta z)\ge\min\{\lambda_A(x),\lambda_A(z)\}$.

Definition 2.6 [7]

Given A a fuzzy subset and let $t\in(0,1]$. Then the crisp set $U(A;t):=\{x\in G:\lambda_A(x)\ge t\}$ is called a level subset of A.

Let t be a fixed point of the interval $(0,1]$ and x be a fixed element of G. Then a fuzzy subset A of G is called a fuzzy point with support x and value t and is denoted by x_t if:

$$\lambda_A(y)=\begin{cases} t, & \text{if} \quad y=x \\ 0, & \text{if} \quad \text{otherwise.}\end{cases}$$

We say that a fuzzy point x_t belongs to a fuzzy subset of A if $\lambda_A(x)\ge t$ and is denoted by $x_t\in A$. On the other hand, if $k\in[0,1)$ and $\lambda_A(x)+t+k>1$, then x_t is quasi coincident with A and is denoted by $x_t q_k A$. If $x_t\in A$ or $x_t q_k A$, then we write $x_t\in\vee q_k A$ and if $x_t\in A$ and $x_t q_k A$, then we write $x_t\in\wedge q_k A$.

Let I be a non-empty subset of G, then the characteristic function χ_I of I is a fuzzy subset of G and is defined by:

$$\chi_I(x)=\begin{cases} 1, & \text{if} \quad x\in I \\ 0, & \text{if} \quad x\notin I.\end{cases}$$

MAIN RESULTS

In this part, our main result is presented, and we introduce an extension of fuzzy generalized bi $\Gamma-$ideals in ordered $\Gamma-$semigroup. Throughout this section, G will represent an ordered $\Gamma-$semigroup and $k\in[0,1)$.

Definition 3.1

Let A be a fuzzy subset of G. If A satisfies the following two conditions, then A is called $(\in,\in\vee q_k)-$fuzzy generalized bi Γ-ideal of G :

(c$_1$) $y_t\in A\to x_t\in A$ for all $x,y\in G$ such that $x\le y$ and $t\in(0,1]$,

(c$_2$) $x_{t_1}\in A, z_{t_2}\in A\to(x\alpha y\beta z)_{\min\{t_1,t_2\}}\in\vee q_k A$ for all $x,y\in G$, $\alpha,\beta\in\Gamma$ and $t_1,t_2\in(0,1]$.

The sufficient conditions for any generalized bi Γ-ideal of G of the type $(\in,\in\vee q_k)$ are provided in the theorem given below.

Theorem 3.2

A fuzzy subset A of G is called $(\in,\in\vee q_k)$-generalized bi $\Gamma-$ideal of G if and only if the following conditions hold for all $a,b,c\in G$ and $\alpha,\beta\in\Gamma$.

(1) $a\le b\to\lambda_A(a)\ge\min\left\{\lambda_A(b),\dfrac{1-k}{2}\right\}$,

(2) $\lambda_A(a\alpha b\beta c)\ge\min\left\{\lambda_A(a),\lambda_A(c),\dfrac{1-k}{2}\right\}$.

Proof: Let A be a $(\in,\in\vee q_k)-$generalized bi $\Gamma-$ideal of G and let there exist $a,b\in G$ such that $a\le b$ and $\lambda_A(a)<\min\left\{\lambda_A(b),\dfrac{1-k}{2}\right\}$.

Then $\lambda_A(a)<t$ and $\lambda_A(a)\le\min\left\{\lambda_A(b),\dfrac{1-k}{2}\right\}$ for some $t\in(0,\dfrac{1-k}{2}]$. It follows that $b_t\in A$ but $a_t\notin A$. And

$\lambda_A(a)+t<t+t\leq\frac{1-k}{2}+\frac{1-k}{2}=1-k$ that is $\lambda_A(a)+t+k<1$ and hence $a_t\overline{q}_k A$, a contradiction with (c_1).

Hence $\lambda_A(a)\geq\min\left\{\lambda_A(b),\frac{1-k}{2}\right\}$ for all $a,b\in G$ with $a\leq b$. For the second case, let $\lambda_A(a\alpha b\beta c)<\min\left\{\lambda_A(a),\lambda_A(c),\frac{1-k}{2}\right\}$ for some $a,b,c\in G$. Then there exist $t\in\left(0,\frac{1-k}{2}\right]$ such that $\lambda_A(a\alpha b\beta c)<t\leq\min\left\{\lambda_A(a),\lambda_A(c),\frac{1-k}{2}\right\}$. It Follows that $a_t\in A, c_t\in A$ but $(a\alpha b\beta c)_t\,\overline{\in\vee q_k}\,A$ and hence again a contradiction with (c_2). Thus $\lambda_A(a\alpha b\beta c)\geq\min\left\{\lambda_A(a),\lambda_A(c),\frac{1-k}{2}\right\}$ for all $a,b,c\in G$ and $\alpha,\beta\in\Gamma$.

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Conversely, consider (1) and (2) hold for a fuzzy subset A of G $a,b\in G$ such that $a\leq b$. If $b_t\in A$, then $a_t\in\vee q_k A$. Indeed: Since $b_t\in A$ so $\lambda_A(b)\geq t$ and by (1)

$$\begin{aligned}\lambda_A(a)&\geq\min\left\{\lambda_A(b),\frac{1-k}{2}\right\}\\&\geq\min\left\{t,\frac{1-k}{2}\right\}\\&=\begin{cases}t, & \text{if}\quad t\leq\frac{1-k}{2}\\ \frac{1-k}{2}, & \text{if}\quad t>\frac{1-k}{2}.\end{cases}\end{aligned}$$

In which it follows that $\lambda_A(a)\geq t,$ alternatively $\lambda_A(a)+t>\frac{1-k}{2}+\frac{1-k}{2}=1-k,$ i.e. $\lambda_A(a)+t+k>1.$ Hence (c_1) holds.

For (c_2) let us consider (2) holds and $a,b,c\in G$ such that $a_{t_1}\in A, c_{t_2}\in A$, then by (2)

$$\begin{aligned}\lambda_A(a\alpha b\beta c)&\geq\min\left\{\lambda_A(a),\lambda_A(c),\frac{1-k}{2}\right\}\\&\geq\min\left\{t_1,t_2\frac{1-k}{2}\right\}\\&=\begin{cases}\min\{t_1,t_2\}, & \text{if}\quad \min\{t_1,t_2\}\leq\frac{1-k}{2}\\ \frac{1-k}{2}, & \text{if}\quad \min\{t_1,t_2\}>\frac{1-k}{2}.\end{cases}\end{aligned}$$

It follows that

$(a\alpha b\beta c)_{\min\{t_1,t_2\}}\in A$ or $(a\alpha b\beta c)_{\min\{t_1,t_2\}}q_k A$ that is $(a\alpha b\beta c)_{\min\{t_1,t_2\}}\in\vee q_k A.$ Hence A is $(\in,\in\vee q_k)-$fuzzy generalized bi $\Gamma-$ideal of $G.$ ■

Example 3.3

Let $G=\{a,b,c,d\}$ with $\Gamma=\{\alpha\}$ and defined an ordered relation $"\leq"$ on G as given in the cayley table 2.

Table 2

α	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

$\leq:=\{(a,a),(b,b),(b,c),(c,c),(d,d),(a,b)\}.$

Then, the ordered set $(G,\Gamma,\leq)$ is an ordered $\Gamma-$semigroup. Likewise, the sets $\{a\}$, $\{a,b\}$, $\{a,c\}$ $\{a,d\}$, $\{a,d,c\}$, $\{a,c,d\}$ and $\{a,b,c,d\}$ are generalized $\Gamma-$ideals of $G.$

Define a fuzzy subset $\lambda:G\to[0,1]$ as:

$$\lambda(x)=\begin{cases}0.2, & if\ x=b,\\ 0.3, & if\ x=c,\\ 0.6, & if\ x=d,\\ 0.7, & if\ x=a,\end{cases}$$

and $$\lambda(x)=\begin{cases}G, & if\ 0<t\leq 0.2,\\ \{a,d\}, & if\ 0.3<t\leq 0.6,\\ \{a,c,d\}, & if\ 0.2<t\leq 0.3,\\ \varnothing, & if\ 0.7<t\leq 1.\end{cases}$$

Then λ is $(\in,\in\vee q_k)-$fuzzy generalized bi $\Gamma-$ideal of G for all $t\in\left(0,\frac{1-k}{2}\right]$ and $k=0.6.$

The link between the generalized bi $\Gamma-$ideal and the new introduced generalization of bi $\Gamma-$ideal is given in the following theorem.

Proposition 3.4

If A is a nonzero fuzzy generalized bi $\Gamma-$ideal of G of the form $(\in,\in\vee q_k),$ then the set $\lambda_0=\{\lambda(a)>0\mid a\in G\}$ is also a generalized bi of G ideal of $G.$

Proof: Suppose Let A is a generalized bi Γ-ideal of G of the form $(\in,\in\vee q_k)$ and let $a,b\in G$ with $a\leq b$ such $b\in\lambda_0.$ Then, $\lambda_A(b)>0$ from the hypothesis. Thus, $\lambda_A(a)\geq\min\left\{\lambda_A(b),\frac{1-k}{2}\right\}>0.$ Therefore, $\lambda_A(a)>0$ thus $a\in\lambda_0.$

Similarly, suppose $a,c \in \lambda_0$ with $\alpha,\beta \in \Gamma$, $\lambda_A(a) > 0$ and $\lambda_A(c) > 0$. Now, $\lambda_A(a\alpha b\beta c) \geq \min\left\{\lambda_A(a), \lambda_A(c), \frac{1-k}{2}\right\} > 0.$

Hence, $a\alpha b\beta c \in \lambda_0$ which shows that A is a generalized bi $\Gamma-$ideal of G. ∎

Theorem 3.5

Let $\varphi \neq I \subseteq G$ and χ_I be a characteristic function of I. Then the following two statements are equivalent:

(1) I is an generalized bi $\Gamma-$ideal of G,

(2) χ_I is a $(\in, \in \vee q_k)-$fuzzy generalized bi $\Gamma-$ideal of G.

Proof: $(1) \Rightarrow (2)$. Let $a,b \in G$ such that $a \leq b \in I$. Then we have $a \in I$ (by (b_3)) $\chi_I(a) = 1 \geq \frac{1-k}{2} = \min\left\{\chi_I(b), \frac{1-k}{2}\right\}$. Let $a,b,c \in G$ and $\alpha,\beta \in \Gamma$. If $a,c \in I$, then by (b_4) $a\alpha b\beta c \in I$. Therefore, $\chi_I(a\alpha b\beta c) = 1 > \frac{1-k}{2} = \min\{\chi_I(a), \chi_I(c), \frac{1-k}{2}\}$. On the other hand, if either $a \notin I$ or $c \notin I$, then we have the following two cases:

(i) If $a\alpha b\beta c \in I$, then $\chi_I(a\alpha b\beta c) = 1 > 0$
$\chi_I(a\alpha b\beta c) = \min\{\chi_I(a), \chi_I(c), \frac{1-k}{2}\},$

(ii) If $a\alpha b\beta c \notin I$, then $\chi_I(a\alpha b\beta c) = 0 = \min\{\chi_I(a), \chi_I(c), \frac{1-k}{2}\}.$

Hence, χ_I is a $(\in, \in \vee q_k)-$fuzzy generalized bi Γ-ideal of G.

$(2) \Rightarrow (1)$. Let $a,b \in G$ such that $a \leq b \in I$. Then $\chi_I(b) = 1$ and by Theorem 3.1 (1)

$$\chi_I(a) \geq \min\left\{\chi_I(b), \frac{1-k}{2}\right\},$$

$$= \min\left\{1, \frac{1-k}{2}\right\} = \frac{1-k}{2} \neq 0.$$

it follows that $a \in I$.

Let $a,b,c \in G$ and $\alpha,\beta \in \Gamma$. If $a,c \in I$, then $\chi_I(a) = 1 = \chi_I(c)$ and by Theorem 3.3 (2)

$$\chi_I(a\alpha b\beta c) \geq \min\left\{\chi_I(a), \chi_I(c), \frac{1-k}{2}\right\} = \min\left\{1, 1, \frac{1-k}{2}\right\} = \frac{1-k}{2} \neq 0,$$

this implies $a\alpha b\beta c \in I$. Hence, I is a generalized bi $\Gamma-$ideal of G.

The equivalent statement on any fuzzy subset in relation to generalized bi $\Gamma-$ideal and level subset are given in the following theorem.

Theorem 3.6

The following two statements are equivalent for any fuzzy subset A of G and for all $t \in (0, \frac{1-k}{2}]$:

(1) The non-empty level subset $U(A;t)$ is an generalized bi $\Gamma-$ideal of G,

(2) A is a $(\in, \in \vee q_k)$-fuzzy generalized bi $\Gamma-$ideal of G.

Proof: $(1) \Rightarrow (2)$. Let $U(A;t) \neq \varphi$ is a generalized bi Γ-ideal of G for all $t \in (0, \frac{1-k}{2}]$. Let $\lambda_A(a) < \min\left\{\lambda_A(b), \frac{1-k}{2}\right\}$ for some $a,b \in G$ with $a \leq b$. Then there exists $t \in (0, \frac{1-k}{2}]$ such that $\lambda_A(a) < t \leq \min\left\{\lambda_A(b), \frac{1-k}{2}\right\}$. It follows that $b \in U(A;t)$ and hence $a \in U(A;t)$ (by (b_3)), but $\lambda_A(a) < t$ implies that $a \notin U(A;t)$. This is a contradiction and hence $\lambda_A(x) \geq \min\left\{\lambda_A(y), \frac{1-k}{2}\right\}$ for all $x,y \in G$ with $x \leq y$.

Next, let $a,b,c \in G$ and $\alpha,\beta \in \Gamma$ such that $\lambda_A(a\alpha b\beta c) < \min\left\{\lambda_A(a), \lambda_A(c), \frac{1-k}{2}\right\}$.

Hence, $\lambda_A(a\alpha b\beta c) < t \leq \min\left\{\lambda_A(a), \lambda_A(c), \frac{1-k}{2}\right\}$ for some $t \in (0, \frac{1-k}{2}]$. So we have $a \in U(A;t)$, $c \in U(A;t)$ and $a\alpha b\beta c \notin U(A;t)$. Again a contradiction and hence we have $\lambda_A(x\alpha y\beta z) \geq \min\left\{\lambda_A(x), \lambda_A(z), \frac{1-k}{2}\right\}$ for all $x,y,z \in G$. By Theorem 3.2 and in light of above discussion A is a $(\in, \in \vee q_k)$-fuzzy generalized bi Γ-ideal of G.

$(2) \Rightarrow (1)$. Let $a,b \in G$ such that $a \leq b \in U(A;t)$. Then $\lambda_A(b) \geq t$ and by Theorem 3.2 (1)

$$\lambda_A(a) \geq \min\left\{\lambda_A(b), \frac{1-k}{2}\right\} \geq \min\left\{t, \frac{1-k}{2}\right\} = t,$$

it follows that $a \in U(A;t)$.

Let $a,b,c \in G$ and $\alpha,\beta \in \Gamma$ such that $a,c \in U(A;t)$. Then $\lambda_A(a) \geq t$, , $\lambda_A(c) \geq t$ and by Theorem 3.1 (2) we have

$$\lambda_A(a\alpha b\beta c) \geq \min\left\{\lambda_A(a), \lambda_A(c), \frac{1-k}{2}\right\} \geq \min\left\{t, t, \frac{1-k}{2}\right\} = t,$$

this implies $a\alpha b\beta c \in U(A;t)$. Hence, $U(A;t)$ is a generalized bi $\Gamma-$ideal of G. ∎

CONCLUSION

The algebraic structure of ordered $\Gamma-$semigroup is considered important in several areas of mathematics such as, robotics, coding and language theory, combinatorics, automata theory and mathematical analysis. Being an ordered $\Gamma-$semigroup a generalization of both ordered semigroup and ordered ternary semigroup, this study provides an extension of fuzzy generalized bi $\Gamma-$ideals and introduces a new generalization of fuzzy generalized bi $\Gamma-$ideals in the structure of ordered $\Gamma-$semigroup. Further, the relation between this new generalization with generalized bi $\Gamma-$ideals using level subset is investigated.

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