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RESEARCH ARTICLE

The unsteady mixed convection flow of rotating second grade fluid in porous medium with ramped wall temperature

Ahmad Qushairi Mohamad [a], Ilyas Khan [b], Nor Athirah Mohd Zin [a], Zulkhibri Ismail [c], Sharidan Shafie [a,*]

[a] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, Skudai 81310, Johor, Malaysia
[b] College of Engineering Majmaah University, P.O. Box 66, Majmaah 11952, Saudi Arabia
[c] Department Faculty of Industrial Sciences & Technology, Universiti Malaysia Pahang, 26300 Kuantan, Pahang, Malaysia

* Corresponding author: sharidan@utm.my

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Abstract

The effects of ramped wall temperature, rotation and porosity on mixed convection flow of incompressible second grade fluid are studied. The momentum equation is modelled in a problem of rotating fluid with constant angular velocity subjected to initial and oscillating boundary conditions. The energy equation is also introduced. Some suitable non-dimensional variables are used to write equations into non-dimensional form. Laplace transform method is used to obtain the analytical solutions of these equations for variables velocity and temperature. The effect of second grade fluid parameter, rotation parameter, porosity parameter, Prandtl number and Grashof number on the fluid flows are presented graphically and analysed. It is found that, for larger values of porosity parameter, the fluid velocity will increase for both primary and secondary velocities. The results also show that, velocity for ramped wall temperature is lower compared to isothermal temperature. The exact solutions obtained in present communication can be applied for verifing of the results obtained through numerical schemes.

***Keywords*:** Mixed convection, second grade fluid, rotating, ramped wall, porous medium

INTRODUCTION

Second grade fluid in a porous medium with the effect of ramped wall temperature has gained extensively attention of researchers in the literature due to its engineering and industrial applications in food and polymer production, fiber and granular insulation and geothermal systems (Samiulhaq *et al*. (2014a, 2014b), Ismail *et al*. (2015a, 2015b)). Further, researchers focused on effect of rotation on the second grade fluid in presence of ramped wall temperature. For example, Mohamad et al. (2014) has investigated the behaviour of velocity of fluid flow that affected by rotation effect and temperature changing. Then, Ismail et al. (2015c) extended the problem of Mohamad et al. (2014) by considering the effects of porosity, magnetic and concentration. They found that, the velocity of fluid flow for ramped wall temperature is always lower compared to isothermal temperature. Recently, Mohamad et al. (2016) studied the heat transfer of second grade fluid through an oscillating plate in a rotating medium. Their results shown that, the effect of oscillation for sine and cosine parts in fluid flow gave a same behaviour of embedded parameters except for phase angle. In all the above studies, Laplace transform technique has been used to obtain the solutions of the problems. Previous study shows that the flow of second grade fluid play an important role in fluid flow problems compared to the Newtonian fluid. Mostly, the theoretically study of unsteady free convection flow of second grade fluid have been conducted in vertical plates.

However, only a few researchers considered the problem of convective flow involve with rotating plate. Even, the expressions of the exact solutions obtained in the previous study for the problem of the flow in infinite rotating vertical plates are conducted without oscillating plate (forced convection) (Hayat et al. (2004), Hayat et al. (2008), Tiwari et al. (2009), Hayat et al. (2010), Salah et al. (2011), Lahurikar (2010), Vijayalakshmi (2010)). Therefore, study to explore the mathematical model for the problem of unsteady free convection secong grade fluid flow through a vertical oscillating plate is significant as well as the effect of rotation is taken into account. This paper emphasized on this matter. The derivation of the mathematical model also included with porosity and ramped wall temperature effects. Motivated by this work, the aim of the present study is to provide the exact solutions for rotating second grade fluid on mixed convection flow in porous medium with the effect of ramped wall temperature.

MATHEMATICAL FORMULATION AND SOLUTION

Mathematical Formulation

In this study, x-axis is taken along the plate in the upward direction and z-axis is taken normal to plate. Initially, both the fluid and plate are at rest to a constant temperature T_∞. At $t = 0^+$, the plate oscillates and fluid starts to rotates with constant angular velocity Ω. At the same time, the temperature of plate is raised or lowered at $T_\infty + (T_w - T_\infty)t/t_0$ when $t \leq 0$ and thereafter, for $t > t_0$ is maintained at the constant temperature T_w. Therefore, under the usual assumption of Boussinesq approximation, the governing equations of momentum and energy are given as

$$\frac{\partial F}{\partial t} + 2i\Omega F = \upsilon \frac{\partial^2 F}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2 \partial t} + gB\left(T - T_\infty\right), \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2}, \tag{2}$$

subjected to initial and boundary conditions (Chandran et al. (2005), Rajesh (2010), Deka & Das (2011), Seth et al. (2011))

$$\begin{gathered} F(0,t) = UH(t)\cos(\omega_1 t), \\ F(z,t) = 0 \text{ as } z \to \infty;\ t > 0, \\ F(z,0) = 0\ ;\ z > 0, \end{gathered} \tag{3}$$

$$\begin{gathered} T(0,t) = T_\infty + \left(T_w - T_\infty\right)\frac{t}{t_0};\ 0 < t \leq t_0, \\ T(0,t) = T_w : t > t_0, \\ T(z,t) \to T_\infty \text{ as } z \to \infty;\ t \geq 0. \\ T(z,0) = T_\infty\ ;\ z \geq 0, \end{gathered} \tag{4}$$

where $F = u + iv$ is the complex velocity where, u is a primary velocity, v is a secondary velocity, ρ designates the density of the fluid, υ is the kinematic viscosity, α_1 is the second grade parameter, $\phi(0 < \phi < 1)$ is the porosity and $k_1 > 0$ is the permeability of the porous medium, μ is the dynamic viscosity, g is the acceleration due to gravity, B is the volumetric coefficient of thermal expansion, T is the temperature of the fluid, k is the thermal conductivity and c_p is the specific heat capacity of the fluid at constant pressure, U is an amplitude of oscillation, $H(t)$ is a Heaviside function, ω_1 is frequency of oscillation and t_0 is a constant value of time.

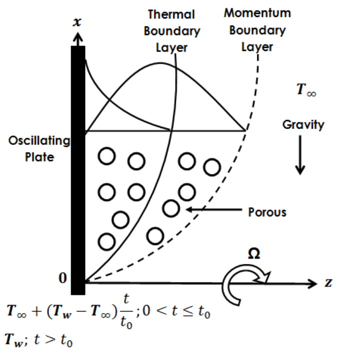

Fig. 1 Physical problem of the study.

Mathematical Solution

The governing Eq. (1) and Eq. (2) with conditions (3) and (4) are reduced to dimensionless form by using the dimensionless variables

$$F^* = \frac{F}{U},\ z^* = \frac{z}{Ut_0},\ t^* = \frac{t}{t_0},\ \omega^* = \omega_1 t_0,\ T^* = \frac{T - T_\infty}{T_w - T_\infty}, \tag{5}$$

as

$$a_1\frac{\partial F}{\partial t} + a_2 F - \frac{\partial^2 F}{\partial z^2} - \alpha\frac{\partial^3 F}{\partial z^2 \partial t} = GrT, \tag{6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial z^2}, \tag{7}$$

Where

$$b = \Omega t_0,\ \ \alpha = \frac{\alpha_1}{\rho U^2 t_0^2},\ \ \frac{1}{K} = \frac{\mu\phi t_0}{\rho k_1},\ \ Gr = \frac{t_0}{U} g\beta\left(T_w - T_\infty\right),$$

$$\Pr = \frac{\mu c_p}{k},\ \ t_0 = \frac{\upsilon}{U^2},\ \ a_1 = 1 + \frac{\alpha}{K},\ \ a_2 = 2ib + \frac{1}{K},$$

are the rotation parameter, second grade parameter, porosity parameter, Grashof number, Prandtl number, characteristics time and constant parameters. The appropriate dimensionless initial and boundary conditions are

$$\begin{gathered} F(z,0) = 0,\ \ T(z,0) = 0;\ z \geq 0, \\ F(0,t) = H(t)\cos(\omega t);\ t > 0, \\ T(0,t) = t;\ 0 < t \leq 1 \text{ or } T(0,t) = 1;\ t > 1, \\ F(z,t) \to 0,\ T(z,t) \to 0 \text{ as } z \to \infty;\ t \geq 0. \end{gathered} \tag{8}$$

The dimensionless governing Eq. (6) and Eq. (7) subject to boundary and initial conditions (8) are solved by using the Laplace transform technique. Hence by taking Laplace transform, the solutions are

$$\begin{aligned} \bar{F}(z,q) = {} & \frac{q}{q^2 + \omega^2} e^{-z a_4\sqrt{\frac{q+a_3}{q+\beta}}} + \\ & \frac{Gr}{q\left[\alpha \Pr q^2 + (\Pr - a_1)q - a_2\right]} \times \\ & \left(1 - e^{-q}\right)\left[\frac{1}{q} e^{-z a_4\sqrt{\frac{q+a_3}{q+\beta}}} - \frac{1}{q} e^{-z\sqrt{q\Pr}}\right] \end{aligned} \tag{9}$$

$$\bar{T}(z,q) = \frac{1}{q^2} e^{-z\sqrt{q\Pr}} - \frac{e^{-q}}{q^2} e^{-z\sqrt{q\Pr}}. \tag{10}$$

Then, the inverse Laplace transform of Eq. (9) and Eq. (10) can be obtained as

$$F(z,t) = F_1(z,t) + F_2(z,t) \tag{11}$$

and

$$T(z,t) = T_1(z,t) - T_1(z,t-1)H(t-1), \tag{12}$$

where

$$\begin{aligned} F_1(z,t) = {} & H(t)\cos(\omega t)e^{-z a_4} + \frac{z a_4\sqrt{a_5}H(t)}{2\sqrt{\pi}} \times \\ & \int_0^t\int_0^\infty \frac{1}{r\sqrt{s}}\cos(\omega t - \omega s)e^{-\frac{z^2 a_4^2}{4r} - r - \beta s} I_1\left(2\sqrt{a_5 r s}\right) dr ds, \end{aligned} \tag{13}$$

$$F_2(z,t) = F_3(z,t) - F_3(z,t-1)H(t-1) \tag{14}$$

and

$$T_1(z,t) = \left(\frac{z^2\Pr}{2} + t\right)erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}\right) - \sqrt{\frac{\Pr t}{\pi}} z e^{-\frac{z^2\Pr}{4t}}. \tag{15}$$

Here, function $F_3(z,t)$ can be written as

$$F_3(z,t) = \int_0^t F_{21}(t-p)F_{22}(z,p)dp, \tag{16}$$

where

$$F_{21}(t-p) = m_3 - m_4 \sinh\left(m_2(t-p)\right)e^{-m_1(t-p)} - m_3 \cosh\left(m_2(t-p)\right)e^{-m_1(t-p)} \tag{17}$$

And

$$F_{22}(z,p) = e^{-za_4} + \frac{za_4\sqrt{a_5}}{2\sqrt{\pi}}\int_0^p\int_0^\infty \frac{1}{r\sqrt{s}} e^{-\frac{z^2a_4^2}{4r}-\beta s - r} \times I_1\left(2\sqrt{a_5 rs}\right)drds - erfc\left(\frac{1}{2}\sqrt{\frac{\Pr}{p}}z\right). \tag{18}$$

Here

$$\beta = \frac{1}{\alpha},\ a_3 = \frac{a_2}{a_1},\ a_4 = \sqrt{\frac{a_1}{\alpha}},\ a_5 = \beta - a_3,\ a_6 = \frac{\Pr - a_1}{\alpha \Pr},$$
$$a_7 = \frac{a_2}{\alpha \Pr},\ m_1 = \frac{a_6}{2},\ m_2 = \frac{\sqrt{a_6^2 - 4a_7}}{2},$$
$$m_3 = \frac{Gr}{\alpha \Pr\left(m_1^2 - m_2^2\right)},\ m_4 = \frac{m_1 m_3}{m_2}.$$

RESULTS AND DISCUSSION

In this section, the obtained solutions are analysed and discussed. The effects of parameters likes second grade parameter α, rotating parameter b, porosity parameter K, Grashof number Gr and Prandtl number Pr on the fluid flow are presented graphically and analysed. The velocity profiles of primary velocity u and secondary velocity v are illustrated in Fig. 2 to Fig. 6 respectively. Further, the comparison of velocity for isothermal and ramped wall temperature is displayed in Fig. 7.

The effect of second grade parameter α on the velocity profiles has been plotted in Fig. 2. It showed that, at the beginning the velocity decreases but then increase when α increases. This is due to the properties of second grade fluid that consist of viscous and elastic behaviour. After that, the behavior of rotation parameter b can be observed in Fig. 3. It is obvious that, for large values of b, the primary velocity is decreased while increases the secondary velocity. This phenomenon occurred due to the effect of Coriolis force. The variation of velocity profiles for porosity parameter K is shown in Fig. 4. Both of the primary and secondary velocities are increased with increasing the values of K. It can be concluded that, the porosity of the medium will reduce the drag force and hence causes the velocity to increase.

In Fig. 5, the distribution of velocity in the fluid are increased when the values of Gr is increased. This scenario is important due to the fact that Gr gives rise to buoyancy effects which results in more induced flow. Fig. 6 illustrates the distribution of the velocity depend on different values of Prandtl number Pr. It is seen that the velocity profiles decrease with increasing values of Pr. Physically, fluids with large Prandtl number have high momentum diffusivity and small thermal conductivity, hence causes a decrease in temperature. In Fig. 7, the velocity for ramped wall temperature is lower compared to isothermal temperature. This is because the heating process in the fluid is more slowly in ramped wall temperature case than in the isothermal case.

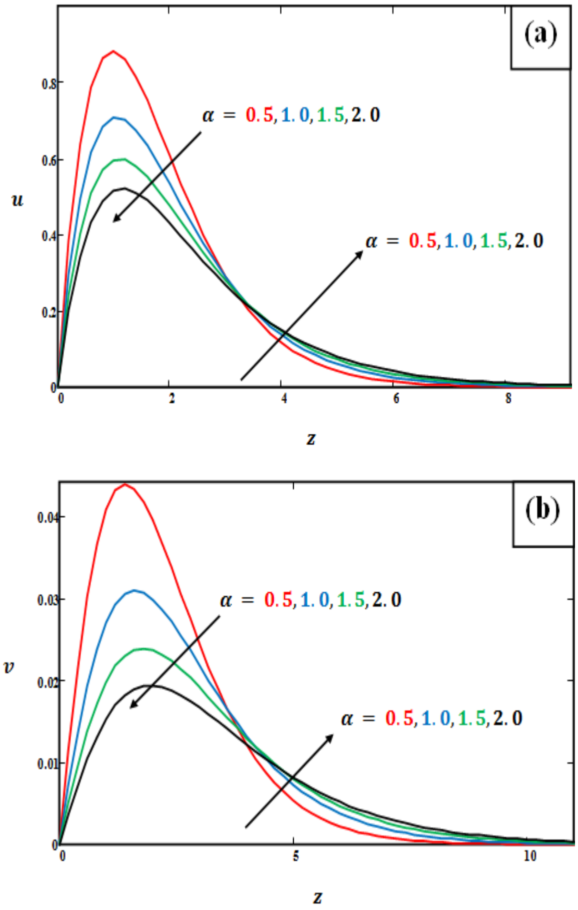

Fig. 2 Velocity profiles for different values of α . **(a)** Primary velocity and **(b)** Secondary velocity.

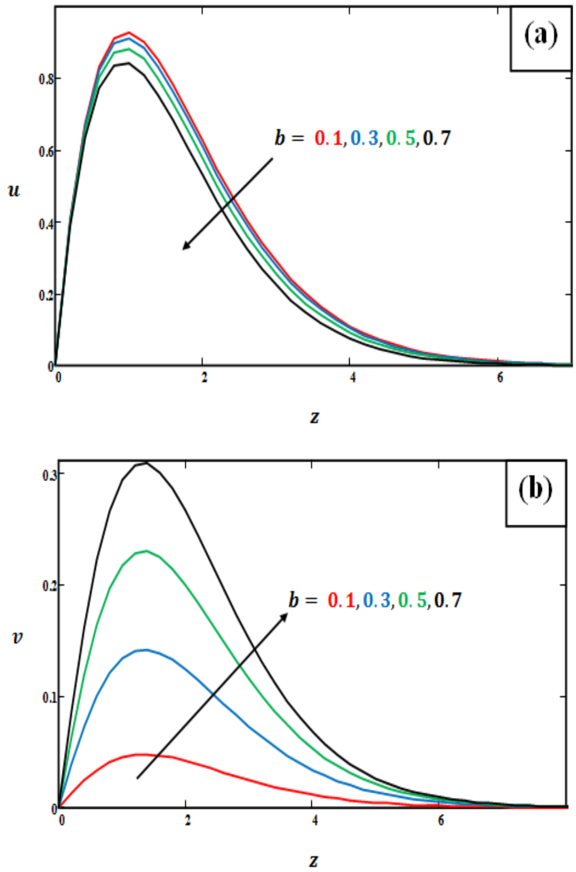

Fig. 3 Velocity profiles for different values of b. **(a)** Primary velocity and **(b)** Secondary velocity.

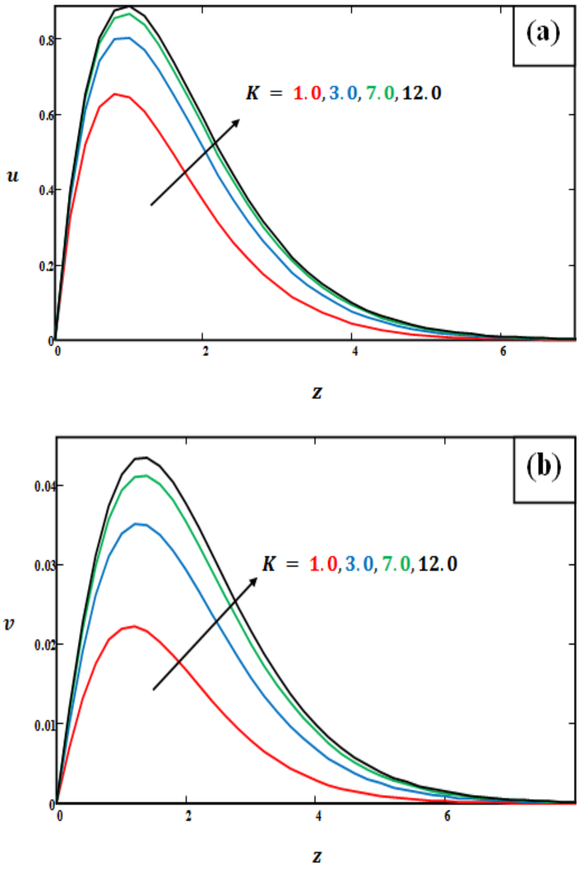

Fig. 4 Velocity profiles for different values of *K*. **(a)** Primary velocity and **(b)** Secondary velocity.

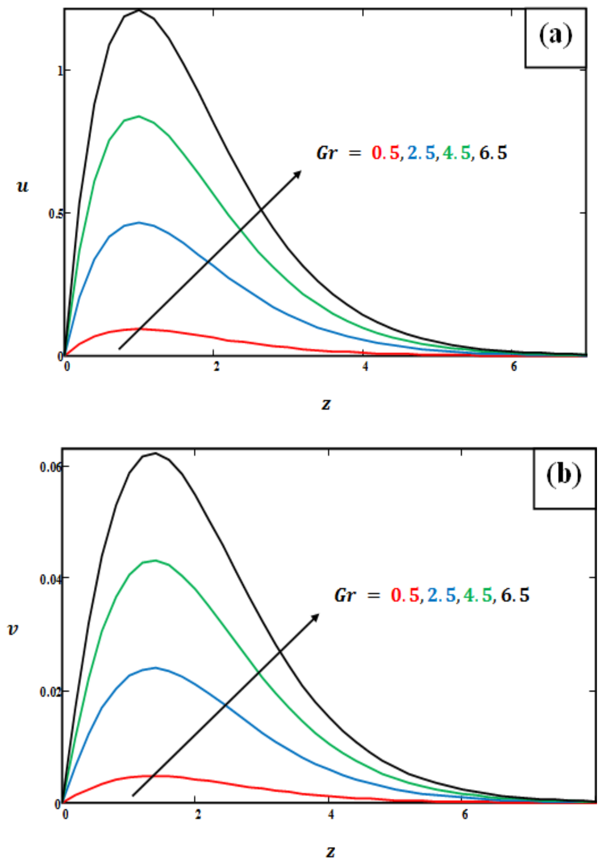

Fig. 5 Velocity profiles for different values of *Gr*. **(a)** Primary velocity and **(b)** Secondary velocity.

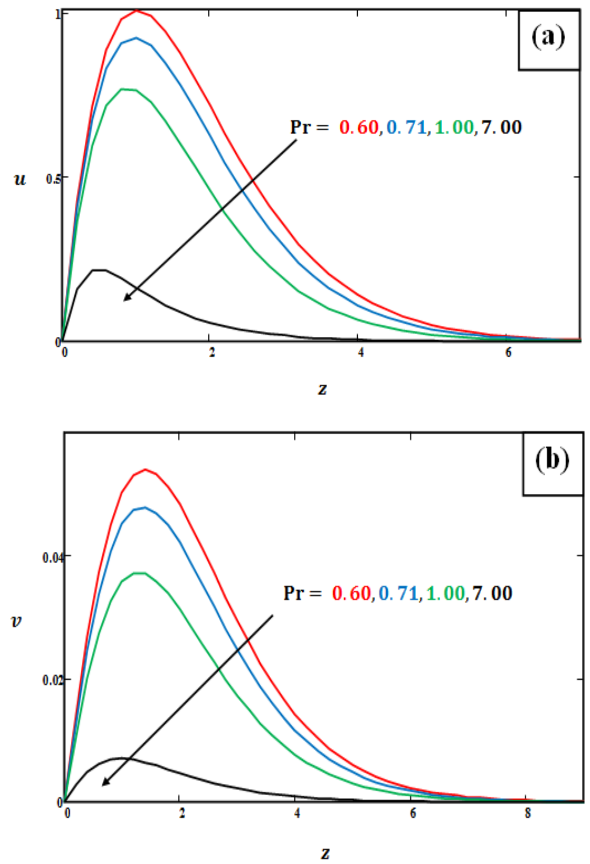

Fig. 6 Velocity profiles for different values of Pr. **(a)** Primary velocity and **(b)** Secondary velocity.

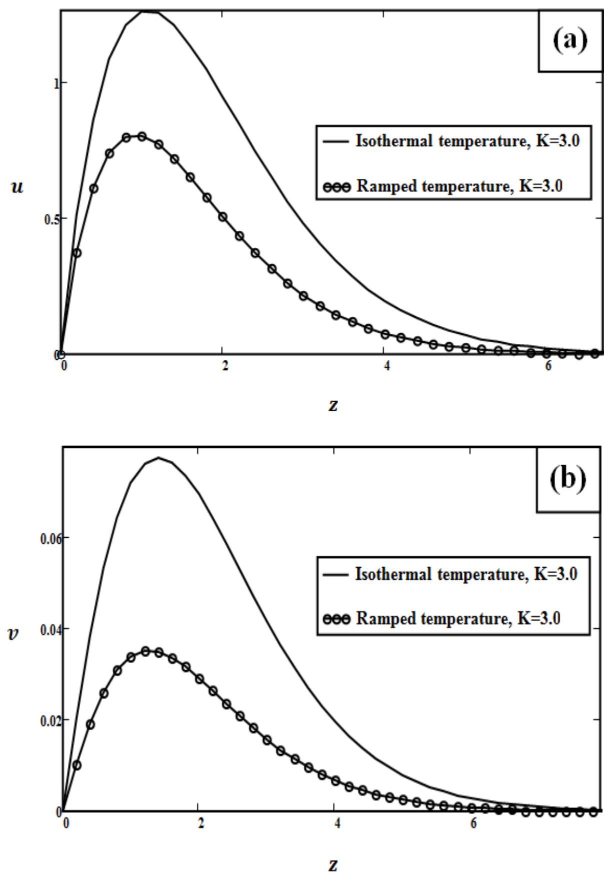

Fig. 7 Comparison of velocity profiles for value of *K*. **(a)** Primary velocity and **(b)** Secondary velocity.

CONCLUSION

In this paper, a mathematical model is presented to investigate the effect of ramped wall temperature on unsteady mixed convection flow of rotating second grade fluid in porous medium. The equations of velocity and temperature are transformed into dimensionless forms and then Laplace transform method is used to determine the exact solutions of these equations. The graphs for velocity and temperature profiles are plotted to investigate the effects of various parameters such as second grade parameter α , rotating parameter *b*, porosity parameter *K*, Grashof number *Gr* and Prandtl number Pr. The following main conclusions are extracted from this study:

(a) The velocity shows an oscillatory behavior first decreases and then increases for both primary and secondary velocities with increasing second grade parameter.
(b) For larger value of rotation parameter, the fluid is decreasing in primary velocity but increasing in secondary velocity.
(c) The velocity increases for large values of porosity parameter Grashof number.
(d) The velocity decreases for larger values of Prandtl number.
(e) This research also provided accurate exact solutions for the mathematical models involving ramped wall temperature. Further, the obtained analytical results can be applied to verify the accuracy of the numerical results.

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