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RESEARCH ARTICLE

Mechanization of the Sturmfel-Salmon resultant method

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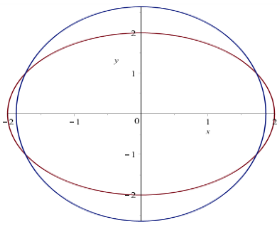
Graphical abstract



Abstract

Designing and implementing a procedure for computing the polynomial resultant provides an avenue for analyzing both the computational complexity and performance of such construction. In this paper a new Maple procedure called *Sturmfelmethod* for computing the Sturmfel-Salmon resultant method is proposed based on existing methods and assumptions. Examples are provided to demonstrate the mechanization of the resulting new algorithm and its computing time. The new procedure can be used to determine whether three polynomials intersect or not and to solve a given system of polynomial equations.

***Keywords*:**Mechanization, procedure, resultant, polynomial resultant

INTRODUCTION

Solving system of polynomial equation is one of the fundamental problem of algebra and algebraic geometry, which comprises both analytical and numerical approaches. One of the important technique is the elimination theory which provide a systematic ways of solving systems of polynomial equations in addition to provide a condition whether the systems have common solution or not. The techniques are Groebner basis, matrix method, characteristics set and homotopy pertubation method. Large storage requirement coupled with high computational complexity of the Groebner basis and set characteristics approaches makes the matrix method of computing resultant more popular and powerful way of solving a system of polynomials.

Several algorithms for computing resultant via the matrix method are presented in (Canny and Emiris, 1993; Canny and Pedersen, 1993; Sulaiman and Aris, 2016; Li *et al*., 2015). If the matrix uses coefficients of the polynomials it is called Sylvester method (Sylvester, 1853; Sturmfels, 2002) while Bezout method has a complicated entries, that is in form of polynomial in terms of the coefficient also (Wang and Lian, 2005, Sulaiman *et al*., 2017)

Availability of the computer algebra system (CAS) such as Maple, Cocoa, Mathematica and Macaulay2 make many problems that are beyond the reach of human being solvable. For example computing the resultant of the system (1)generates a homogenous polynomial of degree 12 in terms of the coefficients of the system with 21,894 different terms (Wang and Lian, 2005). While with the use of CAS, the system can be generated within a few seconds which makes it very important in areas of application such as computer aided design, robotics, geometric modelling and geodesy (Cox et al., 2006).

$$F=\begin{cases} f_1 = 3a_1x^2+2a_2xy+2a_4xz+a_3y^2+a_5yz+a_6z^2 \\ f_2 = b_1x^2+2b_2xy+b_4xz+3b_3y^2+2b_5yz+b_6z^2 \\ f_3 = c_1x^2+c_2xy+2c_4xz+c_3y^2+2c_5yz+3c_6z^2 \end{cases} \quad (1)$$

This paper proposes a new MAPLE procedure called *Sturmfels Method* that can compute the Sturmfel-Salmon resultant and display the resulting polynomial. Naturally, the Sturmfels-Salmon method is like the classical Macaulay method, considering n systemsof homogeneous polynomials with exactly n variables.

The method was proposed by Salmon (1885) for certain class of polynomials in which he had projected that the approach can be extended to higher degree polynomials stating the challenges behind the generalization of the method which until today remain a unsolved. Sturmfel observed that, the method need some modification and proposed a division with a certain constant to reduced the redundancy (Sturmfels, 2002).

The approach was named Sturmfels method in *(Paláncz et al.*, 2008) to acknowledge the contribution of the Bernd Sturmfels, although the real idea was from (Salmon, 1885). The combination Sturmfels-Salmon is due to their vital contributions towards producing that formulation. The choice of the Sturmfels-Salmon resultant method is due to the conciseness of its resultant matrix, which produces only a 6×6 matrix for a three homogenous system of degree two compared to the classical method of Macaulay, which gives up to 15×15 resultant matrix. Therefore, the mechanization of this method for computing the resultant of such homogeneous systems, using a computer algebra system is expected to be effective and efficient.

PRELIMINARIES

Basic notion

Theorem 1 (Sturmfels, 2002) For n system of homogenous polynomial equations in n variables

$$f_1(x_1,\ldots,x_n) = f_2(x_1,\ldots,x_n) = \ldots f_n(x_1,\ldots,x_n) = 0 \quad (2)$$

then any non-trivial common solution is also a solution of the Jacobian polynomial given in (3)

$$J(x_1,\ldots,x_n) = \det\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (3)$$

Moreover, if all f_i have the same degree, then any non-trivial common solution to the system (2) is also a solution of all polynomials(Sturmfels, 1998; Stiller, 1996)

$$\frac{\partial J}{\partial x_i} = (x_1,\ldots,x_n), i = 1,2,\ldots,n \quad (4)$$

Based on Theorem , the Sturmfel-Salmon resultant can be formulated in a series of steps.

Consider the system of n polynomials in n variables given in Eq.(2). From the popular Bezout's Theorem, we shall expect $d_1 d_2 \ldots d_n$ solutions where d_i is a respective degrees of f_i . Although, there may be infinite solutions in a degenerated situation. For the following system of homogeneous equations of degree two

$$f_i = a_i x^2 + b_i y^2 + c_i xy + d_i xz + e_i yz + h_i z^2 = 0 \quad i = 1,2,3 \quad (5)$$

The determinant of the Jocobian matrix given in Eq. (6) is computed, which is another homogeneous polynomial of degree three in 3 variables.

$$J = \det\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix} \quad (6)$$

The partial derivatives of Eq (6) with repect to x, y and z is another set of homogeneous are generated such that

$$\frac{\partial J}{\partial x} := A_1 x^2 + B_1 y^2 + C_1 z^2 + D_1 xy + E_1 xz + H_1 yz = 0,$$
$$\frac{\partial J}{\partial y} := A_2 x^2 + B_2 y^2 + C_2 z^2 + D_2 xy + E_2 xz + H_2 yz = 0,$$
$$\frac{\partial J}{\partial z} := A_3 x^2 + B_3 y^2 + C_3 z^2 + D_3 xy + E_3 xz + H_3 yz = 0,$$

The three partial derivatives above are derived by differentiating Eq. (6) with respect to x, y and z. Considering z as a constant and introduce another variable say w, the following six independent monomials are recorded $x^2, y^2, xy, xw, yw, w^2$. The Sturmfel-Salmon resultant matrix for the system of the homogenous polynomials in Eq. (5),together with the matrix of the monomials are generated from the coefficients of the three homogeneous and that of the coefficients of the pertial derivatives of the determinant of the Jacobian.

$$\begin{bmatrix} a_1 & b_1 & c_1 & zd_1 & ze_1 & z^2h_1 \\ a_2 & b_2 & c_2 & zd_2 & ze_2 & z^2h_2 \\ a_3 & b_3 & c_3 & zd_3 & ze_3 & z^2h_3 \\ A_1 & B_1 & C_1 & D_1 & E_1 & H_1 \\ A_2 & B_2 & C_2 & D_2 & E_2 & H_2 \\ A_3 & B_3 & C_3 & D_3 & E_3 & H_3 \end{bmatrix} \begin{bmatrix} x^2 \\ y^2 \\ xy \\ xw \\ yw \\ w^2 \end{bmatrix} = 0$$

with $A_i, B_i, \ldots, H_i$ as the coefficients of the Jacobian polynomial and the resultant is the determinant of the on the left matrix.

Theorem 2 (Sturmfels, 2002) : For a system of polynomials $f_1,\ldots,f_n$ with respective degrees, $d_1,\ldots,d_n$ there is a unique polynomial $\text{Res}(x_1, x_2,\ldots,x_n) \in \square[c_{i,\alpha}]$ which satisfy the following:-

(a) If $f_1,\ldots,f_n \in \square[x_1, x_2,\ldots,x_n]$ are homogenous of degrees $d_1,\ldots,d_n$ respectively, then Eq(2)has a nontrival solution over $\mathbb{C}$ if and only if $\text{Res}(x_1, x_2,\ldots,x_n) = 0$.

(b) $\text{Res}(x_1^{d_1},\ldots,x_n^{d_n}) = 1$.

(c) $\text{Res}(x_1, x_2,\ldots,x_n)$ is irreducible over $\mathbb{C}$.

Sturmfel-Salmon Algorithm for computing resultant

Input: $f_1(x,y,z)$, $f_2(x,y,z)$ and $f_3(x,y,z)$ with $f_1, f_2, f_3 \in K[x,y,z]$ where K is a field of complex numbers.

Output: C , the determinant of the Sturmfel-Salmon resultant

(1) Convert the system f_1, f_2, f_3 to homogeneous polynomials

(2) Find the Jacobian matrix of f_1, f_2, f_3 and compute its determinant A.

(3) Differentiate A with respect to Var1, Var2 and Var3 where Var1, Var2 and Var3 represent the variables of Eq(1) after homogenization.

(4) Generate the Sturmfel-Salmon resultant matrix B

(5) Compute C the determinant of matrix B

MAPLE procedure for the Sturmfel-Salmon Algorithm:

```
with(linalg):
Sturmfelmethod:=proc(exp1,exp2,exp3,var1,var2,var3)
local R,F,G,H,S,T,Q, Dlim,Dlim1, dlim1, dlim2, dlim3, Elim, Elim1,
elim1, elim2, elim3, Flim, Flim1, flim1, flim2,
flim3,F1,F2,F3,D1,D2,D3,E1,E2,E3,A1,A2,A3,A4,A5,A6,B1,B2,B3,B4
,B5,B6,C1,C2,C3,C4,C5,C6;
R:=matrix(3,3,[[diff(exp1,var1), diff(exp1,var2),
diff(exp1,var3)],[diff(exp2,var1),diff(exp2,var2),diff(exp2,var3)],[diff(
exp3,var1,),diff(exp3,var2),diff(exp3,var3)]]);
Q:=normal(det(R)); if Q ≠ 0 then
F:=diff(Q,var1); G:=diff(Q,var2);H:=diff(Q,var3);
Dlim:=coeff(F,var1);
Dlim1:=coeff(F,var2);dlim1:=coeff(Dlim,var2);
dlim2:=coeff(Dlim,var3);dlim3:=coeff(Dlim1,var3);
Elim:=coeff(G,var1); Elim1:= coeff(G,var2);
elim1:=coeff(Elim,var2); elim2:=coeff(Elim,var3);
elim3:=coeff(Elim1,var3);
Flim:=coeff(H,var1); Flim1:= coeff(H,var2);
flim1:=coeff(Flim,var2); flim2:=coeff(Flim,var3);
flim3:=coeff(Flim1,var3);
F1:=coeff(F,var1^2); F2:=coeff(F,var2^2); F3:=coeff(F,var3^2);
D1:=coeff(G,var1^2); D2:=coeff(G,var2^2); D3:=coeff(G,var3^2);
E1:=coeff(H,var1^2); E2:=coeff(H,var2^2); E3:=coeff(H,var3^2);
A1:=coeff(coeff(exp1,var1,2),var2,0);
A2:=coeff(coeff(exp1,var2,2),var1,0);
A3:=coeff(coeff(exp1,var1,1),var2,1);
A4:=coeff(coeff(exp1,var1,1),var3,1);
A5:=coeff(coeff(exp1,var2,1),var3,1);
A6:=coeff(coeff(exp1,var3,2),var2,0);
```

```
B1:=coeff(coeff(exp2,var1,2),var2,0);
B2:=coeff(coeff(exp2,var2,2),var1,0);
B3:=coeff(coeff(exp2,var1,1),var2,1);
B4:=coeff(coeff(exp2,var1,1),var3,1);
B5:=coeff(coeff(exp2,var2,1),var3,1);
B6:=coeff(coeff(exp2,var3,2),var2,0);
C1:=coeff(coeff(exp3,var1,2),var2,0);
C2:=coeff(coeff(exp3,var2,2),var1,0);
C3:=coeff(coeff(exp3,var1,1),var2,1);
C4:=coeff(coeff(exp3,var1,1),var3,1);
C5:=coeff(coeff(exp3,var2,1),var3,1);
C6:=coeff(coeff(exp3,var3,2),var2,0);
S:=matrix(6,6,[[A1,A2,A3,A4,A5,A6],[B1,B2,B3,B4,B5,B6],[C1,C2,C3,C4,C5,C6],[F1,F2,dlim1,dlim2,dlim3,F3],[D1,D2,elim1,elim2,elim3,D3],[E1,E2,flim1,flim2,flim3,E3]]);
lprint(''The following is a Sturmfel-Salmon  resultant matrix'');
 print(S);
lprint(''The determinant of the above matrix is given below'');
 T:=normal(det(S));
end if;
 end proc:
```

RESULTS AND DISCUSSION

Mechanization of the *Sturmfelmethod*

The MAPLE procedure *Sturmfelmethod* presented in the previous section will be applied to certain systems of multivariate polynomials in two and three variables.The polynomials are homogenized to be of degree 2. The resultant of these polynomials are compared with the results of other resultant matrix method such as the classical Macaulay method. The computing time is observed to indicate the efficiency of the method when applied in an exact computation computer environment such as MAPLE.

Example 1: Consider the following polynomial (Wang and Lian, 2005)

$$\begin{aligned} f_1 &= a_1x^2 - a_2y^2 \\ f_2 &= b_1x^2 - b_2y^2 + b_3xz \\ f_3 &= y - x + z \end{aligned} \tag{7}$$

The system of Eq. (7) is first homogenized and presented in the Maple command given below:

```
f[1]:=a[1]*x^2-a[2]*y^2
f[2]:=b[1]*x^2-b[2]y^2+ z*b[3]x*p
f[3]:=y*p-x*p+z*p^2
Sturmfelmethod(f[1], f[2],f[3],x,y,p)
```

Assuming z is constant with $p$as a homogenizing variable, the procedure *Sturmfelmethod* of the previous section will (i) compute the Jacobian matrix of the system of Example 1, (ii) find the determinant of the Jacobian matrix followed by (iii) finding the partial derivatives of the determinant with repect to varibles x,y and $p$extract the coefficients of the derivatives and the initial system of Example 1, and form the Sturmfel-Salmon resultant matrix and finally (v)compute the determinant of the resultant matrix which is the projection operator. The following output is displayed:

The Sturmfel-Salmon resultant matrix is:

$$\begin{bmatrix} a_1 & -a_2 & 0 & 0 & 0 & 0 \\ b_1 & -b_2 & 0 & zb_3 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & z \\ 0 & -4a_1b_2+4a_2b_1 & 8a_1b_2-8a_2b_1 & -4za_1b_3 & -8za_1b_2+8za_2b_1 & 0 \\ 4a_1b_2-4a_2b_1 & 0 & -8a_1b_2+8a_2b & -8za_1b_2+8za_2b_1 & 4za_2b_3 & 4z^2a_2b_3 \\ -2za_1b_3 & 2za_2b_3 & -8za_1b_2+8za_2b_1 & 0 & 8z^2a_2b_3 & 0 \end{bmatrix}$$

The determintant of the above resultant matrix is:

$$512z^4\ (\ a_1^4b_2^4 - \ 4a_1^3a_2b_1b_2^3 - 2a_1^3a_2b_2^3b_3 - a_1^3a_2b_2^2b_3^2 + \ 6a_1^2a_2^2b_1^2b_2^2 \\ +6a_1^2a_2^2b_1b_2^2b_3 + 2a_1^2a_2^2b_1b_2b_3^2 + a_1^2a_2^2b_2^2b_3^2 - 4a_1a_2^3b_1^3b_2 - 6a_1a_2^3b_1^2b_2b_3 \\ -a_1a_2^3b_1^2b_3^2 - 2a_1a_2^3b_1b_2b_3^2 + a_2^4b_1^4 + 2a_2^4b_1^3b_3 + a_2^4b_1^2b_3^2 + a_2^3b_1b_2b_3^2).$$

Further simplification reveals that the determinant can be given as

$$512z^4(a_1b_2 - a_2b_1)^2(a_1^2b_2^2 - 2a_1a_2b_1b_2 - 2a_1a_2b_2b_3 - a_1a_2b_3^2 + a_2^2b_1^2 \\ +2a_2^2b_1b_3 + a_2^2b_3^2).$$

which coincide with the result obtained using the Macaulay resultant method before the division with the minor matrix which is a systematic approach of reducing the redundant factors. This example shows that, the Sturmfel-Salmon resultant produces an unwanted factor in the projection operator.

Example 2 Consider the following polynomial (Stiller, 1996)

$$\begin{aligned} f_1 &= x^2 + y^2 - 2 \\ f_2 &= x^2 + y^2 + z^2 - 3 \\ f_3 &= x^2 - y^2 \end{aligned} \tag{8}$$

The system (6) is first homogenized and presented in the Maple command given below:

```
f[1]:=x^2+y^2-2w^2
f[2]:=x^2+y^2+( z^2-3)*w^2
f[3]:=x^2-y^2
Sturmfelmethod(f[1], f[2], f[3],x,y,w)
```

Assuming z is constant with w as a homogenizing variable, and implementing the steps as of Example 1, the following output will be displayed.

The Sturmfel-Salmon resultant matrix is:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -2 \\ 1 & 1 & 0 & 0 & 0 & z^2-3 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16z^2-16 & 0 \\ 0 & 0 & 0 & 16z^2-16 & 0 & 0 \\ 0 & 0 & 16z^2-16 & 0 & 0 & 0 \end{bmatrix}$$

The determinant of the above resultant matrix is :

$$8192z^8 - 32768z^6 + 49152z^4 - 32768z^2 + 8192$$

Setting the above expression equal to zere nd further simplification gives the following

Solving the equation gives $z = \pm 1$, four times. By a little back substitution, the solution of the system in this example is $(x = \pm 1, y = \pm 1, z = \pm 1)$.

The result is in agreement with the one obtain in (Stiller, 1996), The size of the resultant matrix is only 6×6 compared to the classical method of Macaulay which gives 15×15 resultant matrix.

Example 3 (Intersection of curve and surface**)** Consider the following polynomial

$$\begin{aligned} f_1 &= x^2 + y^2 - 4 \\ f_2 &= 2x^2 + 3y^2 - 9 \end{aligned} \tag{9}$$

The system after homogenization will have n equation in $n+1$ variables which lead to another notion of u-resultant. The computation of u-resultant is use when finding allcommon isolated roots of underdetermine system of polynomials (Emiris and Mourrain, 1999).

The system of Eq. (9) is first homogenized and presented in the Maple command given below using the method of u-resultant:

f[1]:=x^2+y^2-4w^2
*f[2]:=2*x^2+3*y^2+9*w^2*
*f[3]:=u[1]*x*w+u[2]*y*w+u[3]*w^2*
Sturmfelmethod(f[1], f[2],f[3],x,y,w)

Assuming *w* as a homogenizing variable, and executing the procedure *Sturmfelmethod*, the following results are obtained and displayed:

The Sturmfel-Salmon resultant matrix is:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -4 \\ 2 & 3 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & u_1 & u_2 & u_3 \\ 0 & 4u_2 & 8u_1 & 0 & 8u_3 & 4u_2 \\ 4u_1 & 0 & 8u_2 & 8u_3 & 0 & 12u_1 \\ 0 & 0 & 8u_3 & 8u_2 & 24u_1 & 0 \end{bmatrix}$$

The determintant of the above matrix is:

$$4608u_1^4 - 3072u_1^2u_2^2 - 3072u_1^2u_3^2 + 512u_2^4 - 1024u_2^2u_3^2 + 512u_3^4.$$

Further simplification reveals that the determinant can be given as

$$\frac{512}{9}(\sqrt{3}u_2 - \sqrt{3}u_3 + 3u_1)(\sqrt{3}u_2 - \sqrt{3}u_3 - 3u_1)(\sqrt{3}u_2 + \sqrt{3}u_3 - 3u_1)$$
$$(\sqrt{3}u_2 + \sqrt{3}u_3 + 3u_1)$$

A little work gives the intersection as

$(\sqrt{3},1),(\sqrt{3},-1),(-\sqrt{3},1)$ and $(-\sqrt{3},-1)$.

Example 4: Consider the following polynomial

$$\begin{aligned} f_1 &= a_3xz + a_4y^2 + a_5yz + a_6y^2 \\ f_2 &= b_1x^2 + b_2xy + b_5yz + b_6y^2 \\ f_3 &= c_1x + c_2y + c_3z \end{aligned} \qquad (10)$$

The system of Eq (10) will now be homogenized and presented in the Maple command

*f[1]:=a[3]*x*z*w+a[4]*y^2+a[5]*y*w*z+a[6]*z^2*w^2*
*f[2]:=b[1]*x^2+b[2]*x*y+b[5]*y*w*z+b[6]*z^2*w^2*
*f[3]:=c[1]*x*w+c[2]*y*w+c[3]*z*w^2*
Sturmfelmethod(f[1], f[2],f[3],x,y,w)

Assuming *w* as a homogenizing variable, and executing the procedure *Sturmfelmethod.*

For this example, the resultant matrix is of size 6 by 6 but the equations in the matrix is lengthy and shall not be displayed here. The system generate 10 by 10 resultant matrix using classical Macaulay formulation.

The determinant of the resultant matrix is :

$$\begin{aligned} &-512a_4^2z^4b_1^2(\ a_3a_5b_2b_5c_1^2c_2c_3 - a_3a_4b_1b_2c_2c_3^3 + a_4^2b_1^2c_3^4 - a_4a_5b_1b_2c_1c_3^3 \\ &-a_5^2b_1b_6c_1^2c_2^2 + a_4a_5b_1b_5c_1^2c_3^2 + 2a_4a_6b_1b_6c_1^2c_2^2 + a_4a_5b_5b_6c_1^4 + a_4a_3bcc_3 \\ &-a_4a_6b_2^2c_1^2c_3^2 + a_3^2b_1b_5c_2^3c_3 + a_3a_4b_2^2c_1c_3^3 + 2a_5a_6b_1^2c_2^3c_3 + 2a_6^2b_1b_2c_1c_2^3 \\ &+\ 2a_4a_5b_1^2c_2c_3^3 - 2a_4a_6b_1^2c_2^2c_3^2 -\ a_4a_6b_5^2c_1^4 -\ a_6^2b_2^2c_1^2c_2^2 - b_1a_3^2b_6c_2^4 + \\ &a_3a_5b_1b_2c_2^2c_3^2 - a_3a_6b_1b_2c_2^3c_3 - 2a_4^2b_1b_6c_1^2c_3^2 - a_5^2b_1^2c_2^2c_3^2 + a_5^2b_1b_2c_1c_2c_3^2 \\ &+a_3^2b_2b_6c_1c_2^3\ +\ a_5^2b_2b_6c_1^3c_2 +\ 3a_4a_3b_1b_5c_1c_2c_3^2\ -\ 4a_4a_3b_1b_6c_1c_2^2c_3 + \\ &2a_4a_5b_1b_6c_1^2c_2c_3\ +\ 2a_4a_6b_1b_2c_1c_2c_3^2 - 4a_4a_6b_1b_5c_1^2c_2c_3 - a_4a_3b_5b_6c_1^3c_2 \\ &-a_4a_5b_2b_6c_1^3c_3\ + 2a_4a_6b_2b_5c_1^3c_3 -\ 2a_3a_4b_2b_5c_1^2c_3^2 + 2a_3a_5b_1b_6c_1c_2^3 - \\ &a_3a_6b_1b_5c_1c_2^3 -\ 2a_4a_6b_2b_6c_1^3c_2\ +\ a_5a_6b_1b_5c_1^2c_2^2 +\ 3a_4a_3b_2b_6c_1^2c_2c_3 - \\ &a_3a_5b_1b_5c_1c_2^2c_3\ -\ 3a_5a_6b_1b_2c_1c_2^2c_3\ - a_3^2b_2b_5c_1c_2^2c_3\ -\ a_3a_5b_2^2c_1c_2c_3^2 \\ &-2a_3a_5b_2b_6c_1^2c_2^2\ +\ a_3a_6b_2b_5c_1^2c_2^2\ + a_5a_6b_2^2c_1^2c_2c_3 -\ a_5a_6b_2b_5c_1^3c_2 \\ &+a_3a_6b_2^2c_1c_2^2c_3 - a_4^2b_6^2c_1^4 - a_6^2b_1^2c_2^4) \end{aligned}$$

Limitation and Further Work

The Sturmfel-Salmon resultant method, like other formulations, produces and unwanted factor which is contained in the projection operator. According to Theorem 2(b) the resultant of the leading monomial $\text{Res}(x_1^{d_1},...,x_n^{d_n}) = 1$, but Sturmfel-Salmon resultant method produces a multiple of 512 attached with the resultant.
The input polynomials for the *Sturmfelmethod* procedure must first be homogenized. An algorithm for homogenizing the polynomials need to be constructed. The algorithm can be applicable to other resultant formulation since homogenizing polynomial equations is required whereby the solutions of the homogenizing variable gives an insight to finding the solutions of multivariate polynomials in the underlying projective space.

CONCLUSION

In this paper the MAPLE procedure of computing the Sturmfel-Salmon resultant is constructed and implemented on some examples. The results show that the Sturmfel-Salmon matrix method can produces 6×6 resultant matrix compared to classical methods such Macaulay which produce up to 15×15 resultant for three homogeneous system of degree two. Even though the presence of extraneous factors cannot be eliminated, the implementation of the method on these examples does not take up to a seconds, indicating its efficiency, when appropriate method of computing determinants built in the MAPLE software is applied**.**

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