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RESEARCH ARTICLE

Operations on Neutrosophic $\mathcal{N}$-Hypersoft Sets: A Novel Framework for Decision-Making

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Abstract

Objectives: The aims of this research are to combine the neutrosophic set with the $\mathcal{N}$-hypersoft set framework to develop the Neutrosophic $\mathcal{N}$-Hypersoft Set (N$\mathcal{N}$-HSS), maintaining structural symmetry for the representation and analysis of binary and non-binary data. Furthermore, this study aims to propose a proper mathematical model to handle the uncertainty of multi-sub-attribute structures especially for rating-evaluation-based systems, for improving the accuracy and flexibility of decision making in uncertain and inconsistent environments.The proposed N$\mathcal{N}$-HSS framework establishes fundamental set-theoretic operations, including top-weak and bottom-weak complements, the extended and restricted unions, and the intersections. These operations are formally defined and illustrated by a suitable example to demonstrate their applicability. Additionally, the N$\mathcal{N}$-HSS framework proposes a weighted choice-values-based decision-making approach to evaluate and rank alternatives in uncertain multi-sub-attribute environments. A numerical example is presented to validate the effectiveness and applicability of the proposed approach. The results indicate that the suggested N$\mathcal{N}$-HSS framework efficiently improves decision-making by precisely managing uncertainty in datasets with both binary and non-binary data. The study also emphasizes the usefulness of mathematical ideas in real-world decision-making, where ambiguity and imprecision are common. The suggested framework improves the modeling of intricate multi-sub-attribute structures and offers a reliable method for addressing real-world decision-making issues, especially in rating-based evaluation systems that are marked by inconsistency, uncertainty, and indeterminacy.

Keywords: $\mathcal{N}$-Hypersoft set, Neutrosophic $\mathcal{N}$- Hypersoft set, Basic operations, weight choice values, decision-making.

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Introduction

Fuzzy set theory was developed and established by Zadeh [1] (1965) as a way to represent and control the inherent uncertainty in data from a variety of sectors, including the social sciences, engineering, and medicine. To overcome the restrictions of fuzzy sets in expressing uncertainty, Atanassov [2] (1999) added a degree of non-membership. This resulted in the creation of intuitionistic fuzzy sets, which are generalizations of fuzzy sets to include both membership and non-membership information. As an extension of fuzzy and intuitionistic fuzzy sets, Smarandache [3] created neutrosophic sets in 1998 with the goal of better handling data that is characterized by inaccuracy, indeterminacy, and inconsistency. Soft sets are a major mathematical framework that Molodtsov [4] created in 1999 to handle the ambiguity that arises from the binary assessment of objects and parameters. The intuitionistic fuzzy soft sets framework was created by Tripathy et al. [5] (2016) to improve decision-making procedures and their real-world applications, building on the concept of soft sets. The neutrosophic soft set theory was first put forth by Maji et.al. (2013) [6], and Karaaslan (2014) [7] later improved its operational framework to handle applications involving decision-making in the face of ambiguity. The earlier idea was expanded to matrices by Jafar et al. [8] (2020), who suggested neutrosophic soft matrices used together with a scoring function to efficiently address agricultural decision-making issues. Sophia Porchelvi. R and Subashini.P [9] (2023) addressed maternity diet-related concerns by proposing a novel neutrosophic soft set-based decision-making strategy. Fatimah et al. [10] (2018) extended the soft set theory to address real-world issues with non-binary evaluations by proposing $\mathcal{N}$ - soft sets. Akram et al. (2018) [11] developed the fuzzy $\mathcal{N}$ - soft set framework to improve decision-making analysis. Zhang et al. (2020) [12] then expanded it to create Pythagorean fuzzy $\mathcal{N}$ - soft sets, which made it possible to handle complex multi-attribute decision-making situations. Riaz et al. (2020) [13] developed a TOPSIS technique based on neutrosophic $\mathcal{N}$ - soft sets specifically to manage indeterminacy in medical decision-making applications.

An extension of the soft set, the hypersoft set was created by Smarandache (2018) [14] to address problems related to sub-attribute functions. The hypersoft set structure was extended into the neutrosophic domain by Saqlain et al. (2019 & 2020) [15,16], who also used an accuracy function in addition to the TOPSIS method to handle uncertainty-related problems. Aggregation techniques for neutrosophic hypersoft sets were established by Zulqarnain et al. (2020) [17]. In order to efficiently manage uncertainties, Saqlain et al. (2021) [18] created distance and similarity measures for neutrosophic hypersoft sets, and Jafar et al. (2022) [19] addressed solid waste management issues by using distance and similarity measures of neutrosophic hypersoft sets. The framework of $\mathcal{N}$-hypersoft sets was suggested by Musa et al., (2023) [20] to efficiently handle complex problems with multiple parameters and non-binary object evaluations.

Research Gap

While soft sets, neutrosophic soft sets, and neutrosophic $\mathcal{N}$- soft sets are extensively utilized for managing uncertainty, they mainly concentrate on attribute-level frameworks and struggle to accurately depict intricate uncertainties that emerge in multi-sub-attribute contexts.

Hypersoft sets and their extensions are extensively utilized in multi-attribute decision-making issues due to their capacity to depict attribute-valued data via sub-attributes. Nonetheless, current hypersoft set models are constrained in their capacity to concurrently encapsulate truth, indeterminacy, and falsity information in intricate multi-sub-attribute settings. Neutrosophic hypersoft sets offer a better approach for modeling uncertainty; however, they are still insufficient for managing binary and non-binary data together, especially in decision-making scenarios that involve ratings. This constraint results in a notable research void in creating an adaptable mathematical structure that can depict various data forms while maintaining the expressive strength of neutrosophic information.

To resolve this gap, this research introduces the Neutrosophic N-hypersoft Set (NNHSS) framework, offering a uniform and generalized model for handling intricate uncertain information in multi-sub-attribute decision-making scenarios, particularly in ratings-based evaluations. Additionally, the suggested framework presents essential set-theoretic operations and formulates a weighted choice value-driven decision-making approach to efficiently address practical challenges related to uncertainty, indeterminacy, inconsistency, and diverse data.

Scope of the Paper

A Neutrosophic $\mathcal{N}$-Hypersoft Set (N$\mathcal{N}$-HSS) framework for modeling uncertainty in binary and non-binary multi-sub-attribute data is proposed in this work. To enable precise ranking of options in ambiguous situations, it presents basic set-theoretic operations and a weighted choice-value-based decision-making algorithm. The proposed paradigm can be used for rating-based evaluation systems and other multi-criteria decision-making problems that involve uncertainty, inconsistency, and indeterminacy in different real-world situations.

Organization of the Paper

This article gives decision-making issues more modeling freedom by extending the idea of $\mathcal{N}$-hypersoft sets to neutrosophic $\mathcal{N}$-hypersoft sets. The following is the structure of the paper:

Section 2 presents the main ideas of our recently suggested model. In **Section 3**, the neutrosophic $\mathcal{N}$- hypersoft set is introduced along with its fundamental operations. **Section 4** presents a novel approach based on the neutrosophic $\mathcal{N}-$ hypersoft sets weighted choice values. **Section 5** provides an illustrative scenario to highlight the real-world domains that heavily utilize mathematics in daily decision-making. **Section 6** presents the results of the suggested methods and compares them with other methods. **Section 7** finally concludes the study.

Preliminaries

Definition: 2.1 [4]

Let the universal set be $\mathbb{G}_H$, and the collection of attributes be $\mathcal{F}$, where $\mathcal{L} \subseteq \mathcal{F}$. The power set of $\mathbb{G}_H$ can be considered as $P(\mathbb{G}_H)$. A **soft set** over $\mathbb{G}_H$is defined as a pair $(\chi, \mathcal{L})$ if χ is a mapping from $\mathcal{L}$ to $P(\mathbb{G}_H)$, that is, $\chi\colon\ \mathcal{L} \to P(\mathbb{G}_H)$.

Definition: 2.2 [14]

Let $P(\mathbb{G}_H)$ represent the power set of a universal set $\mathbb{G}_H$. Consider $\mathbb{f}_1, \mathbb{f}_2, \dots, \mathbb{f}_u$as $u \geq 1$ distinct attributes, where each attribute $\mathbb{f}_i$ is related to a collection of attribute values $\mathcal{F}_i$. These sets are mutually disjoint, meaning that for every $i \neq j$, where $i, j \in \{1,2,\dots,u\}$, $\mathcal{F}_i \cap \mathcal{F}_j = \emptyset$. Then, a tuple $(\chi, \mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_u)$ is defined as a **Hypersoft Set (HSS)** over $\mathbb{G}_H$, where χ is a mapping $\chi\colon \mathcal{F}_1 \times \mathcal{F}_2 \times \ \dots \times \mathcal{F}_u \to P(\mathbb{G}_H)$.

Definition: 2.3 [16]

Let $P(\mathbb{G}_H)$ represent the power set of a universal set $\mathbb{G}_H$.Consider $\mathbb{f}_1, \mathbb{f}_2, \dots, \mathbb{f}_u$as $u \geq 1$ distinct attributes, where each attribute $\mathbb{f}_i$ is related to a collection of attribute values $\mathcal{F}_i$. These sets are mutually disjoint, meaning that for every $i \neq j$, where $i, j \in \{1,2,\dots,u\}$, $\mathcal{F}_i \cap \mathcal{F}_j = \emptyset$. Then, a tuple $(\chi, \mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_u)$ is defined as a **Neutrosophic Hypersoft Set (NHSS)** over $\mathbb{G}_H$, where $\chi\colon \mathcal{F}_1 \times \mathcal{F}_2 \times \ \dots \times \mathcal{F}_u \to P(\mathbb{G}_H)$, and

$\chi(\mathcal{F}_1 \times \mathcal{F}_2 \times \ \dots \times \mathcal{F}_u) = \{\mathfrak{g}, \mathbb{T}_l(\mathfrak{g}), I_l(\mathfrak{g}), \mathbb{F}_l(\mathfrak{g}), \mathfrak{g} \in \mathbb{G}_H\}$.

Here $\mathbb{T}_l(\mathfrak{g})$ denotes the degree of truth, and $I_l(\mathfrak{g})$, $\mathbb{F}_l(\mathfrak{g})$ denotes the degree of indeterminacy, and falsity. Also, the functions $\mathbb{T}, I, \mathbb{F} : \mathbb{G}_H \to [0,1]$, and they satisfy the following requirement: $0 \leq \mathbb{T}_l(\mathfrak{g}) + I_l(\mathfrak{g}) + \mathbb{F}_l(\mathfrak{g}) \leq 3$.

Definition: 2.4 [20]

Let $\mathbb{G}_H$ be a universal set and $\mathcal{F} = \mathcal{F}_1 \times \mathcal{F}_2 \times \ \dots \times \mathcal{F}_u$ represents the collection of parameters (or attributes), where $\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_u$ are pairwise disjoint sets and $\mathcal{L} = \ {\mathcal{L}_1}^a \times {\mathcal{L}_2}^b \times \dots \times {\mathcal{L}_u}^z \subseteq \mathcal{F}$. Define $\mathbb{V} = \{0, 1, 2, \dots, \mathcal{N}-1\}$ as a set of ordered grades, where $\mathcal{N} \in \{2,3,\dots\}$, that is, $\mathcal{N} \geq 2$.

The triple $(\chi, \mathcal{L}, \mathcal{N})$ represents a **$\mathcal{N}$-Hypersoft Set ($\mathcal{N}$ -HSS)** over $\mathbb{G}_H$, where $\chi\colon\ \mathcal{L} \to \mathrm{P}(\mathbb{G}_\mathrm{H} \times \mathbb{V})$ and the mapping χ satisfies the condition: The pair $(\mathfrak{g}, v_l) \in \ \chi(l)$, for every $l \in \mathcal{L}$ and $\mathfrak{g} \in \mathbb{G}_H$, and there exists a unique grade $v_l \in \ \mathbb{V}$ such that $(\mathfrak{g}, v_l) \in \ \chi(l)$.

Definition: 2.5 [20]

An **$\mathcal{N}$-Hypersoft set weak complement** of $(\chi, \mathcal{L}, \mathcal{N})$ is any $\mathcal{N}$ -Hypersoft set

$$(\chi, \mathcal{L}, \mathcal{N})^{\ wc} = \ (\chi^{wc}, \mathcal{L}, \mathcal{N})$$

where $\chi^{wc}(\mathfrak{g}) \cap \ \chi(\mathfrak{g}) = \ \emptyset$ for each $\mathfrak{g} \in \mathbb{G}_H$.

Definition: 2.6 [20]

An **$\mathcal{N}$-Hypersoft set top - weak complement** of $(\chi, \mathcal{L}, \mathcal{N})$ is $(\chi, \mathcal{L}, \mathcal{N})^{\ twc} = \ (\chi^{twc}, \mathcal{L}, \mathcal{N})$. where,

$$\chi^{twc}(l)(\mathfrak{g}) = \begin{cases} \mathcal{N}-1, & if\ v(l)(\mathfrak{g}) < \mathcal{N}-1 \\ 0, & if\ v(l)(\mathfrak{g}) = \ \mathcal{N}-1 \end{cases}$$

for every $v \in \mathbb{V}$, $\mathfrak{g} \in \mathbb{G}_H$.

Definition: 2.7 [20]
An **$\mathcal{N}$-Hypersoft set bottom - weak complement** of $(\chi, \mathcal{L}, \mathcal{N})$ is $(\chi, \mathcal{L}, \mathcal{N})^{bwc} = (\chi^{bwc}, \mathcal{L}, \mathcal{N})$. where,

$$\chi^{bwc}(l)(\mathrm{g}) = \begin{cases} 0, & if\ v(l)(\mathrm{g}) > 0 \\ \mathcal{N}-1, & if\ v(l)(\mathrm{g}) = 0 \end{cases}$$

for every $v \in \mathbb{V}$, $\mathrm{g} \in \mathbb{G}_H$.

Neutrosophic $\mathcal{N}$-Hypersoft Set

Let $\mathbb{G}_H$ denote the universal set, $\mathcal{F}_1 \times \mathcal{F}_2 \times \ldots \times \mathcal{F}_u = \mathcal{F}$ be the collection of parameters (attributes), where $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_u$ are the pairwise disjoint sets and $\mathcal{L}_1 \times \mathcal{L}_2 \times \ldots \times \mathcal{L}_u = \mathcal{L} \subseteq \mathcal{F}$. We define $\mathbb{V} = \{0, 1, \ldots, \mathcal{N}-1\}$ as a collection of ordered grades, where $\mathcal{N} \in \{2,3,\ldots\}$. $\mathrm{P}(\mathbb{G}_\mathrm{H} \times \mathbb{V})$ denotes the set of all NHSSs (neutrosophic hypersoft sets) over $\mathbb{G}_\mathrm{H} \times \mathbb{V}$. A **Neutrosophic $\mathcal{N}$ –Hypersoft Set (N $\mathcal{N}$ -HSS)** might be expressed as a triple $(\omega, \gamma, \mathcal{N})$, where $\gamma = (\chi, \mathcal{L}, \mathcal{N})$ is an $\mathcal{N}$ -HSS and ω: $\mathcal{L} \to \mathrm{P}(\mathbb{G}_\mathrm{H} \times \mathbb{V})$ fulfills the given requirement: for all $l \in \mathcal{L}$ and $\mathrm{g} \in \mathbb{G}_H$ there exist a unique $(l, v_l) \in \mathbb{G}_\mathrm{H} \times \mathbb{V}$ such that $v_l \in \mathbb{V}$ and $[\prec \mathrm{g}, \mathbb{T}_l(\mathrm{g}), I_l(\mathrm{g}), \mathbb{F}_l(\mathrm{g}) \succ, v_l(\mathrm{g})] \in \omega(l)$.

Mathematically,

$$(\omega, \gamma, \mathcal{N}) = \{[\prec \mathrm{g}, \mathbb{T}_l(\mathrm{g}), I_l(\mathrm{g}), \mathbb{F}_l(\mathrm{g}) \succ, v_l(\mathrm{g})], \mathrm{g} \in \mathbb{G}_H, \mathbb{T}_l, I_l, \mathbb{F}_l \in [0,1], v \in \mathbb{V}, l \in \mathcal{L}\}.$$

Where, v_l indicates the element attribute's level and $\mathbb{T}_l(\mathrm{g}), I_l(\mathrm{g}), \mathbb{F}_l(\mathrm{g})$ represents the truth, indeterminacy and falsity values of the element $\mathrm{g} \in \mathbb{G}_H$ to the attribute $l \in \mathcal{L}$ with $0 \leq \mathbb{T}_l(\mathrm{g}) + I_l(\mathrm{g}) + \mathbb{F}_l(\mathrm{g}) \leq 3$.

Let us assume that both $\mathbb{G}_H = \{\mathrm{g}_i, i = 1, 2, \ldots, t\}$ and $\mathcal{L} = \{l_j, j = 1, 2, \ldots, u\}$ are finite.

Example: 3.1
A person is looking to purchase a smartwatch for everyday workouts. The brands offered are $\mathbb{G}_H = \{\mathbb{G}_{H_1} = Samsung, \mathbb{G}_{H_2} = Realme, \mathbb{G}_{H_3} = boAt\}$. Their performance is assessed according to their application; such as, $\mathcal{F} = \{\mathcal{F}_1 = Battery\ Duration, \mathcal{F}_2 = Cost, \mathcal{F}_3 = Health\ \&\ Fitness\}$.

Every attribute is additionally split into the subsequent sub-attributes:

$$\mathcal{F}_1{}^a = Battery\ Duration = \{one - two\ days, more\ than\ seven\ days\},$$
$$\mathcal{F}_2{}^b = Cost = \{Below\ Rs.2000, Rs.3000 - Rs.4000, above\ Rs.5000\},$$
$$\mathcal{F}_3{}^c = Health\ \&\ Fitness = \{Sports\ mode, Sleep\ monitor, Heart\ rate\ tracking\}.$$

Let $\omega(\mathcal{F}_1{}^a \times \mathcal{F}_2{}^b \times \mathcal{F}_3{}^c) = \{more\ than\ seven\ days, Rs.3000 - Rs.4000, Sports\ mode\}$ indicate the individuals preference. In this example ($\mathcal{N} = 4$); therefore, the corresponding 4-HSS is described in Table 1 below.

Table 1. Tabular Format of the 4-HSS $(\chi_1, \mathcal{L}, 4)$

$(\chi_1, \mathcal{L}, 4)$	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$
$\mathbb{G}_{H_1}$	0	1	1
$\mathbb{G}_{H_2}$	2	3	3
$\mathbb{G}_{H_3}$	1	0	2

The N4-HSS $(\omega_1, \gamma_1, 4)$ is described in Table 2.

Table 2. Tabular Format of the N4-HSS $(\omega_1, \gamma_1, 4)$

$(\omega_1, \gamma_1, 4)$	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$
$\mathbb{G}_{H_1}$	$(\prec 0.2, 0.1, 0.8 \succ, 0)$	$(\prec 0.3, 0.15, 0.7 \succ, 1)$	$(\prec 0.4, 0.2, 0.6 \succ, 1)$
$\mathbb{G}_{H_2}$	$(\prec 0.55, 0.28, 0.45 \succ, 2)$	$(\prec 0.8, 0.4, 0.2 \succ, 3)$	$(\prec 0.9, 0.45, 0.1 \succ, 3)$
$\mathbb{G}_{H_3}$	$(\prec 0.4, 0.2, 0.6 \succ, 1)$	$(\prec 0.1, 0.05, 0.9 \succ, 0)$	$(\prec 0.65, 0.33, 0.35 \succ, 2)$

The numerical grade values used in Tables 1 and 2 are for illustrative purposes only and are assumed to show the proposed methodology.

Definition: 3.1
Let $(\omega_1, \gamma_1, \mathcal{N}_1)$ be a N$\mathcal{N}$-HSS. The notation $({\omega_1}^{twc}, \gamma_1, \mathcal{N}_1)$ represents the **top-weak complement of the N$\mathcal{N}$-HSS** and may be represented formally as follows:

$$ {\omega_1}^{twc} = \{[\prec \mathfrak{g},\ \mathbb{F}_l(\mathfrak{g}),\ 1 - I_l(\mathfrak{g}),\ \mathbb{T}_l(\mathfrak{g}) \succ, {v_l}^{twc}(\mathfrak{g})]: \mathfrak{g}\ \in\ \mathbb{G}_H, l \in \mathcal{L}\ \}. $$

In this case, ${v_l}^{twc}$ stands for the top-weak complement of the $\mathcal{N}$-HSS according to definition 2.6.

Example: 3.2
According to the N4-HSS values presented in Example 3.1, the top-weak complement of the N4-HSS $({\omega_1}^{twc}, \gamma_1, 4)$ is specified in Table 3.

Table 3. Top - weak complement of the N4-HSS

$(\boldsymbol{\omega_1}^{twc}, \boldsymbol{\gamma_1}, \mathbf{4})$	$\boldsymbol{\mathcal{F}_1}$	$\boldsymbol{\mathcal{F}_2}$	$\boldsymbol{\mathcal{F}_3}$
$\mathbb{G}_{H_1}$	$(\prec 0.8, 0.9, 0.2 \succ, 3)$	$(\prec 0.7, 0.85, 0.3 \succ, 3)$	$(\prec 0.6, 0.8, 0.4 \succ, 3)$
$\mathbb{G}_{H_2}$	$(\prec 0.45, 0.72, 0.55 \succ, 3)$	$(\prec 0.2, 0.6, 0.8 \succ, 0)$	$(\prec 0.1, 0.55, 0.9 \succ, 0)$
$\mathbb{G}_{H_3}$	$(\prec 0.6, 0.8, 0.4 \succ, 3)$	$(\prec 0.9, 0.95, 0.1 \succ, 3)$	$(\prec 0.35, 0.67, 0.65 \succ, 3)$

Definition: 3.2
Let $(\omega_1, \gamma_1, \mathcal{N}_1)$ be a N$\mathcal{N}$-HSS. The notation $({\omega_1}^{bwc}, \gamma_1, \mathcal{N}_1)$ represents the **bottom-weak complement of the N$\mathcal{N}$-HSS** and may be represented formally as follows:

$$ {\omega_1}^{bwc} = \{[\prec \mathfrak{g},\ \mathbb{F}_l(\mathfrak{g}),\ 1 - I_l(\mathfrak{g}),\ \mathbb{T}_l(\mathfrak{g}) \succ, {v_l}^{bwc}(\mathfrak{g})]: \mathfrak{g}\ \in\ \mathbb{G}_H, l \in \mathcal{L}\ \}. $$

In this case, ${v_l}^{bwc}$ stands for the bottom-weak complement of the $\mathcal{N}$-HSS according to definition 2.7.

Example: 3.3
According to the N4-HSS values presented in Example 3.1, the bottom-weak complement of the N4-HSS $({\omega_1}^{bwc}, \gamma_1, 4)$ is specified in Table 4.

Table 4. Bottom - weak complement of the N4-HSS

$(\boldsymbol{\omega_1}^{bwc}, \boldsymbol{\gamma_1}, \mathbf{4})$	$\boldsymbol{\mathcal{F}_1}$	$\boldsymbol{\mathcal{F}_2}$	$\boldsymbol{\mathcal{F}_3}$
$\mathbb{G}_{H_1}$	$(\prec 0.8, 0.9, 0.2 \succ, 3)$	$(\prec 0.7, 0.85, 0.3 \succ, 0)$	$(\prec 0.6, 0.8, 0.4 \succ, 0)$
$\mathbb{G}_{H_2}$	$(\prec 0.45, 0.72, 0.55 \succ, 0)$	$(\prec 0.2, 0.6, 0.8 \succ, 0)$	$(\prec 0.1, 0.55, 0.9 \succ, 0)$
$\mathbb{G}_{H_3}$	$(\prec 0.6, 0.8, 0.4 \succ, 0)$	$(\prec 0.9, 0.95, 0.1 \succ, 3)$	$(\prec 0.35, 0.67, 0.65 \succ, 0)$

Definition: 3.3
Let $(\omega_1, \gamma_1, \mathcal{N}_1)$ and $(\omega_2, \gamma_2, \mathcal{N}_2)$ be two N$\mathcal{N}$-HSS, where γ_1=$(\chi_1, \mathcal{L}, \mathcal{N}_1)$ and γ_2=$(\chi_2, \mathcal{B}, \mathcal{N}_2)$ are $\mathcal{N}$-HSS. Their **restricted intersection** is represented and defined as follows: $(\omega_1, \gamma_1, \mathcal{N}_1) \cap_{Ri} (\omega_2, \gamma_2, \mathcal{N}_2) = (s, \gamma_1 \cap_{Ri} \gamma_2, \min(\mathcal{N}_1, \mathcal{N}_2))$, where $\gamma_1 \cap_{Ri} \gamma_2 = (\mathbb{S}, \mathcal{L} \cap \mathcal{B}, \min(\mathcal{N}_1, \mathcal{N}_2))$ and

$$ \mathbb{S}(l)(\mathfrak{g}) = [\prec \min(\ \mathbb{T}_{\mathcal{L}}(\mathfrak{g}),\ \mathbb{T}_{\mathcal{B}}(\mathfrak{g})\ ), \max(\ I_{\mathcal{L}}(\mathfrak{g}),\ I_{\mathcal{B}}(\mathfrak{g})\ ), \max(\ \mathbb{F}_{\mathcal{L}}(\mathfrak{g}),\ \mathbb{F}_{\mathcal{B}}(\mathfrak{g})\ ) \succ, \min(\ v_{\mathcal{L}}(\mathfrak{g}),\ v_{\mathcal{B}}(\mathfrak{g}))] $$

for every $l \in \mathcal{L} \cap \mathcal{B}$ and $\mathfrak{g}\ \in\ \mathbb{G}_H$.

Example: 3.4
Using the N4-HSS values from Example 3.1, let $\omega\left({\mathcal{F}_2}^b \times {\mathcal{F}_3}^c\right) = \{Rs.3000 - Rs.4000, Sports\ mode\}$ and define the 4-HSS $(\chi_2, \mathcal{B}, 4)$ as presented in Table 5.

Table 5. Tabular Format of the 4-HSS $(\chi_2, \mathcal{B}, \mathbf{4})$

$(\boldsymbol{\chi_2}, \boldsymbol{\mathcal{B}}, \mathbf{4})$	$\boldsymbol{\mathcal{F}_2}$	$\boldsymbol{\mathcal{F}_3}$
$\mathbb{G}_{H_1}$	1	0
$\mathbb{G}_{H_2}$	2	3
$\mathbb{G}_{H_3}$	1	2

The N4-HSS $(\omega_2, \gamma_2, 4)$ is described in Table 6.

Table 6. Tabular Format of the N4-HSS $(\omega_2, \gamma_2, 4)$

$(\omega_2, \gamma_2, 4)$	$\mathcal{F}_2$	$\mathcal{F}_3$
$\mathbb{G}_{H_1}$	$(\prec 0.45, 0.23, 0.55 \succ, 1)$	$(\prec 0.2, 0.1, 0.8 \succ, 0)$
$\mathbb{G}_{H_2}$	$(\prec 0.7, 0.35, 0.3 \succ, 2)$	$(\prec 0.8, 0.4, 0.2 \succ, 3)$
$\mathbb{G}_{H_3}$	$(\prec 0.3, 0.15, 0.7 \succ, 1)$	$(\prec 0.6, 0.3, 0.4 \succ, 2)$

The restricted intersection of $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$ is defined in table 7.

Table 7. restricted intersection of the $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$

$(\omega_1, \gamma_1, 4) \cap_{Ri} (\omega_2, \gamma_2, 4)$	$\mathcal{F}_2$	$\mathcal{F}_3$
$\mathbb{G}_{H_1}$	$(\prec 0.3, 0.23, 0.7 \succ, 1)$	$(\prec 0.2, 0.2, 0.8 \succ, 0)$
$\mathbb{G}_{H_2}$	$(\prec 0.7, 0.4, 0.3 \succ, 2)$	$(\prec 0.8, 0.45, 0.2 \succ, 3)$
$\mathbb{G}_{H_3}$	$(\prec 0.1, 0.15, 0.9 \succ, 0)$	$(\prec 0.6, 0.33, 0.4 \succ, 2)$

Definition: 3.4
Let $(\omega_1, \gamma_1, \mathcal{N}_1)$ and $(\omega_2, \gamma_2, \mathcal{N}_2)$ be two N$\mathcal{N}$-HSS, where γ_1=$(\chi_1, \mathcal{L}, \mathcal{N}_1)$ and γ_2=$(\chi_2, \mathcal{B}, \mathcal{N}_2)$ are $\mathcal{N}$-HSS. Their **extended intersection** is represented and defined as follows: $(\omega_1, \gamma_1, \mathcal{N}_1) \cap_{Ei} (\omega_2, \gamma_2, \mathcal{N}_2) = (s, \gamma_1 \cap_{Ei} \gamma_2, \min(\mathcal{N}_1, \mathcal{N}_2))$, where $\gamma_1 \cap_{Ei} \gamma_2 = (\mathbb{S}, \mathcal{L} \cup \mathcal{B}, \min(\mathcal{N}_1, \mathcal{N}_2))$ and

$$\mathbb{S}(l)(\mathrm{g}) = [\prec \min(\mathbb{T}_{\mathcal{L}}(\mathrm{g}), \mathbb{T}_{\mathcal{B}}(\mathrm{g})), \max(I_{\mathcal{L}}(\mathrm{g}), I_{\mathcal{B}}(\mathrm{g})), \max(\mathbb{F}_{\mathcal{L}}(\mathrm{g}), \mathbb{F}_{\mathcal{B}}(\mathrm{g})) \succ, \min(v_{\mathcal{L}}(\mathrm{g}), v_{\mathcal{B}}(\mathrm{g}))]$$

for every $l \in \mathcal{L} \cup \mathcal{B}$ and $\mathrm{g} \in \mathbb{G}_H$.

Example: 3.5
Using the N4-HSS values from Example 3.1 & 3.4, the extended intersection of $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$ is specified in Table 8.

Table 8. extended intersection of the $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$

$(\omega_1, \gamma_1, 4) \cap_{Ei}$ $(\omega_2, \gamma_2, 4)$	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$
$\mathbb{G}_{H_1}$	$(\prec 0.2, 0.1, 0.8 \succ, 0)$	$(\prec 0.3, 0.23, 0.7 \succ, 1)$	$(\prec 0.2, 0.2, 0.8 \succ, 0)$
$\mathbb{G}_{H_2}$	$(\prec 0.55, 0.28, 0.45 \succ, 2)$	$(\prec 0.7, 0.4, 0.3 \succ, 2)$	$(\prec 0.8, 0.45, 0.2 \succ, 3)$
$\mathbb{G}_{H_3}$	$(\prec 0.4, 0.2, 0.6 \succ, 1)$	$(\prec 0.1, 0.15, 0.9 \succ, 0)$	$(\prec 0.6, 0.33, 0.4 \succ, 2)$

Definition: 3.5
Let $(\omega_1, \gamma_1, \mathcal{N}_1)$ and $(\omega_2, \gamma_2, \mathcal{N}_2)$ be two N$\mathcal{N}$-HSS, where γ_1=$(\chi_1, \mathcal{L}, \mathcal{N}_1)$ and γ_2=$(\chi_2, \mathcal{B}, \mathcal{N}_2)$ are $\mathcal{N}$-HSS. Their **restricted union** is represented and defined as follows: $(\omega_1, \gamma_1, \mathcal{N}_1) \cup_{Ru} (\omega_2, \gamma_2, \mathcal{N}_2) = (s, \gamma_1 \cup_{Ru} \gamma_2, \max(\mathcal{N}_1, \mathcal{N}_2))$, where $\gamma_1 \cup_{Ru} \gamma_2 = (\mathbb{S}, \mathcal{L} \cap \mathcal{B}, \max(\mathcal{N}_1, \mathcal{N}_2))$ and

$$\mathbb{S}(l)(\mathrm{g}) = [\prec \max(\mathbb{T}_{\mathcal{L}}(\mathrm{g}), \mathbb{T}_{\mathcal{B}}(\mathrm{g})), \min(I_{\mathcal{L}}(\mathrm{g}), I_{\mathcal{B}}(\mathrm{g})), \min(\mathbb{F}_{\mathcal{L}}(\mathrm{g}), \mathbb{F}_{\mathcal{B}}(\mathrm{g})) \succ, \max(v_{\mathcal{L}}(\mathrm{g}), v_{\mathcal{B}}(\mathrm{g}))]$$

for every $l \in \mathcal{L} \cap \mathcal{B}$ and $\mathrm{g} \in \mathbb{G}_H$.

Example: 3.6
Using the N4-HSS values from Example 3.1 & 3.4, the restricted union of $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$ is specified in Table 9.

Table 9. restricted union of the $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$

$(\boldsymbol{\omega_1}, \boldsymbol{\gamma_1}, \mathbf{4}) \cup_{Ru} (\boldsymbol{\omega_2}, \boldsymbol{\gamma_2}, \mathbf{4})$	$\mathcal{F}_2$	$\mathcal{F}_3$
$\mathbb{G}_{H_1}$	$(\prec 0.45, 0.15, 0.55 \succ, 1)$	$(\prec 0.4, 0.1, 0.6 \succ, 1)$
$\mathbb{G}_{H_2}$	$(\prec 0.8, 0.35, 0.2 \succ, 3)$	$(\prec 0.9, 0.4, 0.1 \succ, 3)$
$\mathbb{G}_{H_3}$	$(\prec 0.3, 0.05, 0.7 \succ, 1)$	$(\prec 0.65, 0.3, 0.35 \succ, 2)$

Definition: 3.6
Let $(\omega_1, \gamma_1, \mathcal{N}_1)$ and $(\omega_2, \gamma_2, \mathcal{N}_2)$ be two N$\mathcal{N}$-HSS, where $\gamma_1 = (\chi_1, \mathcal{L}, \mathcal{N}_1)$ and $\gamma_2 = (\chi_2, \mathcal{B}, \mathcal{N}_2)$ are $\mathcal{N}$-HSS. Their **extended union** is represented and defined as follows: $(\omega_1, \gamma_1, \mathcal{N}_1) \cup_{Eu} (\omega_2, \gamma_2, \mathcal{N}_2) = (s, \gamma_1 \cup_{Eu} \gamma_2, \max(\mathcal{N}_1, \mathcal{N}_2))$, where $\gamma_1 \cup_{Eu} \gamma_2 = (\mathbb{S}, \mathcal{L} \cup \mathcal{B}, \max(\mathcal{N}_1, \mathcal{N}_2))$ and

$$\mathbb{S}(l)(\mathfrak{g}) = [\prec \max(\mathbb{T}_{\mathcal{L}}(\mathfrak{g}), \mathbb{T}_{\mathcal{B}}(\mathfrak{g})), \min(I_{\mathcal{L}}(\mathfrak{g}), I_{\mathcal{B}}(\mathfrak{g})), \min(\mathbb{F}_{\mathcal{L}}(\mathfrak{g}), \mathbb{F}_{\mathcal{B}}(\mathfrak{g})) \succ, \max(v_{\mathcal{L}}(\mathfrak{g}), v_{\mathcal{B}}(\mathfrak{g}))]$$

for every $l \in \mathcal{L} \cup \mathcal{B}$ and $\mathfrak{g} \in \mathbb{G}_H$.

Example: 3.7
Using the N4-HSS values from Example 3.1 & 3.4, the extended union of $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$ is specified in Table 10.

Table 10. extended union of the $(\omega_1, \gamma_1, 4)$ and $(\omega_2, \gamma_2, 4)$

$(\boldsymbol{\omega_1}, \boldsymbol{\gamma_1}, \mathbf{4}) \cup_{Eu} (\boldsymbol{\omega_2}, \boldsymbol{\gamma_2}, \mathbf{4})$	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$
$\mathbb{G}_{H_1}$	$(\prec 0.2, 0.1, 0.8 \succ, 0)$	$(\prec 0.45, 0.15, 0.55 \succ, 1)$	$(\prec 0.4, 0.1, 0.6 \succ, 1)$
$\mathbb{G}_{H_2}$	$(\prec 0.55, 0.28, 0.45 \succ, 2)$	$(\prec 0.8, 0.35, 0.2 \succ, 2)$	$(\prec 0.9, 0.4, 0.1 \succ, 3)$
$\mathbb{G}_{H_3}$	$(\prec 0.4, 0.2, 0.6 \succ, 1)$	$(\prec 0.3, 0.05, 0.7 \succ, 1)$	$(\prec 0.65, 0.3, 0.35 \succ, 2)$

Methodology

In this part, a novel method is presented based on weighted choice values in the framework of N$\mathcal{N}$-HSS, and an application is given to show its applicability. The new method is developed and improved from the previous decision-making framework introduced for hypersoft sets, as presented in [20].

To verify the reliability and validity of the suggested MCDM framework, a panel of experts [20] with 5 to 6 years of experience was taken into account. Their assessments were gathered and combined to assess the significance of the criteria and analyze the options.

The methodology's specific steps are described in the following.

Step: 1 Input the universal set as $\mathbb{G}_H = \{\mathfrak{g}_i, i = 1,2, \ldots, t\}$ and the set of attributes as $\mathcal{L} = \{l_j, j = 1,2, \ldots, u\}$.

Step: 2 Enter the N$\mathcal{N}$-HSS and the weight $\Omega_j (0 \le \Omega_j \le 1)$ that has been allocated to each parameter.

$$(\omega, \gamma, \mathcal{N}) = \left\{\left[\prec \mathfrak{g}_i, \mathbb{T}_{l_j}(\mathfrak{g}_i), I_{l_j}(\mathfrak{g}_i), \mathbb{F}_{l_j}(\mathfrak{g}_i) \succ, v_{l_j}(\mathfrak{g}_i)\right], \mathfrak{g}_i \in \mathbb{G}_H, \mathbb{T}_{l_j}, I_{l_j}, \mathbb{F}_{l_j} \in [0,1], v_{l_j} \in \mathbb{V}\right\}$$

Step: 3 Compute $\lambda_i = \left(\left|\sum_{j=1}^{u} \frac{\mathbb{T}_{l_j} - I_{l_j} - \mathbb{F}_{l_j}}{3}\right|, \sum_{j=1}^{u} v_{l_j}\right) \Omega_j$ for every $\mathfrak{g}_i \in \mathbb{G}_H$.

Step: 4 Determine the index κ such that $\lambda_\kappa{}^{\Omega} = \max \lambda_i{}^{\Omega}$.

Step: 5 The output is any $\mathfrak{g}_\kappa$ satisfying the condition $\lambda_\kappa{}^{\Omega} = \max \lambda_i{}^{\Omega}$ maybe selected.

Figure 1 illustrates the diagram of the suggested approach.

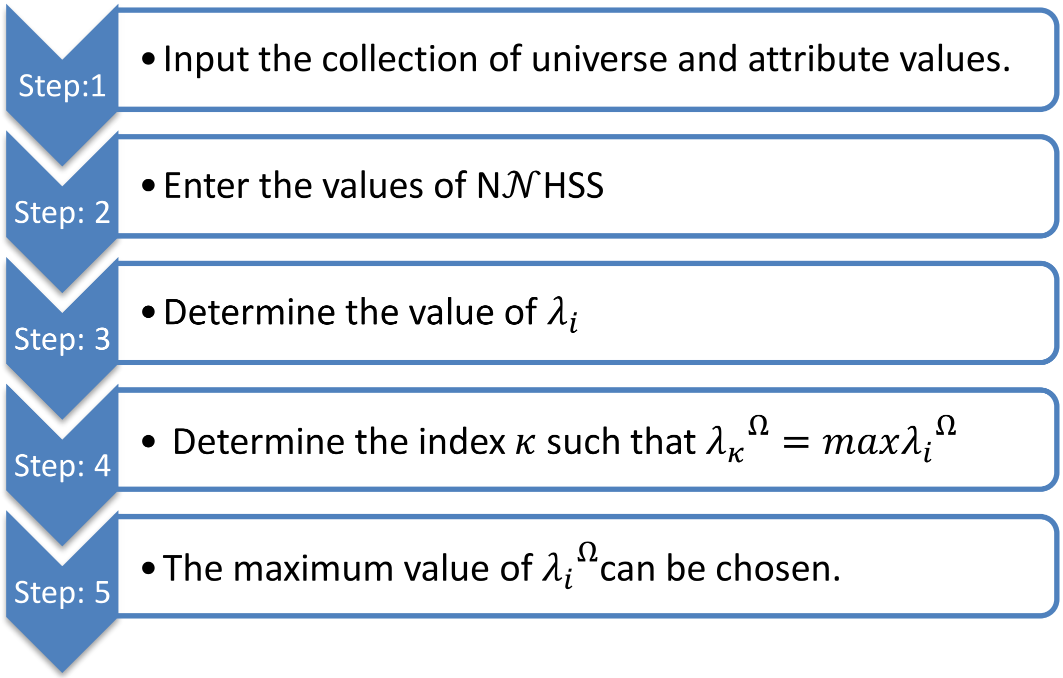

Figure 1. Diagram of the proposed approach.

Case Study

Our everyday lives depend heavily on mathematics, frequently without our knowledge. Math helps us solve problems and comprehend the world around us, from effectively managing finances and shopping to cooking, traveling, and making decisions. It is an indispensable talent for daily life since it fosters critical thinking and makes it possible for us to manage practical activities effectively. The goal of this study is to determine which area of daily life most frequently uses mathematics to make decisions. Decision-makers consider the following fields of mathematics to be the most commonly used, and their level of importance is explained.

Budgeting and Investment

In financial planning and investing, mathematics is essential because it enables people and organizations to make well-informed, calculated financial decisions. It is employed in the creation of budgets, interest rate computation, risk analysis, financial outcome forecasting, and investment return optimization. Mathematical tools like percentages, ratios, probability, and compound interest calculations are crucial for making wise financial decisions, whether they are used for managing everyday costs, retirement planning, or assessing investment opportunities.

Shopping and Consumption

People use mathematics to make informed selections when they shop and consume. Math helps customers make the most of their money in a variety of ways, from comparing costs and calculating discounts to managing budgets and comprehending taxes. Basic math and percentage computations help consumers make wise decisions and efficiently manage their expenditure, whether they are purchasing expensive goods or everyday necessities.

Construction/ Architecture

In the domains of architecture and building, mathematics is essential to guaranteeing precision, security, and usefulness. Structure design, dimension calculation, material estimation, and layout planning all include the use of concepts like geometry, algebra, and measurement. From preliminary plans to finished construction, mathematics aids in converting concepts into sturdy, well-built structures.

Business and Economics

In business and economics, mathematics is essential because it gives us the means to analyse data, optimize processes, and make strategic choices. Calculus, algebra, and statistics are among the concepts that aid in managing finances, predicting economic growth, assessing market trends, and calculating profits and losses. In order to increase productivity and make wise decisions in a competitive market, economists and businesses can use mathematical models.

Navigation/ Travel

Planning effective routes, estimating trip times, and calculating distances are all made possible by mathematics, which is essential to navigation and travel. Directions, speed, and fuel consumption are determined using topics like geometry, trigonometry, and arithmetic. Math guarantees correct and safe travel, making navigation dependable and effective whether on land, at sea, or in the air.

Banking

Mathematics is essential in banking for managing transactions, calculating interest, and assessing risks. It helps banks determine loan repayments, savings growth, and currency conversions. By applying mathematical principles such as percentages, ratios, and financial formulas, banking operations become accurate and efficient, supporting both customers and financial institutions.

Numerical Example

To determine the real-life domain where mathematics is most commonly utilized in daily decision-making, six different domains are taken into account as assessment standards by the decision-makers:

CA_1- Navigation / Travel,

CA_2 - Shopping and Consumption,

CA_3 - Construction / Architecture,

CA_4 - Business and Economics,

CA_5 - Budgeting and Investment,

CA_6 - Banking.

These categories reflect common areas where mathematical principles and approaches help practical decision-making processes.

Every chosen domain is examined through five key attributes that define the level and type of mathematical application: B_1- Frequency of use, B_2 - Math domain, B_3 - Decision-making Involvement, B_4 - Complexity of Tasks, B_5 - Accuracy Requirement.

These characteristics offer an organized system for assessing and contrasting the significance of mathematics in various real-world situations.

Additionally, these attributes have transformed into several distinct sub-attributes that are grouped under the following categories:

${B_1}^a$- Frequency of use = {Daily, Weekly},

${B_2}^b$ - Math domain = {Arithmetic, Algebra, Geometry, Statistics},

${B_3}^c$ - Decision-making Involvement = {Low, Medium, High},

${B_4}^d$ - Complexity of Tasks = {Simple, Moderate, Complex},

${B_5}^e$ - Accuracy Requirement = {Low, Medium, High}.

The values of the membership function will be used to calculate the grading scores, as indicated in Table 11.

Table 11. Grade scale levels

Membership value	Grading scores
$0 \leq \mathbb{T}_l(\mathfrak{g}) < 0.2$	$v_l = 0$
$0.2 \leq \mathbb{T}_l(\mathfrak{g}) < 0.45$	$v_l = 1$
$0.45 \leq \mathbb{T}_l(\mathfrak{g}) < 0.55$	$v_l = 2$
$0.55 \leq \mathbb{T}_l(\mathfrak{g}) < 0.65$	$v_l = 3$
$0.65 \leq \mathbb{T}_l(\mathfrak{g}) < 0.75$	$v_l = 4$
$0.75 \leq \mathbb{T}_l(\mathfrak{g}) < 0.85$	$v_l = 5$
$0.85 \leq \mathbb{T}_l(\mathfrak{g}) < 1.0$	$v_l = 6$

Assume that $\omega\left(B_1{}^a \times B_2{}^b \times B_3{}^c \times B_4{}^d \times B_5{}^e\right) = \omega(Daily, Algebra, Medium, Complex, High)$ denotes the combination required by the researchers.

Step: 1 To illustrate everyday situations where mathematics is most frequently employed, the universal set is defined as $\mathbb{G}_H = \{CA_1, CA_2, CA_3, CA_4, CA_5, CA_6\}$. The suitable collection of attributes is represented as $\mathcal{L} = \{B_1, B_2, B_3, B_4, B_5\}$.

Step: 2 The decision-makers use N$\mathcal{N}$-HSS numbers to communicate their opinions based on the researchers requirements $\omega\left(B_1{}^a \times B_2{}^b \times B_3{}^c \times B_4{}^d \times B_5{}^e\right) = \omega(Daily, Algebra, Medium, Complex, High)$ as indicated in Table 12 below:

Table 12. Tabular representation of N7-HSS

$(\omega, \gamma, 7)$	B_1	B_2	B_3	B_4	B_5
CA_1	$(< 0.35, 0.0, 0.45 >, 1)$	$(< 0.1, 0.05, 0.6 >, 0)$	$(< 0.25, 0.1, 0.5 >, 1)$	$(< 0.15, 0.03, 0.4 >, 0)$	$(< 0.8, 0.0, 0.2 >, 5)$
CA_2	$(< 0.75, 0.33, 0.25 >, 5)$	$(< 0.5, 0.45, 0.5 >, 2)$	$(< 0.7, 0.35, 0.3 >, 4)$	$(< 0.4, 0.53, 0.6 >, 1)$	$(< 0.35, 0.13, 0.65 >, 1)$
CA_3	$(< 0.6, 0.2, 0.4 >, 3)$	$(< 0.35, 0.03, 0.6 >, 1)$	$(< 0.3, 0.05, 0.7 >, 1)$	$(< 0.35, 0.1, 0.6 >, 1)$	$(< 0.4, 0.06, 0.6 >, 1)$
CA_4	$(< 0.55, 0.73, 0.45 >, 3)$	$(< 0.7, 0.85, 0.3 >, 4)$	$(< 0.6, 0.85, 0.4 >, 3)$	$(< 0.65, 0.9, 0.35 >, 4)$	$(< 0.65, 0.9, 0.35 >, 4)$
CA_5	$(< 0.6, 0.55, 0.4 >, 3)$	$(< 0.6, 0.55, 0.4 >, 3)$	$(< 0.55, 0.53, 0.45 >, 3)$	$(< 0.55, 0.53, 0.45 >, 3)$	$(< 0.35, 0.33, 0.65 >, 1)$
CA_6	$(< 0.4, 0.65, 0.6 >, 1)$	$(< 0.5, 0.75, 0.5 >, 2)$	$(< 0.6, 0.3, 0.4 >, 3)$	$(< 0.6, 0.3, 0.4 >, 3)$	$(< 0.45, 0.65, 0.55 >, 2)$

Step: 3 To compute $\lambda_i = \left(\left|\sum_{j=1}^{u} \frac{\mathbb{T}_{l_j} - I_{l_j} - \mathbb{F}_{l_j}}{3}\right|, \sum_{j=1}^{u} v_{l_j}\right)$ for every $\mathfrak{g}_i \in \mathbb{G}_H$, as shown in Table 13.

Table 13. The values of λ_i for N7-HSS

$(\omega, \gamma, 7)$	B_1	B_2	B_3	B_4	B_5
CA_1	(-0.033, 1)	(-0.183, 0)	(-0.1167, 1)	(-0.093, 0)	(0.2, 5)
CA_2	(0.057,5)	(-0.15, 2)	(0.0167, 4)	(-0.243, 1)	(-0.43, 1)
CA_3	(0.06, 3)	(-0.09,1)	(-0.15, 1)	(-0.1167, 1)	(-0.26,1)
CA_4	(-0.21, 3)	(-0.15, 4)	(-0.2167, 3)	(-0.2, 4)	(-0.6, 4)
CA_5	(-0.117, 3)	(-0.117, 3)	(-0.143, 3)	(-0.14, 3)	(-0.63, 1)
CA_6	(-0.283, 1)	(-0.25, 2)	(-0.03,3)	(-0.03, 3)	(-0.3, 2)

In table 14, let $\Omega_j (0 \leq \Omega_j \leq 1, \sum \Omega_j = 1)$ represent the weight given to the parameters (attributes) $\boldsymbol{B_j}$ and the weights are, Ω_1 = 0.15, Ω_2= 0.1, Ω_3= 0.2, Ω_4= 0.4, Ω_5= 0.09.

Table 14. The weight choice values of λ_i for N7-HSS

$(\omega, \gamma, 7)$	B_1	B_2	B_3	B_4	B_5
CA_1	(-0.005, 0.15)	(-0.0183, 0)	(-0.023, 0.2)	(-0.037, 0)	(0.018, 0.45)
CA_2	(0.0085, 0.75)	(-0.015, 0.2)	(0.003, 0.8)	(-0.097, 0.4)	(-0.0387, 0.09)
CA_3	(0.009, 0.45)	(-0.009, 0.1)	(-0.03, 0.2)	(-0.05, 0.4)	(-0.0234, 0.09)
CA_4	(-0.0315, 0.45)	(-0.015, 0.4)	(-0.043, 0.6)	(-0.08, 1.6)	(-0.05, 0.36)
CA_5	(-0.0175, 0.45)	(-0.012, 0.3)	(-0.03, 0.6)	(-0.06, 1.2)	(-0.057, 0.09)
CA_6	(-0.043, 0.15)	(-0.025, 0.2)	(-0.007, 0.6)	(-0.013, 1.2)	(-0.027, 0.18)

Step: 4 To determine the index κ such that $\lambda_\kappa{}^\Omega = \max \lambda_i{}^\Omega$. Table 15 shows the computation for all values of ${\lambda_i}^{\Omega}$ for every $\mathfrak{g}_i \in \mathbb{G}_H$.

Table 15. Values of ${\lambda_i}^{\Omega}$ for every $\mathfrak{g}_i \in \mathbb{G}_H$.

$(\boldsymbol{\omega}, \boldsymbol{\gamma}, \mathbf{7})$	${\boldsymbol{\lambda}_i}^{\boldsymbol{\Omega}}$
$\boldsymbol{CA_1}$	(0.07, 0.8)
$\boldsymbol{CA_2}$	(0.14, 2.24)
$\boldsymbol{CA_3}$	(0.1, 1.24)
$\boldsymbol{CA_4}$	(0.22, 3.41)
$\boldsymbol{CA_5}$	(0.17, 2.64)
$\boldsymbol{CA_6}$	(0.11, 2.33)

Step: 5 Table 15 shows that CA_4 has the highest ranking of all the categories, indicating that CA_4 - business and economics - most frequently uses mathematics for decision-making.

Results and Discussion

Modern science and technology are based on mathematics, which provide the basic concepts and tools needed to recognize patterns, solve issues, and make logical outcomes. According to our suggested weight choice values in N$\mathcal{N}$-HSS method, the field of business and economics most commonly uses mathematical decision-making. This usage, based on the researchers' needs, is characterized by its everyday frequency, high decision-making complexity, a strong emphasis on accuracy, and algebra as the principal mathematical domain. All of these factors suggest business and economics as the field where mathematics is used the most. We also found the same result when we compared our suggested method with the current approach of Tripathy, B. K., et al. [5]. Table 16 shows a comparison of the suggested and existing methods.

Table 16. Method comparison table

Methods	Ranking alternatives	Optimum solution
Methodology (our proposed)	$CA_4 > CA_5 > CA_6 > CA_2 > CA_3 > CA_1$	CA_4
Tripathy, B. K. et al., [5]	$CA_4 > CA_5 > CA_6 > CA_2 > CA_3 > CA_1$	CA_4

Sensitivity Analysis

To examine the robustness of the proposed decision-making approach, a sensitivity analysis was conducted by altering the criterion weights (Ω_j) throughout the interval [0,1] while preserving the normalization constraint $\sum \Omega_j$=1. Specifically, the task complexity criterion's (B_4) weight was lowered, and the remaining weights were modified proportionately. The results show that these weight modifications have no effect on the alternatives' ranking. This shows that the suggested method is stable and that slight variations in the weights of each criterion do not significantly affect the final choice. As a result, in real-world decision-making applications, the suggested model demonstrates excellent durability and reliability.

Advantages of the Proposed Method

The suggested hybrid framework, $\mathcal{N}$-Hypersoft sets within neutrosophic fuzzy environments, offers a thorough and adaptable method for uncertain knowledge modeling and analysis. The suggested framework, in contrast to traditional approaches, can concurrently handle binary, non-binary, uncertain, inconsistent, and incomplete data in multi-sub-attribute decision structures. The model offers a more sophisticated depiction of indeterminacy by integrating the neutrosophic viewpoint, allowing for a more thorough examination of ambiguous data and intricate rating-based assessment systems.

A significant benefit of the suggested method is its effectiveness in overcoming the shortcomings of current uncertainty management strategies by providing improved flexibility, adaptability, and information-processing abilities. The framework is especially appropriate for intricate applications that include multi-attribute decision-making, assessment systems, and data-processing settings where uncertainty and inconsistency are fundamental. Additionally, the suggested weighted choice value approach offers an effective and practical way to rank and choose options, making it very relevant to actual decision-making challenges. The combination of $\mathcal{N}$-hypersoft structures with neutrosophic fuzzy theory creates a robust modeling tool that enhances decision reliability, captures underlying uncertainty more effectively, and facilitates more precise and logical decision results.

Conclusions

The main objective of this study is to introduce the concept of neutrosophic $\mathcal{N}$-hypersoft sets. The research also explores a number of set-theoretic operations, including the restricted and extended union and intersection of neutrosophic $\mathcal{N}$- hypersoft sets, as well as the top-weak and bottom-weak complements accompanied by relevant examples. Furthermore, a decision-making procedure based on neutrosophic $\mathcal{N}$- hypersoft sets weighted choice values is introduced, and its importance is demonstrated with an example. To evaluate their efficacy, the suggested outcomes are examined and compared with existing strategies.

The relevance of the suggested framework in real-life scenarios like medical diagnosis, investment choice, risk evaluation, and resource management highlights its practical importance. Comparative studies using current techniques verify its improved adaptability, dependability, and ability to handle intricate decision-making situations. Therefore, neutrosophic $\mathcal{N}$-hypersoft sets act as a valuable mathematical framework for facilitating precise and logical decision-making in uncertain situations, with upcoming research aimed at expanding topics like bipolar neutrosophic $\mathcal{N}$-hypersoft sets and their roles in wider decision-support frameworks.

Conflicts of Interest

The writers have declared themselves to have no competing interests.

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