

# Mixed Convective Flow of Williamson Ternary Hybrid Ferrofluid Through a Moving Vertical Flat Plate

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**Abstract** Present paper studies the characteristics of an upgraded fluid called Williamson ternary hybrid ferrofluid. This fluid comprises three types of nanoparticles which are magnetite, gold and aluminium oxide in a Williamson based fluid in hopes to improve the fluidity and heat transfer of based fluid. Thus, the objective of this research is to understand the capabilities of this upgraded fluid and to determine whether it performs better than the less nanoparticles hybrid fluid. The blood is taken as a based fluid to integrate the pseudoplastic behaviour of Williamson fluid. The physical model developed is interpreted into non-linear partial differential equations then transformed into ordinary differential equations using similarity transformations. Using Runge-Kutta-Fehlberg (RKF45) method, the transformed equations then coded in Maple software. Parameter used to study the behaviour of the fluid are the nanoparticles volume fraction, the magnetic parameter, the moving plate parameter and buoyancy parameter. Comparison with different types of ferroparticle volume fractions are also included in this research. In summary, the Williamson ternary hybrid ferrofluid demonstrates a 2.81% enhancement in fluidity relative to the Williamson hybrid ferrofluid, with both showing comparable heat transfer characteristics when evaluated using the same value moving plate parameter. Magnetic parameter as predicted do reduced the thermal and velocity boundary layer. Buoyancy parameter also showed similar result with magnetic parameter.

**Keywords:** Williamson ternary hybrid ferrofluid, blood based, vertical moving plate, Williamson fluid model.

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## Introduction

Fluids with inconsistency viscosity are called non-Newtonian fluid. Non-Newtonian fluid can be characterized by their shear rate characteristics which are either shear thinning (pseudoplastic) or shear thickening (dilatant). Both characteristics demonstrate unique behaviours relative to Newtonian fluids in thermal transfer processes. Both characteristics has its own advantages and application such as heat exchangers, reactor, natural polymers and damping applications [1]. There are many fluid models produce by past researchers to mathematically study the rheology of non-Newtonian fluid, but this study will focus on Williamson fluid model to understand the fluid pseudoplastic behaviour. Latest research that considered blood and Williamson fluid model was done by Thenmozhi *et al.* [2] where they claimed the novelty of the research is in the application of blood infused Williamson hybrid ferrofluid in real time

applications of magnetic hyperthermia for cancer treatment. Rosli *et al.* [3-5] studied on the blood-based Williamson hybrid ferrofluid under different parameter and plate conditions. Their research novelty was using Williamson fluid model and copper nanoparticles to help with the heat transfer performance under different fluid parameters and plate conditions. They concluded that the presence of copper does improve the heat transfer performance of blood-based ferrofluid.

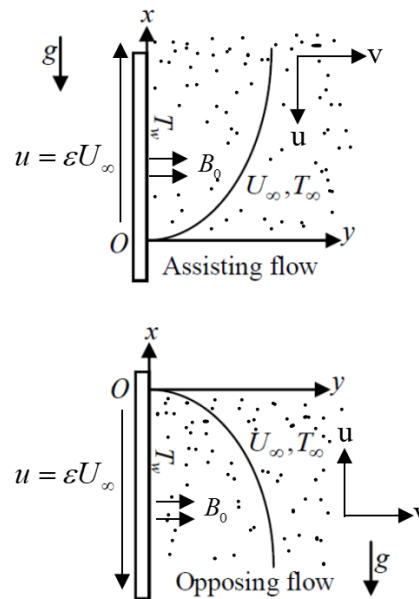
As claimed by Yasin *et al.* [6], the surface area is important in determining the fluid flow characteristics. They provide examples where the best heat exchange is when the fins is placed vertically compared to the horizontal surface. Ramachandran *et al.* [7] pioneered the researched for the heat exchange on a vertical surface problem. They reported that both the local Nusselt number and the wall shear stress exhibit an increasing trend with respect to the buoyancy parameter. Recent research on this problem was done Pradhan *et al.* [8] where they studied the free convective time-dependent MHD flow over a vertical porous flat plate under the effect of heat sink and chemical reactions while applying Laplace transformation technique. They claimed that buoyant forces enhance the velocity profile, emphasising surface cooling. The characteristics of water based hybrid ferrofluid ( $Fe_3O_4 - CoFe_2O_4 / H_2O$ ) with the effects of chemical reaction and slip conditions analysis was done Zainodin *et al.* [9]. Using bvp4c numerical approach they found that the hybrid ferrofluid heat transfer rate do affect by different order of chemical reaction. Next, Yasin *et al.* [6] studies the characteristics of mixed convection stagnation point flow water based ferrofluid over a vertical flat plate. Their findings using Keller-box method was ferrofluid volume fraction do influence the ferrofluid velocity, reduced skin friction coefficient and reduced Nusselt number.

Ternary hybrid nanofluid is a relatively new concepts in the mathematically theoretical research industries. Combination of three nanoparticles rather than two or one in a single fluid is expected to provide improvements in fluidity and heat transfer performance. Such research investigate this new type of fluid is from Ullah *et al.* [10], where they studied ternary hybrid, hybrid and nanofluid containing copper, silver and alumina nanoparticles under the effects of thermal radiation, heat source, joule heating and constant distance. Using the Karman similarity transformation and bvp4c numerical approach, they reflected that the ternary hybrid ferrofluid provide 28% heat transfer performance compared to the hybrid fluid and nanofluid. Using gold, graphene, and copper in a carboxymethyl cellulose (CMC)-water base fluid, Mahanta and Sharma [11] investigate the fluid characteristics of time dependent flow between two horizontal plates using bvp4c code to produce the graphical results. They deduced that fluid ternary nanofluid produce larger Nusselt number that hybrid nanofluid when radiative heat transport amplifies. Response Surface Methodology (RSM) was employed by Abu Bakar *et al.* [12] to maximize the heat transfer rate of ternary hybrid ferrofluid flow past a shrinking surface under the influence of magnetic dipole and velocity slip. They considered iron oxide, cobalt ferrite, and copper for the ternary nanoparticles and concluded provide analysis where ternary hybrid nanoparticles outshined the conventional ferrofluid and hybrid ferrofluid. Based on this past research, it can be concluded that the ternary hybrid nanofluid do provide better fluidity and heat transfer performance than the current popular hybrid nanofluid.

Enlightenment from this past research, there is gap that can be explored in ternary hybrid nanofluid studies in terms of fluid model, plate condition and type of nanoparticles. Therefore, present study investigates the flow of Williamson ternary hybrid ferrofluid consisting of magnetite, gold and aluminium nanoparticles passes through a vertical moving plate. The based fluid for ferrofluid is blood as it represents the pseudoplastic characteristics of Williamson fluid. The incorporation of gold and aluminium oxide nanoparticles into the Williamson ferrofluid is anticipated to enhance its fluidity and heat transfer performance relative to the conventional hybrid ferrofluid under the influence of moving vertical plate.

## Mathematical Formulation

This study considers incompressible free stream,  $U_\infty$ , and ambient temperature,  $T_\infty$ , of Williamson ternary-hybrid ferrofluid flowing through a moving vertical flat plate,  $u = \varepsilon U_\infty$ . The surface temperature of the vertical flat plate is represented by  $T_w$  while  $B_0$ ,  $g$ ,  $\beta$  are the magnetic field strength, gravitational acceleration, and thermal expansion coefficient respectively. Since the plate is in vertical position, the cartesian coordinate  $x$  axis will become vertical while  $y$  axis is orthogonal to it. The model of this study is depicted in Figure 1.



**Figure 1.** Two-dimensional physical model and coordinate system of incompressible Williamson ternary-hybrid ferrofluid flowing through the vertical flat plate

From Figure 1, the governing equations for 2-dimentional steady convective boundary layer flow in a Williamson ternary hybrid ferrofluid can be written as [3, 13-17]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{Thnf} \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \nu_{Thnf} \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} + \frac{(\rho\beta)_{Thnf}}{\rho_{Thnf}} g(T - T_\infty) - \frac{\sigma B_0^2(x)}{\rho_{Thnf}} (u - U_\infty), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{Thnf}}{(\rho C_p)_{Thnf}} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

with boundary conditions:

$$\begin{aligned} u = \varepsilon U_\infty, v = 0, T = T_w \text{ at } y = 0 \\ u \rightarrow U_\infty, T \rightarrow T_\infty, \text{ as } y \rightarrow \infty. \end{aligned} \tag{4}$$

The velocity components along the  $x$  and  $y$  axes are designated as  $u$  and  $v$  with  $\varepsilon$ , the define as moving plate velocity. Equations (1) - (3) have many dependent and independent variables. The similarity transformation can be applied to simplify the equations, thus the similarity variable considered are as follows [18, 19]:

$$\eta = \left( \frac{U_\infty}{\nu x} \right)^{1/2} y, \psi = (U_\infty \nu x)^{1/2} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{5}$$

Referring to Equation (5)  $\eta$ ,  $\psi$  and  $\theta$  are define as non-dimensional variable, dimensional stream function and temperature. Equation (5) satisfies the continuity Equation (1) by definition:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{6}$$

substitute the similarity variables Equation (5) and (6) into governing Equation (2) and (3) gives the following transformed ordinary differential equations:

$$\frac{\nu_{Thnf}}{\nu_f} (f''' + \lambda f'' f''') + \frac{1}{2} f f'''' + \frac{(\rho\beta)_{Thnf}}{\rho_{Thnf} \beta_f} \Omega \theta - M (f' - 1) = 0, \tag{7}$$

$$\frac{1}{Pr} \frac{k_{Thnf}}{k_f} \frac{(\rho C_p)_f}{(\rho C_p)_{Thnf}} \theta'' + \frac{1}{2} f \theta' = 0. \tag{8}$$

In order to solve the similarity solution of Equation (7) and (8), parameters  $\lambda$  and  $\Omega$  must be constants with no functions of  $x$  [20-22], therefore it is assumed that:

$$\Gamma = \alpha_1 \sqrt{x} \text{ and } \beta_f = \frac{\alpha_2}{x} \tag{9}$$

Next, boundary conditions (4) become:

$$\begin{aligned} f(0) = 0, \quad f'(0) = \varepsilon, \quad \theta(0) = 1, \\ f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty. \end{aligned} \tag{10}$$

Where  $\lambda = \alpha_1 \sqrt{\frac{2U_\infty^3}{\nu_f}}$  is the Williamson parameter,  $\Omega = \frac{Gr}{Re^2}$  as the buoyancy parameter, with Grashoff number,  $Gr = \frac{g\beta_f(T_w - T_\infty)x^3}{\nu_f^2}$ , Reynolds number,  $Re = \frac{U_\infty x}{\nu_f}$ , magnetic parameter,  $M = \frac{\sigma B_0^2}{\alpha_2 \rho_{Thnf}}$  and  $Pr = \frac{\nu_f (\rho C_p)_f}{k_f}$  as the Prandtl number. Other quantities related to ternary-hybrid nanofluid are shown in

Table 1 [13, 15, 23]:

**Table 1.** Thermophysical properties of fluid and particles

Thermophysical Properties	Ternary-Hybrid Nanofluid
Density $(\rho)_{Thnf}$	$(1 - \phi_3) \left[ (1 - \phi_2) \left[ (1 - \phi_1) \rho_f + \phi_1 \rho_{s1} \right] + \phi_2 \rho_{s2} \right] + \phi_3 \rho_{s3}$
Heat capacity $(\rho C_p)_{Thnf}$	$(1 - \phi_3) \left[ (1 - \phi_2) \left[ (1 - \phi_1) (\rho C_p)_f + \phi_1 (\rho C_p)_{s1} \right] + \phi_2 (\rho C_p)_{s2} \right] + \phi_3 (\rho C_p)_{s3}$
Dynamic viscosity $(\mu)_{Thnf}$	$\frac{\mu_f}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} (1 - \phi_3)^{2.5}}$
Thermal conductivity $(k)_{Thnf}$	$k_{Thnf} = k_{hnf} \left( \frac{k_{s3} + 2k_{hnf} - 2\phi_{s3} (k_{hnf} - k_{s3})}{k_{s3} + 2k_{hnf} + \phi_{s3} (k_{hnf} - k_{s3})} \right),$ $k_{hnf} = k_{nf} \left( \frac{k_{s2} + 2k_{nf} - 2\phi_{s2} (k_{nf} - k_{s2})}{k_{s2} + 2k_{nf} + \phi_{s2} (k_{nf} - k_{s2})} \right),$ and $k_{nf} = k_f \left( \frac{k_{s1} + 2k_f - 2\phi_{s1} (k_f - k_{s1})}{k_{s1} + 2k_f + \phi_{s1} (k_f - k_{s1})} \right).$
Thermal expansion coefficient $(\rho\beta)_{Thnf}$	$(1 - \phi_3) \left[ (1 - \phi_2) \left[ (1 - \phi_1) (\rho\beta)_f + \phi_1 (\rho\beta)_{s1} \right] + \phi_2 (\rho\beta)_{s2} \right] + \phi_3 (\rho\beta)_{s3}$

In Table 1,  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  represents the different type of nanoparticles volume fraction that will used for this research. The subscripts  $f$ ,  $nf$ ,  $hnf$  and  $Thnf$  defined as base fluid, nanofluid, hybrid nanofluid, and ternary hybrid nanofluid respectively. The physical quantities of interest are the skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_x$  [3, 5, 15, 23]:

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \tag{11}$$

where the wall shear stress  $\tau_w$  and heat flux from the surface  $q_w$  are given by

$$\tau_w = \mu_{Thnf} \left( \frac{\partial u}{\partial y} \right), \text{ and } q_w = -k_{Thnf} \left( \frac{\partial t}{\partial y} \right) \tag{12}$$

Substituting Equation (5) and (12) into Equation (11), the following equations are obtained:

$$C_f Re_x^{1/2} = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}} \left( f''(0) + \frac{\lambda}{2} f'^2(0) \right) \text{ and} \tag{13}$$

$$Nu_x Re_x^{-1/2} = -\frac{k_{Thnf}}{k_f} \theta'(0)$$

## Results and Discussion

To produce the results, the non-linear ordinary differential Equation (7) and (8) with boundary conditions (10) are solved using Runge-Kutta-Fehlberg (RFK45) method in Maples software. The value for boundary layer thickness,  $\eta_\infty$ , employed for this research is set to 7 as it provides the optimal results. Taking blood into consideration as a based fluid, Prandtl number of 21 is the default value throughout the parameter tested following research done by Rosli *et al.* [3-5], Chu *et al.* [24] and Subbarayudu *et al.* [25]. Furthermore, the default value of nanoparticle volume fractions ( $\phi_1, \phi_2, \phi_3$ ) for Williamson ternary hybrid ferrofluid is 0.01 [13, 15, 23]. Table 2 shows the thermophysical characteristics of nanoparticles and base fluids used for this research [3-5, 16, 26-29].

**Table 2.** Thermophysical characteristics of nanoparticles and base fluids

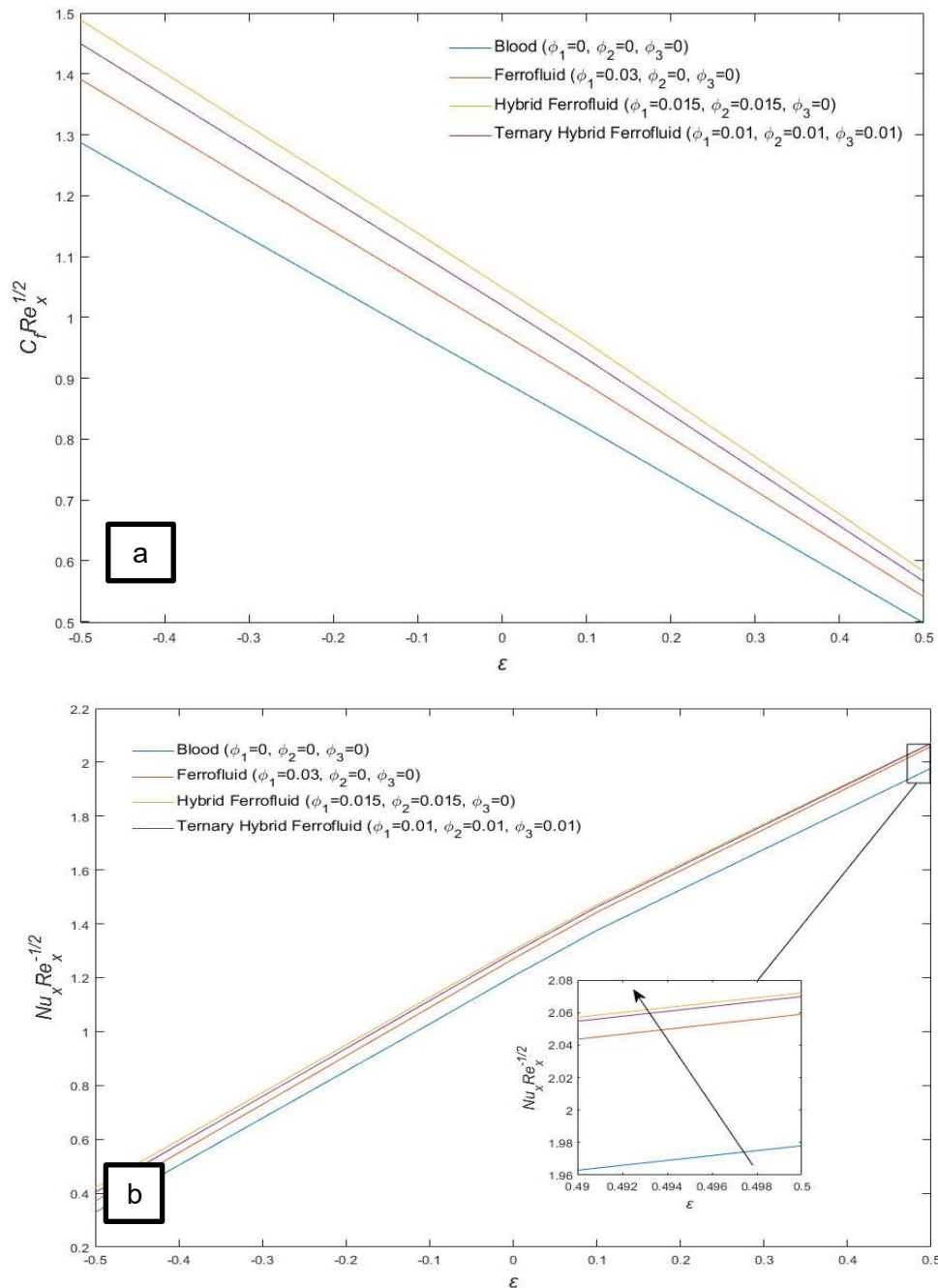
Fluid/ Nanoparticles	Physical Properties				
	$\rho(kg / m^3)$	$C_p(J / kg.K)$	$k(W / m.K)$	$\beta \times 10^{-5}(1 / k)$	Pr
Water	997	4179	0.613	0.00021	6.2
Blood	1053	3594	0.492	0.000074	21
Magnetite ( $Fe_3O_4$ )	5180	670	9.7	0.000013	-
Gold ( $Au$ )	19300	129	318	0.0000142	-
Copper ( $Cu$ )	8933	385	400	0.0000167	-
Aluminium oxide ( $Al_2O_3$ )	3970	765	40	0.0000085	-

To validate the numerical method and equation for this research, Table 3 presents the comparison results between the present with previous research from Bachok *et al.* [26] using copper nanoparticles in a water based fluid where Pr is set to 6.2 and  $\phi_1 = \phi_3 = \varepsilon = \lambda = M = \Omega = 0$ . The comparison was in good agreement for both present and previous research.

**Table 3.** Comparison  $f''(0)$  values with previously published result when  $\phi_1 = \phi_3 = M = \lambda = \varepsilon = \Omega = 0$

$\phi_2$	$f''(0)$	
	Bachok et al. [26]	Present
1	0.3321	0.3321
2	0.3901	0.3901
3	0.4045	0.4045

Next, the comparison distributions of  $Nu_x Re_x^{-1/2}$  and  $C_f Re_x^{1/2}$  for different types of fluid against the moving plate parameter,  $\varepsilon$ , are presented in Figure 2.



**Figure 2.** Distribution of  $C_f Re_x^{1/2}$  (a) and  $Nu_x Re_x^{-1/2}$  (b) for different type of fluid with various values of  $\varepsilon$  when  $Pr = 21, \lambda = 0.1, M = 0.5, \Omega = 0.3$ .

The moving plate parameter,  $\varepsilon$ , defined as the ratio of plate velocity over the free stream velocity [30] and the positive values indicate the upwards movement of the plate while negative values indicate downwards movement of the plate. The four types of ferrofluid volume fractions compared are blood based fluid ( $\phi_1=0, \phi_2=0, \phi_3=0$ ), 3%  $Fe_3O_4$ /blood ferrofluid ( $\phi_1=0.03, \phi_2=0, \phi_3=0$ ), 3%  $Fe_3O_4 - Au$ /blood hybrid ferrofluid ( $\phi_1=0.015, \phi_2=0.015, \phi_3=0$ ), and 3%  $Fe_3O_4 - Au - Al_2O_3$ /blood ternary hybrid ferrofluid ( $\phi_1=0.01, \phi_2=0.01, \phi_3=0.01$ ). Figure 2(a) shows that all the ferrofluid volume fractions tested have a reduction in skin friction coefficient distribution as the parameter  $\varepsilon$  increases, due to the increasing plate velocity over the free stream velocity. Noticed that the distribution of skin friction coefficient values for Williamson ternary hybrid ferrofluid are comparable with the hybrid ferrofluid as the  $\varepsilon$  increases despite the similar number of ferrofluid volume fraction. It is due to the low density of  $Al_2O_3$  nanoparticles compared to  $Au$  nanoparticles which reduce the friction in a fluid. Blood based fluid is expected to have the lowest skin friction coefficient value as it has no nanoparticle volume fraction to resist the flow. Observing Figure 2(b), it can be concluded that the heat transfer performance of 3%  $Fe_3O_4 - Au - Al_2O_3$ /blood ternary hybrid ferrofluid ( $\phi_1=0.01, \phi_2=0.01, \phi_3=0.01$ ) is comparable with 3%  $Fe_3O_4 - Au$ /blood hybrid ferrofluid ( $\phi_1=0.015, \phi_2=0.015, \phi_3=0$ ), when plate is moving upwards. This shows that less gold in ternary hybrid ferrofluid, consumes less in production cost but still provide comparable result heat transfer performance.

Figure 3 illustrated the velocity and the temperature profile for different types of fluid tested. The ferrofluid volume fraction used are similar with Figure 2 blood based fluid ( $\phi_1=0, \phi_2=0, \phi_3=0$ ), 3%  $Fe_3O_4$ /blood ferrofluid ( $\phi_1=0.03, \phi_2=0, \phi_3=0$ ), 3%  $Fe_3O_4 - Au$ /blood hybrid ferrofluid ( $\phi_1=0.015, \phi_2=0.015, \phi_3=0$ ), and 3%  $Fe_3O_4 - Au - Al_2O_3$ /blood ternary hybrid ferrofluid ( $\phi_1=0.01, \phi_2=0.01, \phi_3=0.01$ ). Considering the velocity profile in Figure 3(a), it is shown that the Williamson hybrid ferrofluid has thinner momentum boundary layer compared to Williamson ternary hybrid ferrofluid meaning that high density and volume fraction of gold in hybrid ferrofluid contributed to a high deceleration of fluid thus thinning the thicknesses of the momentum boundary layer. Next, the temperature profile in Figure 3(b) justifies the comparable heat transfer performance of the tested fluids in Figure 2(b). From the figure, the thermal boundary layer for nanofluid, hybrid and ternary hybrid ferrofluid is larger than the blood based fluid due to the presence of nanoparticles improving heat transfer capability, which is similar to the result from Prasannakumara *et al.* [31] and Zhao *et al.* [32]. Based on the data, the Williamson ternary hybrid ferrofluid achieves a 2.81% decrease in the skin-friction coefficient relative to the Williamson hybrid ferrofluid, with both exhibiting similar heat-transfer performance.

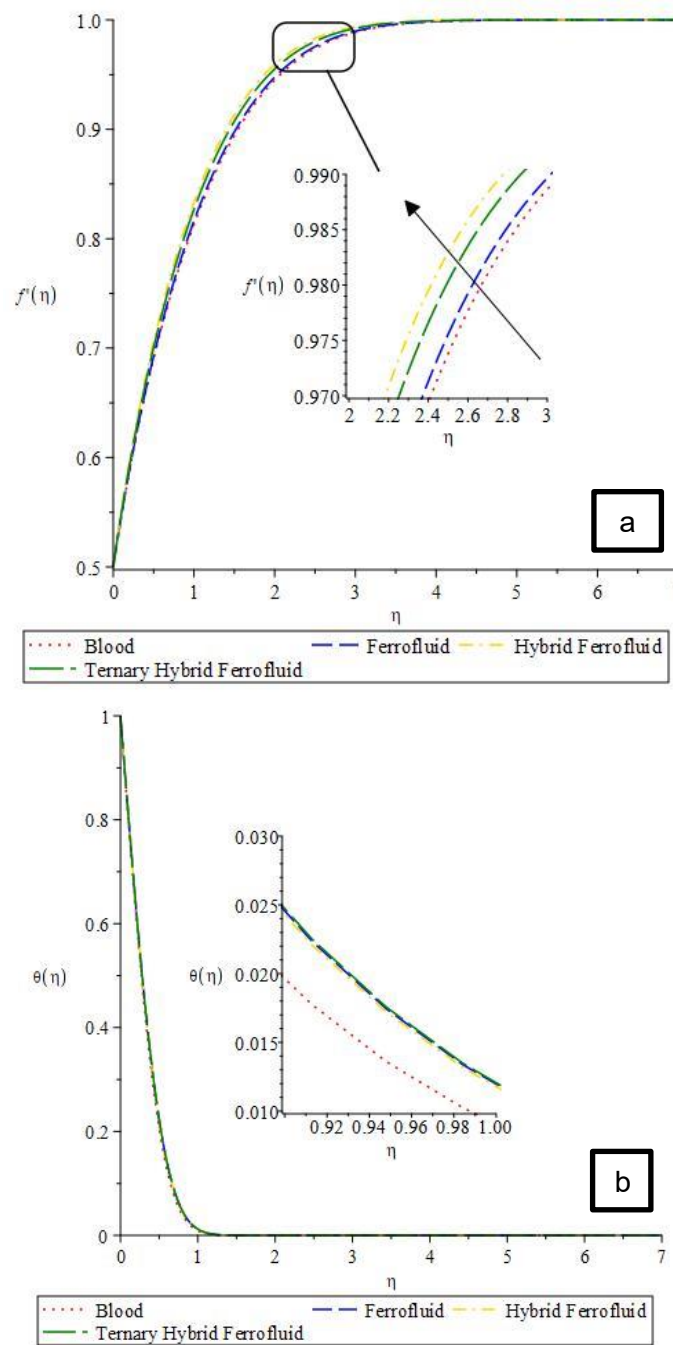
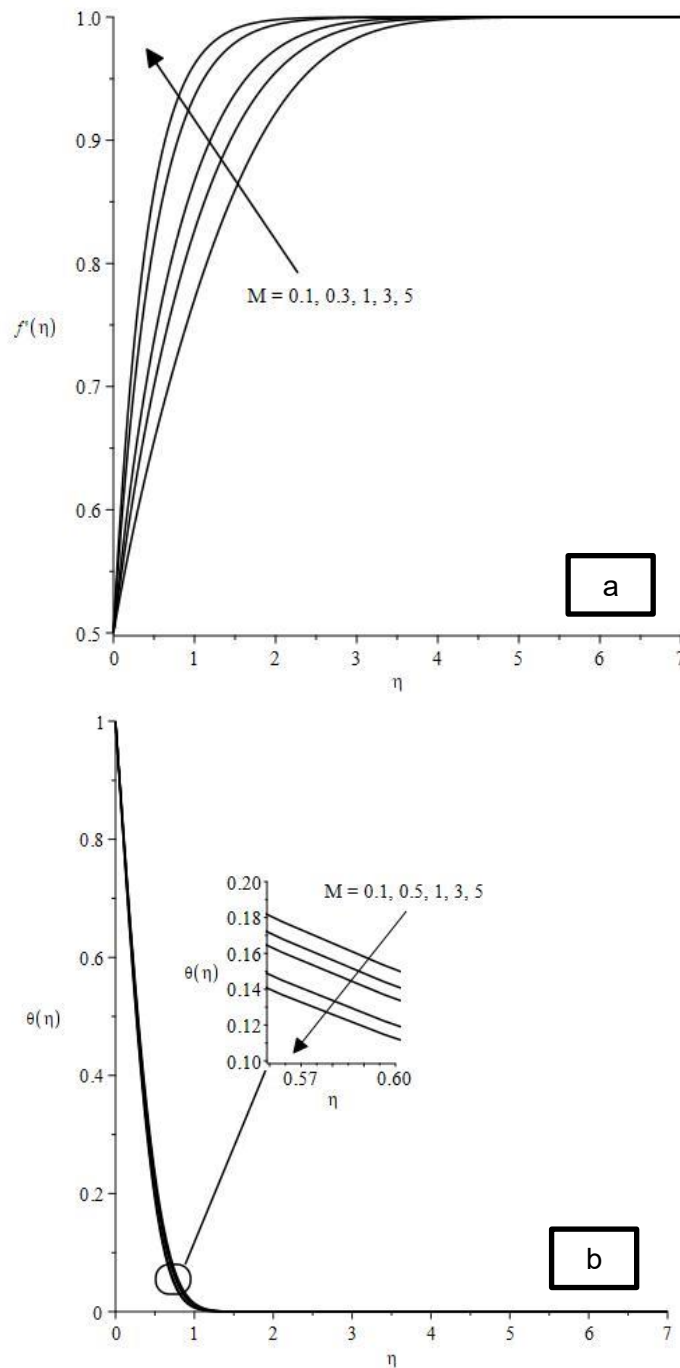


Figure 3. Velocity profile(a) and temperature profile(b) for various types of fluid with  $Pr = 21$ ,  $\lambda = 0.1$ ,  $M = 0.5$ ,  $\varepsilon = 0.5$ ,  $\Omega = 0.3$ .

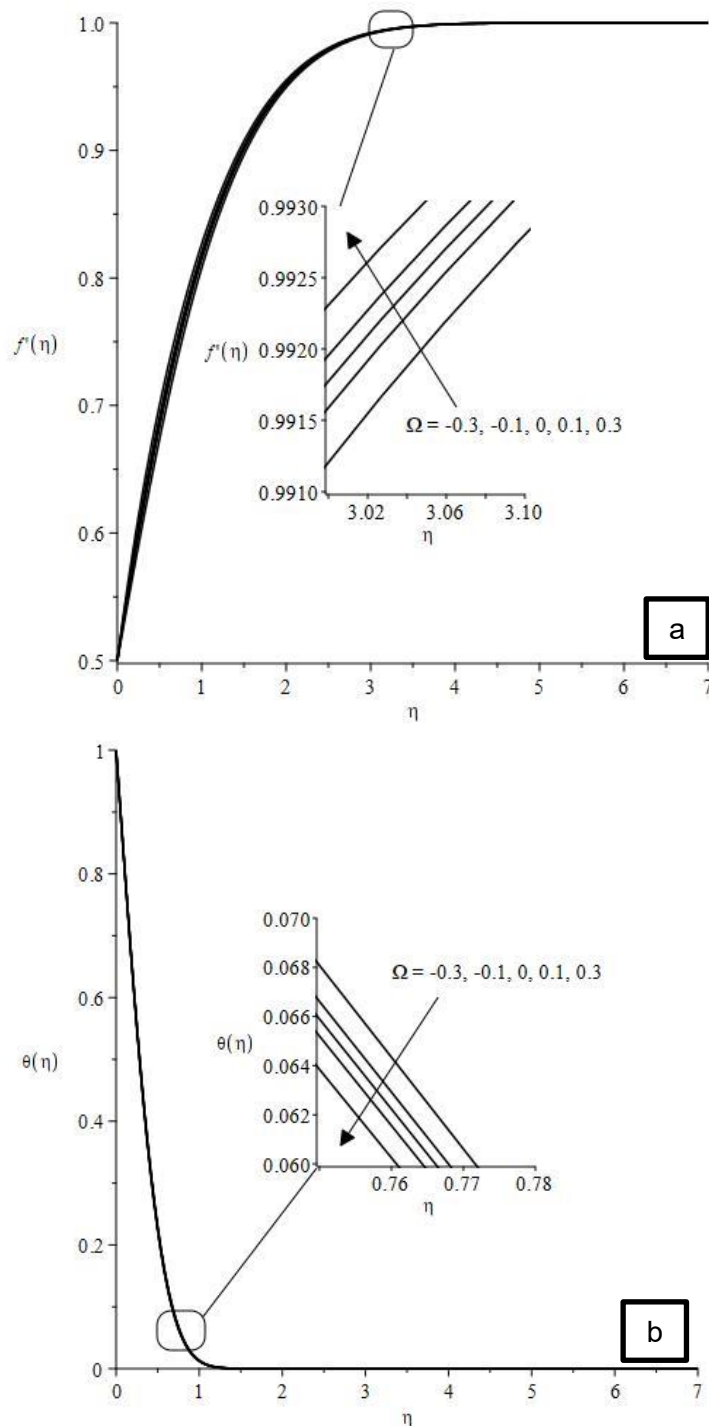
The influence of magnetic parameter,  $M$ , towards the Williamson ternary hybrid ferrofluid is illustrated in Figure 4. Figure 4(a) shows that the boundary layer thickness is decreasing for the velocity profile as the magnetic parameter increases. Increasing the magnetic force will induce the Lorentz effect which then increases the skin friction coefficient of the fluid and retarded the fluid flow. The result is expected as ferroparticles inside the ternary hybrid ferrofluid are highly magnetized. From Figure 4(b), it is evident that the thermal boundary layer decreases only slightly rather than showing any significant variation. This minor reduction corresponds to the small changes in the Nusselt number, which indicate only slight variations in the temperature gradient. Since magnetic force is present, the ferroparticles will gather at the plate thus induced conductive heat transfer rather than convective heat transfer due to the ferroparticles compact and vibrating with each other at the plate where the magnetic force presence.



**Figure 4.** Velocity profile(a) and temperature profile(b) for various values of  $M$  when  $Pr = 21, \lambda = 0.1, \varepsilon = 0.5, \Omega = 0.3$ .

Figures 5 display the characteristics of thermal and momentum boundary layer due to the effects of various buoyancy parameter values,  $\Omega$ . The negative (-) and positive values (+) of  $\Omega$  parameter indicates the opposing and assisting flow of the fluid [33]. It can be seen that both figure shows that  $\Omega$  gives out small effect for both assisting flow ( $\lambda > 0$ ) and opposing flow ( $\lambda < 0$ ) to the thermal boundary layer and velocity boundary layer. Mohamed *et al.* [16] mentioned in their studies the buoyancy effect in fluids with large  $Pr$  numbers have a minimal influence, which supports the findings of this work. Fluid with large  $Pr$ , which in this case 21 has low thermal diffusivity that led to momentum diffuse faster than

heat. Therefore, forced convection plays the dominant role in determining both thermal and velocity boundary layer thickness in Figure 5.



**Figure 5.** Velocity profile (a) and temperature profile (b) for various values of  $\Omega$  when  $Pr = 21$ ,  $\lambda = 0.1$ ,  $\varepsilon = 0.5$ ,  $M = 0.5$ .

## Conclusions

In this research, the mathematical fluid model for Williamson ternary hybrid ferrofluid is numerically studied. Magnetite ( $Fe_3O_4$ ), Gold ( $Au$ ), and aluminium oxide ( $Al_2O_3$ ) are taken as the nanoparticles. RKF45 method is used to numerically solve the transformed equation. This study employs the moving-plate parameter alongside different combinations of ferroparticle volume fractions, magnetic parameters, and buoyancy parameters. The findings for this research are provided below:

- The increase in moving plate results to decreasing in skin friction coefficient values.
- The presence of  $Al_2O_3$  ternary hybrid ferrofluid provides the comparable Nusselt number performance with gold containing hybrid ferrofluid.
- Momentum boundary layer thickness of ternary hybrid fluid is larger than hybrid ferrofluid but comparable in terms of thermal boundary layer thicknesses.
- Magnetic parameter reduces the velocity and thermal boundary layer of Williamson ternary hybrid ferrofluid.
- Buoyancy parameter does not produce significant effect on the velocity and temperature profile for the fluid tested due to dominant role of forced convection.

## Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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