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Neutrosophic Type Reduction for Type-2 Neutrosophic B-spline Curve Model

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Abstract The neutrosophic type reduction process converts type-2 neutrosophic values into their type-1 counterparts. Type-2 neutrosophic set (T2NS) theory is a comprehensive framework that encompasses intuitionistic fuzzy sets (IFS), fuzzy sets (FS), type-2 fuzzy sets (T2FS), and neutrosophic sets (NS). Within this framework, a T2NS employs six membership functions—truth, indeterminacy, and falsity- each represented with both primary and secondary values. The reduction process simplifies these memberships into a type-1 neutrosophic set (NS), which consists solely of the primary truth, indeterminacy, and falsity memberships. However, generating a geometric representation, such as a B-spline curve, presents challenges when using the T2NS theory due to the complexity involved in the reduction process. Therefore, this study introduces an enhanced type reduction technique inspired by the T2FS framework. The method employs the Liu and Karnik-Mendel algorithms to transform type-2 neutrosophic B-spline curves (T2NBsCs) into type-1 neutrosophic B-spline curves (T1NBsCs) for approximation. To construct the model, the triangular T2NS concept is utilized to define a type-2 neutrosophic control point relationship (T2NCPR), which is subsequently integrated with the B-spline basis functions. This integration produces T2NBsCs through an approximation process. The study then presents several numerical examples of T2NBsCs and illustrates the resulting T1NBsCs models along with the corresponding algorithm.

Keywords: Type-2 Neutrosophic Set Theory, Neutrosophic Type Reduction, B-spline Curve Model, Approximation Method, Enhanced Liu and Karnik Mendel Algorithm.

Introduction

Various theories have been proposed to address uncertainty, including probability theory, fuzzy sets, intuitionistic fuzzy sets, and rough set theory, among others. In 1998, Smarandache [1, 2] introduced the concept of neutrosophic sets (NS), which uniquely includes an independent membership of indeterminacy. This framework serves as a generalization of classical sets, fuzzy sets [3], and intuitionistic fuzzy sets [4]. Within the NS framework, uncertainty is accurately quantified through three independent membership functions: truth (T), indeterminacy (I), and falsity (F).

According to Mendel and John [5], type-1 fuzzy sets are inadequate for directly modeling uncertainty due to their precise, crisp membership functions. In contrast, type-2 fuzzy sets (T2FS) are more suitable as their membership functions are fuzzy. While type-1 fuzzy sets are two-dimensional, type-2 fuzzy sets extend into a third dimension through a secondary grade of membership [6]. As a result, traditional type-1 fuzzy set techniques are limited in their ability to handle complex uncertainty data. Someyama [7] highlighted that T2FS comprise fuzzy sets with fuzzy-valued memberships, making them more capable of managing intricate or large-scale uncertain data. The concept of type-2 neutrosophic numbers (T2NN) was initially introduced by Basset *et al*. [8], and Bakali and Broumi [9] provided a visual representation of type-2 neutrosophic sets (T2NS), enabling the treatment of truth, indeterminacy, and falsity components within a footprint of uncertainty (FOU). Consequently, this study adopts foundational ideas from both T2FS and T2NS to develop a type-2 neutrosophic geometric model that incorporates a neutrosophic type reduction process.

Neutrosophic type reduction involves converting type-2 neutrosophic values into either type-1 neutrosophic or simplified neutrosophic values [10]. Wang *et al*. [10] introduced this reduction for interval

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neutrosophic sets by distinguishing between the upper and lower bounds of the FOU. Subsequently, Yeh *et al*. [11] advanced the method by proposing a refined type reduction algorithm for T2FS, which utilizes the α-cut of the secondary membership as the main α-cut to generate type-1 fuzzy values using the Liu and Karnik-Mendel algorithm. Building on this approach, the current work employs an enhanced version of the Liu algorithm, inspired by Yeh *et al*. [11], to perform the neutrosophic type reduction.

Neutrosophic sets offer promising solutions for constructing curves and surfaces within computer-aided geometric design (CAGD), especially in handling ambiguous control or data points. This capability is essential for achieving smoother geometric models. Tas and Topal were pioneers in exploring the integration of NS in geometric modeling, beginning with Bézier curves [12] and later expanding to Bézier surfaces [13]. The properties and structure of the type-2 neutrosophic control point have been introduced in [30]. Following this, Rosli and Zulkifly [14–17] investigated neutrosophic Bézier curves by incorporating neutrosophic control point relations (NCPR) and neutrosophic control net relations (NCNR) with Bernstein basis functions. They extended their work by developing several neutrosophic B-spline models, achieved by combining B-spline basis functions with both NCPR and NCNR [18–20, 31]. Furthermore, their research explored the neutrosophication and deneutrosophication of neutrosophic Bézier and B-spline surface approximations to convert type-1 NS into crisp outputs [21–23, 32]. They also delved into interval-valued and interval type-2 neutrosophic set theories in the context of geometric modeling [24–27]. Recently, they proposed a fuzzification and defuzzification approach for intuitionistic fuzzy B-spline curve approximations [28]. Despite these advancements, none have developed a geometric model based on type-2 neutrosophic sets or applied a neutrosophic type reduction process to transition from general T2NS to standard NS values.

The structure of this paper is outlined as follows: the introductory section provides foundational background. Section 2 reviews key properties of the T2NS theory. Section 3 introduces the construction of type-2 neutrosophic control point relations (T2NCPR) through the type reduction process. Section 4 presents numerical examples illustrating the T2NB-sCs approximation using T2NCPR. Section 5 details the reduction of T2NB-sCs to T1NB-sCs models for selected αβγ-cut values. Finally, Section 6 concludes the paper with discussions and final remarks.

Type-2 Neutrosophic Set Theory Concepts

By integrating the principles of neutrosophic sets and type-2 fuzzy set environments, type-2 neutrosophic sets (T2NS) illustrate the interconnections among truth, indeterminacy, and falsity in data, along with an added third-dimensional perspective [9]. These sets also reveal that the footprint of uncertainty (FOU) captures the inherent ambiguity in truth, indeterminacy, and falsity. In this context, Karaaslan and Hunu [29] outline the core concept of T2NS as follows:

Definition 1 [29]

A type-2 neutrosophic set, denoted $\hat{\hat{A}}$ is characterized by a secondary truth membership function, $\mu_{\hat{\hat{A}}}(x,T)$, secondary indeterminacy membership function, $\eta_{\hat{\hat{A}}}(x,I)$ and secondary falsity membership function, $\nu_{\hat{\hat{A}}}(x,F)$ where $x \in X$ and T,I,F as primary truth, indeterminacy, and falsity memberships function respectively in $J_x^T, J_x^I, J_x^F \subseteq [0,1]$. i. e.,

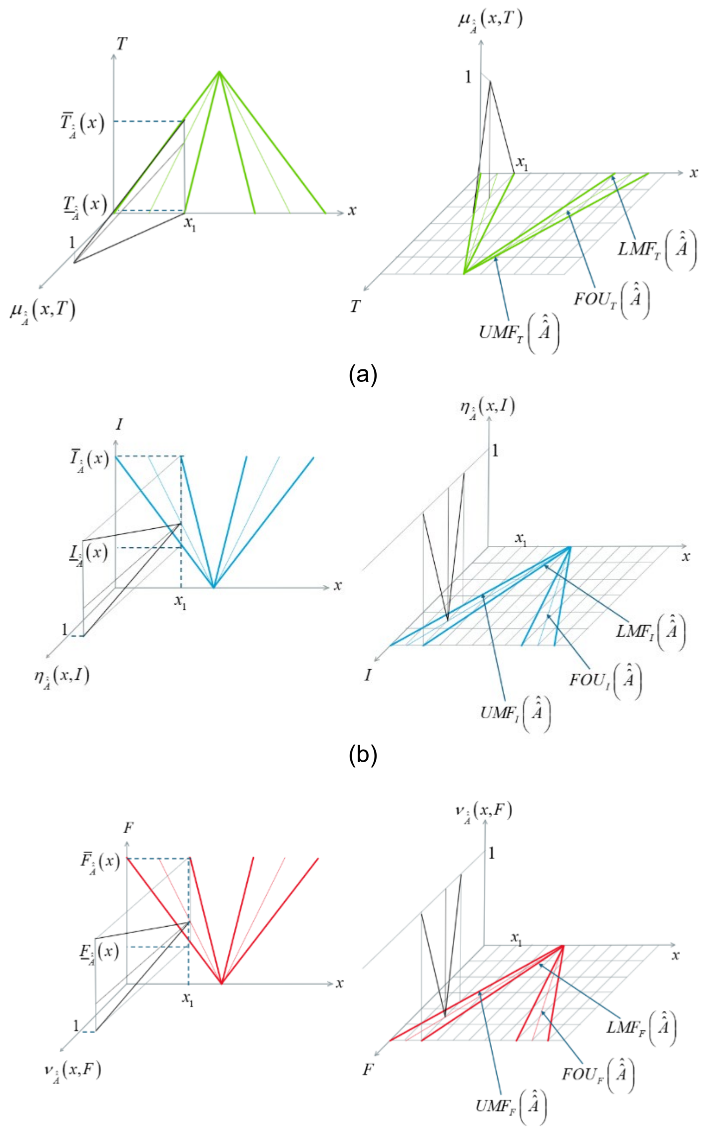
$$\hat{\hat{A}} = \left\{ \left((x,T,I,F), \left(\mu_{\hat{\hat{A}}}(x,T), \eta_{\hat{\hat{A}}}(x,I), \nu_{\hat{\hat{A}}}(x,F) \right) \right) \middle| \forall x \in X, T \in J_x^T, I \in J_x^I, F \in J_x^F \right\} \tag{1}$$

In which $0 \le \mu_{\tilde{A}}(x,u) \le 1$, $0 \le \eta_{\hat{\hat{A}}}(x,I) \le 1$ and $0 \le \nu_{\hat{\hat{A}}}(x,F) \le 1$. $\hat{\hat{A}}$ can also be expressed as

$$\begin{aligned} \hat{\hat{A}}_T &= \int_{x\in X} \int_{T\in J_x^T} \frac{\mu_{\hat{\hat{A}}}(x,T)}{(x,T)} \quad ; \quad J_x^T \subseteq [0,1] \\ \hat{\hat{A}}_I &= \int_{x\in X} \int_{I\in J_x^I} \frac{\eta_{\hat{\hat{A}}}(x,I)}{(x,I)} \quad ; \quad J_x^I \subseteq [0,1] \\ \hat{\hat{A}}_F &= \int_{x\in X} \int_{T\in J_x^F} \frac{\nu_{\hat{\hat{A}}}(x,F)}{(x,F)} \quad ; \quad J_x^F \subseteq [0,1] \end{aligned} \tag{2}$$

A Graphical Depiction of Type-2 Neutrosophic Sets

Let $\hat{\hat{A}}$ represent a type-2 neutrosophic set (T2NS). In this context, the *x*-axis refers to the parameters or variables, denoted by *x*; the *y*-axis corresponds to the primary memberships—truth (*T*), indeterminacy (*I*), and falsity (*F*); and the ***z***-axis represents the secondary memberships—truth $\mu_{\hat{\hat{A}}}(x,T)$, indeterminacy $\eta_{\hat{\hat{A}}}(x,I)$, and falsity $\nu_{\hat{\hat{A}}}(x,F)$, respectively. The formal definition of a T2NS is presented in **Definition 1** and illustrated as follows:



(c)

Figure 1. Triangular T2NS in 3-dimensional (3D) form for; (**a**) Truth; (**b**) Indeterminacy and (**c**) Falsity

In T2NS, two footprints for each membership will also be defined. The 2-dimensional (2D) support of $\mu_{\hat{\hat{A}}}(x,T)$, $\eta_{\hat{\hat{A}}}(x,I)$, and $\nu_{\hat{\hat{A}}}(x,F)$ are called the *FOU* of $\hat{\hat{A}}$ for truth, indeterminacy, and falsity memberships. Thus, the $FOU_T\left(\hat{\hat{A}}\right)$, $FOU_I\left(\hat{\hat{A}}\right)$ and $FOU_F\left(\hat{\hat{A}}\right)$ represented as follows:

$$FOU_T\left(\hat{\hat{A}}\right)=\left\{(x,T)\in X\times[0,1] \middle| \mu_{\hat{\hat{A}}}(x,T)>0\right\}$$
$$FOU_I\left(\hat{\hat{A}}\right)=\left\{(x,I)\in X\times[0,1] \middle| \eta_{\hat{\hat{A}}}(x,I)>0\right\} \tag{3}$$
$$FOU_F\left(\hat{\hat{A}}\right)=\left\{(x,F)\in X\times[0,1] \middle| \nu_{\hat{\hat{A}}}(x,F)>0\right\}$$

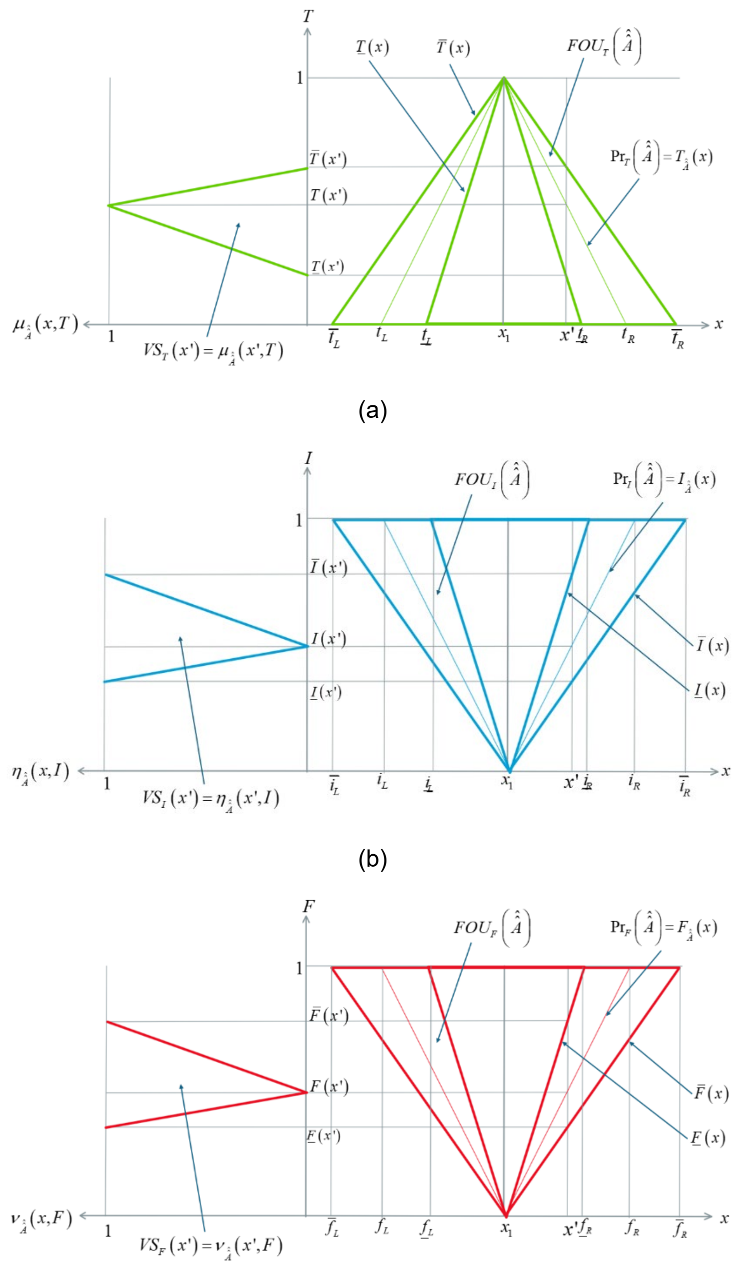
The $FOU_T\left(\hat{\hat{A}}\right)$, $FOU_I\left(\hat{\hat{A}}\right)$ and $FOU_F\left(\hat{\hat{A}}\right)$ are bounded by lower and upper truth, indeterminacy, and falsity membership functions denoted as $\overline{T}_{\hat{\hat{A}}}(x)$, $\underline{T}_{\hat{\hat{A}}}(x)$, $\overline{I}_{\hat{\hat{A}}}(x)$, $\underline{I}_{\hat{\hat{A}}}(x)$, $\overline{F}_{\hat{\hat{A}}}(x)$ and $\underline{F}_{\hat{\hat{A}}}(x)$ or called as $UMF_T\left(\hat{\hat{A}}\right)$, $LMF_T\left(\hat{\hat{A}}\right)$, $UMF_I\left(\hat{\hat{A}}\right)$, $LMF_I\left(\hat{\hat{A}}\right)$, $UMF_F\left(\hat{\hat{A}}\right)$, and $LMF_F\left(\hat{\hat{A}}\right)$ respectively as presented in Figure 1, where:

$$UMF_T\left(\hat{\hat{A}}\right)=\overline{T}_{\hat{\hat{A}}}(x)=\sup\left\{T \middle| T\in[0,1],\mu_{\hat{\hat{A}}}(x,T)>0\right\}$$
$$LMF_T\left(\hat{\hat{A}}\right)=\underline{T}_{\hat{\hat{A}}}(x)=\inf\left\{T \middle| T\in[0,1],\mu_{\hat{\hat{A}}}(x,T)>0\right\} \tag{4}$$

$$UMF_I\left(\hat{\hat{A}}\right)=\overline{I}_{\hat{\hat{A}}}(x)=\sup\left\{I \middle| I\in[0,1],\eta_{\hat{\hat{A}}}(x,I)>0\right\}$$
$$LMF_I\left(\hat{\hat{A}}\right)=\underline{I}_{\hat{\hat{A}}}(x)=\inf\left\{I \middle| I\in[0,1],\eta_{\hat{\hat{A}}}(x,I)>0\right\} \tag{5}$$

$$UMF_F\left(\hat{\hat{A}}\right)=\overline{F}_{\hat{\hat{A}}}(x)=\sup\left\{F \middle| F\in[0,1],\nu_{\hat{\hat{A}}}(x,F)>0\right\}$$
$$LMF_F\left(\hat{\hat{A}}\right)=\underline{F}_{\hat{\hat{A}}}(x)=\inf\left\{F \middle| F\in[0,1],\nu_{\hat{\hat{A}}}(x,F)>0\right\} \tag{6}$$

The VS is the 2D plane in the $T-\mu_{\hat{\hat{A}}}(x,T)$, $I-\eta_{\hat{\hat{A}}}(x,I)$, and $F-\nu_{\hat{\hat{A}}}(x,F)$ axes for a single of $x=x'$ and denoted as $VS_T(x')=\mu_{\hat{\hat{A}}}(x',T)$, $VS_I(x')=\eta_{\hat{\hat{A}}}(x',I)$, and $VS_F(x')=\nu_{\hat{\hat{A}}}(x',F)$ for truth, indeterminacy, and falsity membership respectively. The PrMF will also be introduced to each membership and denoted as $\Pr_T\left(\hat{\hat{A}}\right)$ equivalent to $T_{\hat{\hat{A}}}(x)$, $\Pr_I\left(\hat{\hat{A}}\right)$ equivalent to $I_{\hat{\hat{A}}}(x)$ and $\Pr_I\left(\hat{\hat{A}}\right)$ equivalent to $I_{\hat{\hat{A}}}(x)$. Therefore, Figure 2 shows the 2D representation of a T2NS with triangular vertical slices as follows:



(a)

(b)

(c)

Figure 2. Triangular T2NS in 2D form for; (a) Truth; (b) Indeterminacy and **(c)** Falsity memberships

Triangular Type-2 Neutrosophic Numbers

This section outlines the essential properties and conditions that define type-2 neutrosophic numbers (T2NNs). Building upon **Definition 1**, the triangular form of a T2NN will be formally introduced. Furthermore, the components illustrated in Figure 2 will be explained through **Definition 2**.

Definition 2 (Type-2 Neutrosophic Number)

Let $\hat{\hat{A}}$ said to be a T2NN if the following conditions are satisfied:

- $\hat{\hat{A}}$ is a normal T2NS.
- $FOU_T\left(\hat{\hat{A}}\right)$ is a convex region while $FOU_I\left(\hat{\hat{A}}\right)$ and $FOU_F\left(\hat{\hat{A}}\right)$ are concave regions.

- The support of $\hat{\hat{A}}$ is $\left\{(x,T):\left|\mu_{\hat{\hat{A}}}(x,T)>0; x\in\mathbb{R}, T\in J_x^T\subseteq[0,1]\right.\right\}$, $\left\{(x,I):\left|\eta_{\hat{\hat{A}}}(x,I)>0; x\in\mathbb{R}, I\in J_x^I\subseteq[0,1]\right.\right\}$ and $\left\{(x,F):\left|\eta_{\hat{\hat{A}}}(x,F)>0; x\in\mathbb{R}, F\in J_x^F\subseteq[0,1]\right.\right\}$ are a closed and bounded set.
- $\Pr_T\left(\hat{\hat{A}}\right)$, $\Pr_I\left(\hat{\hat{A}}\right)$ and $\Pr_I\left(\hat{\hat{A}}\right)$ are a fuzzy number.
- The secondary membership functions are a fuzzy number for each fixed $x\in\mathbb{R}$.

Definition 3 [9]

A triangular T2NS $\hat{\hat{A}}$ is defined by

$$\hat{\hat{A}}=\left\{\begin{array}{l}\hat{\hat{A}}_T=\left\{\left(x,\underline{T}_{\hat{\hat{A}}}(x),T_{\hat{\hat{A}}}(x),\overline{T}_{\hat{\hat{A}}}(x)\right):x\in\mathbb{R},\underline{T}_{\hat{\hat{A}}}(x)\le T_{\hat{\hat{A}}}(x)\le\overline{T}_{\hat{\hat{A}}}(x)\right\},\\ \hat{\hat{A}}_I=\left\{\left(x,\underline{I}_{\hat{\hat{A}}}(x),I_{\hat{\hat{A}}}(x),\overline{I}_{\hat{\hat{A}}}(x)\right):x\in\mathbb{R},\underline{I}_{\hat{\hat{A}}}(x)\le I_{\hat{\hat{A}}}(x)\le\overline{I}_{\hat{\hat{A}}}(x)\right\},\\ \hat{\hat{A}}_F=\left\{\left(x,\underline{F}_{\hat{\hat{A}}}(x),F_{\hat{\hat{A}}}(x),\overline{F}_{\hat{\hat{A}}}(x)\right):x\in\mathbb{R},\underline{F}_{\hat{\hat{A}}}(x)\le F_{\hat{\hat{A}}}(x)\le\overline{F}_{\hat{\hat{A}}}(x)\right\}\end{array}\right\}\tag{7}$$

where $0\le\hat{\hat{A}}_T+\hat{\hat{A}}_I+\hat{\hat{A}}_F\le3$ for $\hat{\hat{A}}_T\in[0,1]$, $\hat{\hat{A}}_I\in[0,1]$, and $\hat{\hat{A}}_F\in[0,1]$. The functions $\underline{T}_{\hat{\hat{A}}}(x),T_{\hat{\hat{A}}}(x)$, and $\overline{T}_{\hat{\hat{A}}}(x)$ are triangular truth memberships, while $\underline{I}_{\hat{\hat{A}}}(x),I_{\hat{\hat{A}}}(x)$, and $\overline{I}_{\hat{\hat{A}}}(x)$ are triangular indeterminacy memberships, and $\underline{F}_{\hat{\hat{A}}}(x),F_{\hat{\hat{A}}}(x)$, and $\overline{F}_{\hat{\hat{A}}}(x)$ are triangular falsity memberships for LMF, PrMF, and UMF respectively. Based on Figure 2, $\hat{\hat{A}}$ are denoted by $\hat{\hat{A}}=\left(\hat{\hat{A}}_T,\hat{\hat{A}}_I,\hat{\hat{A}}_F\right)$ where $\hat{\hat{A}}_T,\hat{\hat{A}}_I$, and $\hat{\hat{A}}_F$ describe as follows:

$$\begin{aligned}\hat{\hat{A}}_T&=\left[\underline{T}_{\hat{\hat{A}}}(x),T_{\hat{\hat{A}}}(x),\overline{T}_{\hat{\hat{A}}}(x)\right]\\ \hat{\hat{A}}_I&=\left[\underline{I}_{\hat{\hat{A}}}(x),I_{\hat{\hat{A}}}(x),\overline{I}_{\hat{\hat{A}}}(x)\right]\\ \hat{\hat{A}}_F&=\left[\underline{F}_{\hat{\hat{A}}}(x),F_{\hat{\hat{A}}}(x),\overline{F}_{\hat{\hat{A}}}(x)\right]\end{aligned}\tag{8}$$

where;

$$\begin{aligned}\left[\underline{T}_{\hat{\hat{A}}}(x),T_{\hat{\hat{A}}}(x),\overline{T}_{\hat{\hat{A}}}(x)\right]&=\left[\left(\underline{t}_L,x_1,\underline{t}_R\right),\left(t_L,x_1,t_R\right),\left(\overline{t}_L,x_1,\overline{t}_R\right)\right]\\ \left[\underline{I}_{\hat{\hat{A}}}(x),I_{\hat{\hat{A}}}(x),\overline{I}_{\hat{\hat{A}}}(x)\right]&=\left[\left(\underline{i}_L,x_1,\underline{i}_R\right),\left(i_L,x_1,i_R\right),\left(\overline{i}_L,x_1,\overline{i}_R\right)\right]\\ \left[\underline{F}_{\hat{\hat{A}}}(x),F_{\hat{\hat{A}}}(x),\overline{F}_{\hat{\hat{A}}}(x)\right]&=\left[\left(\underline{f}_L,x_1,\underline{f}_R\right),\left(f_L,x_1,f_R\right),\left(\overline{f}_L,x_1,\overline{f}_R\right)\right]\end{aligned}\tag{9}$$

Hence, the triangular $\hat{\hat{A}}=\left(\hat{\hat{A}}_T,\hat{\hat{A}}_I,\hat{\hat{A}}_F\right)$ are obtained as follows:

$$\underline{T}_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-\underline{t}_L)}{(x_1-\underline{t}_L)}, & \underline{t}_L\le x\le x_1,\\ \dfrac{(\underline{t}_R-x)}{(\underline{t}_R-x_1)}, & x_1\le x\le\underline{t}_R,\\ 0, & \text{otherwise}\end{cases};\overline{T}_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-\overline{t}_L)}{(x_1-\overline{t}_L)}, & \overline{t}_L\le x\le x_1,\\ \dfrac{(\overline{t}_R-x)}{(\overline{t}_R-x_1)}, & x_1\le x\le\overline{t}_R,\\ 0, & \text{otherwise.}\end{cases}\tag{10}$$

$$T_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-t_L)}{(x_1-t_L)}, & t_L\le x\le x_1,\\[2mm] \dfrac{(t_R-x)}{(t_R-x_1)}, & x_1\le x\le t_R,\\[2mm] 0, & \text{Otherwise.}\end{cases}$$

$$\underline{I}_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-\underline{i}_L)}{(x_1-\underline{i}_L)}, & \underline{i}_L\le x\le x_1,\\[2mm] \dfrac{(\underline{i}_R-x)}{(\underline{i}_R-x_1)}, & x_1\le x\le \underline{i}_R,\\[2mm] 0, & \text{otherwise.}\end{cases};;\overline{I}_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-\overline{i}_L)}{(x_1-\overline{i}_L)}, & \overline{i}_L\le x\le x_1,\\[2mm] \dfrac{(\overline{i}_R-x)}{(\overline{i}_R-x_1)}, & x_1\le x\le \overline{i}_R,\\[2mm] 0, & \text{otherwise}\end{cases}$$

$$I_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-i_L)}{(x_1-i_L)}, & i_L\le x\le x_1,\\[2mm] \dfrac{(i_R-x)}{(i_R-x_1)}, & x_1\le x\le i_R,\\[2mm] 0, & \text{Otherwise.}\end{cases}$$

$$\underline{F}_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-\underline{f}_L)}{(x_1-\underline{f}_L)}, & \underline{f}_L\le x\le x_1,\\[2mm] \dfrac{(\underline{f}_R-x)}{(\underline{f}_R-x_1)}, & x_1\le x\le \underline{f}_R,\\[2mm] 0, & \text{otherwise.}\end{cases};\overline{F}_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-\overline{f}_L)}{(x_1-\overline{f}_L)}, & \overline{f}_L\le x\le x_1,\\[2mm] \dfrac{(\overline{f}_R-x)}{(\overline{f}_R-x_1)}, & x_1\le x\le \overline{f}_R,\\[2mm] 0, & \text{otherwise.}\end{cases}$$

$$F_{\hat{\hat{A}}}(x)=\begin{cases}\dfrac{(x-f_L)}{(x_1-f_L)}, & f_L\le x\le x_1,\\[2mm] \dfrac{(f_R-x)}{(f_R-x_1)}, & x_1\le x\le f_R,\\[2mm] 0, & \text{Otherwise.}\end{cases}$$

While for any fixed $x'\in J_x^T,J_x^I,J_x^F$. The secondary triangular membership functions based on Figure 2 can be obtained as follows:

$$\mu_{\hat{\hat{A}}}(x',T)=\begin{cases}\dfrac{\left(x-\underline{T}(x')\right)}{\left(T(x')-\underline{T}(x')\right)}, & \underline{T}(x')\leq x\leq T(x'),\\ \dfrac{\left(\overline{T}(x')-x\right)}{\left(\overline{T}(x')-T(x')\right)}, & T(x')\leq x\leq \overline{T}(x'),\\ 0, & \text{Otherwise.}\end{cases}$$

$$\eta_{\hat{\hat{A}}}(x',I)=\begin{cases}\dfrac{\left(x-\underline{I}(x')\right)}{\left(I(x')-\underline{I}(x')\right)}, & \underline{I}(x')\leq x\leq I(x'),\\ \dfrac{\left(\overline{I}(x')-x\right)}{\left(\overline{I}(x')-I(x')\right)}, & I(x')\leq x\leq \overline{I}(x'),\\ 0, & \text{Otherwise.}\end{cases} \tag{11}$$

$$\nu_{\hat{\hat{A}}}(x',F)=\begin{cases}\dfrac{\left(x-\underline{F}(x')\right)}{\left(F(x')-\underline{F}(x')\right)}, & \underline{F}(x')\leq x\leq F(x'),\\ \dfrac{\left(\overline{F}(x')-x\right)}{\left(\overline{F}(x')-F(x')\right)}, & F(x')\leq x\leq \overline{F}(x'),\\ 0, & \text{Otherwise.}\end{cases}$$

Neutrosophic Type Reduction of Type-2 Neutrosophic Control Point Relation (T2NCPR)

This section will introduce the type-2 neutrosophic point relation (T2NPR), the T2NCPR, and the neutrosophic type reduction of T2NCPR. The T2NCPR is built based on the concept of T2NS, which was addressed in the previous section. Therefore, the terms were defined as follows:

Definition 4 (Type-2 Neutrosophic Point Relation)

Let $\hat{\hat{A}}_i$ be a T2NS over the universal set *X*, thus, $\hat{\hat{R}}_i$ is T2NPR where $i=0,1,2,...,n,$ and the position vector of *n* as control polygon vertices defined as

$$\hat{\hat{R}}_i=\left\{\begin{matrix}\left\langle (x_i),T_R(x_i),I_R(x_i),F_R(x_i),\mu_R(x_i),\eta_R(x_i),\nu_R(x_i)\right\rangle \\ \left| T_R(x_i),I_R(x_i),F_R(x_i),\mu_R(x_i),\eta_R(x_i),\nu_R(x_i)\in S\right.\end{matrix}\right\} \tag{12}$$

where $S\in[0,1]$, $T_R(x_i):X\to S$, $I_R(x_i):X\to S$, $F_R(x_i):X\to S$, $\mu_R(x_i):X\to S$, $\eta_R(x_i):X\to S$ and $\nu_R(x_i):X\to S$ that obey the condition

$$0\leq T_R(x_i)+I_R(x_i)+F_R(x_i)\leq 3 \tag{13}$$

$$0\leq \mu_R(x_i)+\eta_R(x_i)+\nu_R(x_i)\leq 3 \tag{14}$$

where $T_R(x_i)$, $I_R(x_i)$, $F_R(x_i),\mu_R(x_i),\eta_R(x_i),\nu_R(x_i)\to[0,1]$ for all $(x_i)\in X$.

From Equations (26), $\hat{\hat{R}}_i$ are represented as T2NPR and consist of the neutrosophic relation point denoted as R^T, R^I, R^F, R^μ, R^η and R^ν for truth, indeterminacy, and falsity for primary and secondary memberships. If Equations (13) and (14) are fulfilled, there exists R^T, R^I, R^F, R^μ, R^η and R^ν that is stated as follows:

$$\begin{aligned} R^T &= \left\{ \langle (x_i), T_R(x_i) \rangle \middle| T_R(x_i) \in S \right\}; \quad R^\mu = \left\{ \langle (x_i), \mu_R(x_i) \rangle \middle| \mu_R(x_i) \in S \right\}; \\ R^I &= \left\{ \langle (x_i), I_R(x_i) \rangle \middle| I_R(x_i) \in S \right\}; \quad R^\eta = \left\{ \langle (x_i), \eta_R(x_i) \rangle \middle| \eta_R(x_i) \in S \right\}; \\ R^F &= \left\{ \langle (x_i), F_R(x_i) \rangle \middle| F_R(x_i) \in S \right\}; \quad R^\nu = \left\{ \langle (x_i), \nu_R(x_i) \rangle \middle| \nu_R(x_i) \in S \right\} \end{aligned} \tag{15}$$

with $T_R(x_i)$, $I_R(x_i)$, $F_R(x_i)$, $\mu_R(x_i)$, $\eta_R(x_i)$, and $\nu_R(x_i)$ are truth, indeterminacy, and falsity of primary and secondary memberships in S respectively.

Definition 5 (Type-2 Neutrosophic Control Point Relation)

Suppose $\hat{\hat{R}}_i$ is T2NPR, then $\hat{\hat{P}}_i^T$, $\hat{\hat{P}}_i^I$, $\hat{\hat{P}}_i^F$, $\hat{\hat{P}}_i^\mu$, $\hat{\hat{P}}_i^\eta$ and $\hat{\hat{P}}_i^\nu$ denoted as T2NCPRs for truth, indeterminacy, and falsity of primary and secondary memberships respectively, where $i = 0,1,2,...,n,$ and the position vector of n as control polygon vertices.

$$\begin{aligned} \hat{\hat{P}}_i^T &= \left\{ \hat{\hat{p}}_0^T, \hat{\hat{p}}_1^T, \hat{\hat{p}}_2^T, \dots \hat{\hat{p}}_n^T \right\}; \quad \hat{\hat{P}}_i^\mu = \left\{ \hat{\hat{p}}_0^\mu, \hat{\hat{p}}_1^\mu, \hat{\hat{p}}_2^\mu, \dots \hat{\hat{p}}_n^\mu \right\}; \\ \hat{\hat{P}}_i^I &= \left\{ \hat{\hat{p}}_0^I, \hat{\hat{p}}_1^I, \hat{\hat{p}}_2^I, \dots \hat{\hat{p}}_n^I \right\}; \quad \hat{\hat{P}}_i^\eta = \left\{ \hat{\hat{p}}_0^\eta, \hat{\hat{p}}_1^\eta, \hat{\hat{p}}_2^\eta, \dots \hat{\hat{p}}_n^\eta \right\}; \\ \hat{\hat{P}}_i^F &= \left\{ \hat{\hat{p}}_0^F, \hat{\hat{p}}_1^F, \hat{\hat{p}}_2^F, \dots \hat{\hat{p}}_n^F \right\}; \quad \hat{\hat{P}}_i^\nu = \left\{ \hat{\hat{p}}_0^\nu, \hat{\hat{p}}_1^\nu, \hat{\hat{p}}_2^\nu, \dots \hat{\hat{p}}_n^\nu \right\} \end{aligned} \tag{16}$$

T2NS consists of a union of T2NCPR. This section discusses the properties and structure of T2NCPR to prove the statement. Therefore, Remark 1 describes the nature of T2NCPR in T2NS based on the nature of T2NN.

Remark 1

Let $\hat{\hat{P}}$ be the T2NCPR and $\hat{\hat{A}}$ is a T2NS in the universal set X. $\hat{\hat{P}} \in \hat{\hat{A}}$ if and only if $T_P(x_i) \le T_A(x_i)$, $I_P(x_i) \ge I_A(x_i)$, $F_P(x_i) \ge F_A(x_i)$, $\mu_P(x_i) \le \mu_A(x_i)$, $\eta_P(x_i) \ge \eta_A(x_i)$ and $\nu_P(x_i) \ge \nu_A(x_i)$ for all $x_i \in X$. Each $\hat{\hat{A}}$ can be expressed as a union of $\hat{\hat{P}}$ if $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $\mu_A(x_i)$, $\eta_A(x_i)$ and $\nu_A(x_i)$ is not an empty set for $x_i \in X$. Therefore, the elements denoted as follows:

$$T_A(x_i)=\sup\left\{\begin{array}{l}\alpha : T_P(x_i) \text{ is NPs of primary truth membership and}\\ \alpha \le T_A(x_i)\end{array}\right\}$$
$$I_A(x_i)=\inf\left\{\begin{array}{l}\beta : I_P(x_i) \text{ is NPs of primary indeterminacy membership and}\\ \beta \ge I_P(x_i)\end{array}\right\}$$
$$F_A(x_i)=\inf\left\{\begin{array}{l}\gamma : F_P(x_i) \text{ is NPs of primary falsity membership and}\\ \gamma \ge F_A(x_i)\end{array}\right\}$$
$$\mu_A(x_i)=\sup\left\{\begin{array}{l}\tilde{\alpha} : \mu_P(x_i) \text{ is NPs of secondary truth membership and}\\ \tilde{\alpha} \le \mu_A(x_i)\end{array}\right\} \quad (17)$$
$$\eta_A(x_i)=\inf\left\{\begin{array}{l}\tilde{\beta} : \eta_P(x_i) \text{ is NPs of secondary indeterminacy membership and}\\ \tilde{\beta} \ge \eta_P(x_i)\end{array}\right\}$$
$$\nu_A(x_i)=\inf\left\{\begin{array}{l}\tilde{\gamma} : \nu_P(x_i) \text{ is NPs of secondary falsity membership and}\\ \tilde{\gamma} \ge \nu_A(x_i)\end{array}\right\}$$

By using Remark 1, Theorem 1 shows that T2NS, $\hat{\hat{A}}$ is a union of T2NCPR, $\hat{\hat{P}}$. Therefore, the theorem is as follows:

Theorem 1

If $\hat{\hat{A}}=\bigcup_{c\in C}\hat{\hat{A}}_c$ with $C=\{0,1,\ldots,n\}$ and C is an index set, then $\hat{\hat{P}}\in\hat{\hat{A}}$ if and only if $\hat{\hat{P}}\in\hat{\hat{A}}_c$ for some $c\in C$.

Proof: Let $x_0\in\hat{\hat{A}}$. By using Remark 1,

$$\begin{aligned}&T_A(x_0)=\sup_{c\in C}T_{A_c}(x_0),\quad \mu_A(x_0)=\sup_{c\in C}\mu_{A_c}(x_0),\\&I_A(x_0)=\inf_{c\in C}I_{A_c}(x_0),\quad \eta_A(x_0)=\inf_{c\in C}\eta_{A_c}(x_0),\\&F_A(x_0)=\inf_{c\in C}F_{A_c}(x_0),\quad \nu_A(x_0)=\inf_{c\in C}\nu_{A_c}(x_0).\end{aligned} \quad (18)$$

There exist some $c_0\in C$ such as $T_{A_{C_0}}(x_0)=T_A(x_0)$, $I_{A_{C_0}}(x_0)=I_A(x_0)$, $F_{A_{C_0}}(x_0)=F_A(x_0)$, $\mu_{A_{C_0}}(x_0)=\mu_A(x_0)$, $\eta_{A_{C_0}}(x_0)=\eta_A(x_0)$ and $\nu_{A_{C_0}}(x_0)=\nu_A(x_0)$.
And $T_{A_C}(x_0)\le T_A(x_0)$, $I_{A_C}(x_0)\ge I_A(x_0)$, $F_{A_C}(x_0)\ge F_A(x_0)$, $\mu_{A_C}(x_0)\le\mu_A(x_0)$, $\eta_{A_C}(x_0)\ge\eta_A(x_0)$ and $\nu_{A_C}(x_0)\ge\nu_A(x_0)$ for all $c\in C$.
For (a), $\hat{\hat{P}}\in\hat{\hat{A}}_{c_0}$. For (b), $\hat{\hat{P}}\in\hat{\hat{A}}$ imply that $T_{A_C}(x_0)\le T_A(x_0)$, $I_{A_C}(x_0)\ge I_A(x_0)$, $F_{A_C}(x_0)\ge F_A(x_0)$, $\mu_{A_C}(x_0)\le\mu_A(x_0)$, $\eta_{A_C}(x_0)\ge\eta_A(x_0)$ and $\nu_{A_C}(x_0)\ge\nu_A(x_0)$ and considering $T_A(x_0)=\sup_{c\in C}T_{A_c}(x_0)$, $I_A(x_0)=\inf_{c\in C}I_{A_c}(x_0)$, $F_A(x_0)=\inf_{c\in C}F_{A_c}(x_0)$, $\mu_A(x_0)=\sup_{c\in C}\mu_{A_c}(x_0)$, $\eta_A(x_0)=\inf_{c\in C}\eta_{A_c}(x_0)$ and $\nu_A(x_0)=\inf_{c\in C}\nu_{A_c}(x_0)$, then $T_{A_C}(x_0)\le T_A(x_0)$, $I_{A_C}(x_0)\ge I_A(x_0)$, $F_{A_C}(x_0)\ge F_A(x_0)$, $\mu_{A_C}(x_0)\le\mu_A(x_0)$, $\eta_{A_C}(x_0)\ge\eta_A(x_0)$ and $\nu_{A_C}(x_0)\ge\nu_A(x_0)$ obeyed for some c_0. Therefore, $\hat{\hat{P}}\in\hat{\hat{A}}_{c_0}$. ■

Algorithm 1 is about a neutrosophic type reduction process for selected $(\alpha,\beta,\gamma)-\text{cut}$ values in triangular forms. The neutrosophic type reduction for general T2NS was inspired by enhanced type reduction for T2FS by Yeh *et al*. [11]. Figure 3 demonstrates the neutrosophic type reduction of T2NCPRs.

Algorithm 1 (Neutrosophic Type-Reduction of T2NCPR)

Let $\hat{\alpha}=[\hat{\alpha}_1,\hat{\alpha}_2,...,\hat{\alpha}_k]$, $\hat{\beta}=[\hat{\beta}_1,\hat{\beta}_2,...,\hat{\beta}_k]$ and $\hat{\gamma}=[\hat{\gamma}_1,\hat{\gamma}_2,...,\hat{\gamma}_k]$,

Procedure Improved $\left(\hat{\hat{A}},Z,\hat{\alpha},\hat{\beta},\hat{\gamma}\right)$

For $i=k$ **down to** 1

Take $\left\langle\hat{\alpha}_i,\hat{\beta}_i,\hat{\gamma}_i\right\rangle-\text{cuts}$ of secondary membership functions of $\hat{\hat{A}}$,

Let these $\left\langle\hat{\alpha}_i,\hat{\beta}_i,\hat{\gamma}_i\right\rangle-\text{cuts}$ be $\left[{}^{\hat{\alpha},\hat{\beta},\hat{\gamma}}P^{\mu,\eta,\nu}_{i(L)},{}^{\hat{\alpha},\hat{\beta},\hat{\gamma}}P^{\mu,\eta,\nu}_{i(R)}\right]$,

Form these $\left\langle\hat{\alpha}_i,\hat{\beta}_i,\hat{\gamma}_i\right\rangle-\text{cuts}$ as the $\langle\alpha,\beta,\gamma\rangle-\text{plane}$ for $\hat{P}^{T,I,F}_i$;

If $(i=k)$

Call Karnik-Mendel and obtain $\left[\underline{b}^{T,I,F}_k,\overline{b}^{T,I,F}_k\right]$, $\underline{L}^{T,I,F}_k,\overline{L}^{T,I,F}_k$ for $\langle\alpha_k,\beta_k,\gamma_k\rangle$ using **Equation (19);**

Else

Initialize $\underline{L}^{T,I,F}_i$ and $\overline{L}^{T,I,F}_i$ by **Equation (20),**

Initialize $\underline{b}^{T,I,F}_i$ and $\overline{b}^{T,I,F}_i$ by **Equations (21) and (22)** , respectively.

Call Karnik-Mendel and obtain $\left[\underline{b}^{T,I,F}_k,\overline{b}^{T,I,F}_k\right]$, $\underline{L}^{T,I,F}_k,\overline{L}^{T,I,F}_k$ for $\langle\alpha_k,\beta_k,\gamma_k\rangle$ using **Equations (21) and (22);**

Endif;

End for;

Obtain the type-reduced set $\hat{P}^{T,I,F}_i$ as T1NCPR.

End procedure.

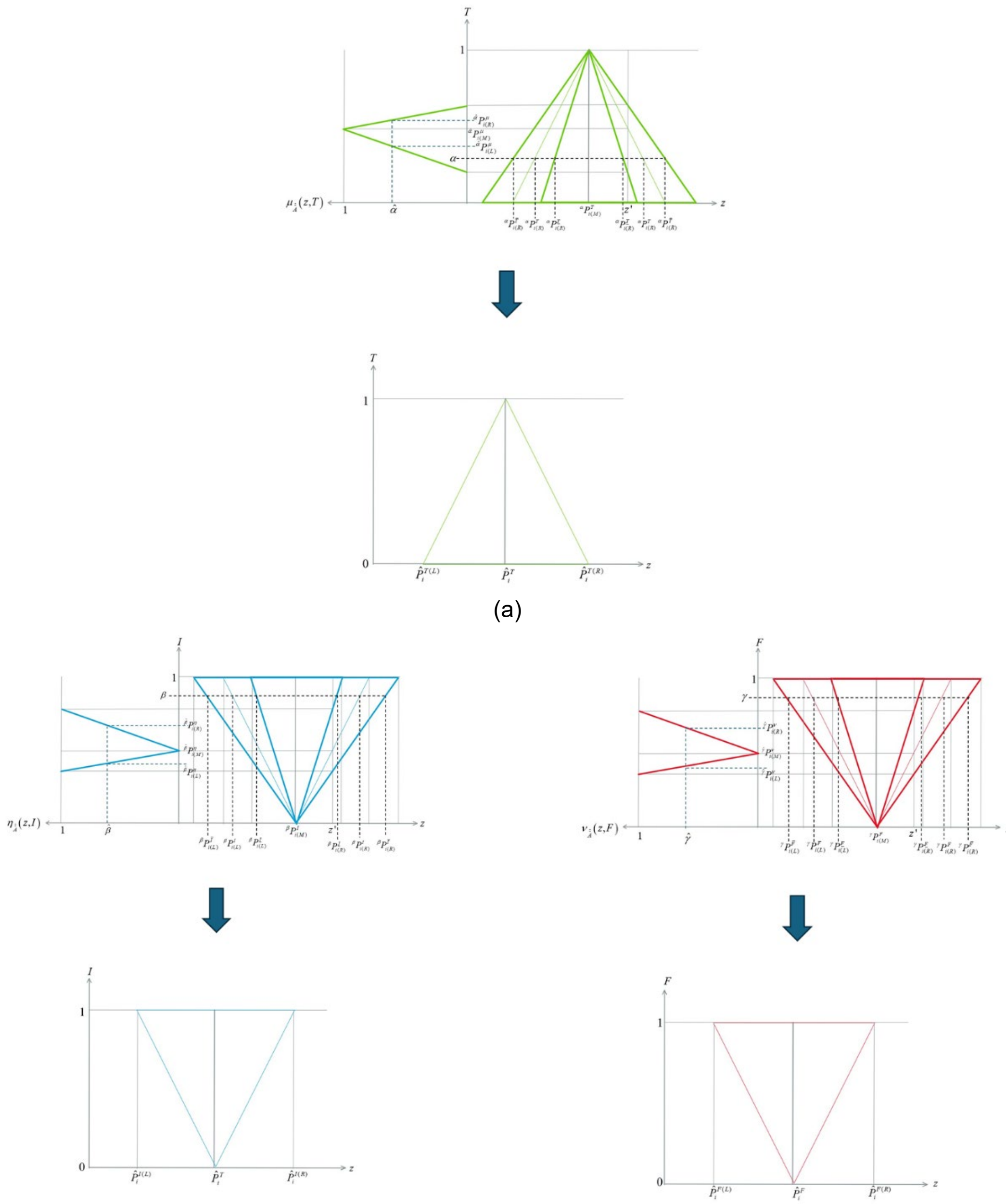
Where $\hat{\hat{P}}_i$ is the type-2 neutrosophic control point relation. Supposed $\alpha-\text{cut}$ as the cut set operation for $\hat{\hat{P}}^T_i$, $\beta-\text{cut}$ as the cut set operation for $\hat{\hat{P}}^I_i$, and $\gamma-\text{cut}$ as the cut set operation for $\hat{\hat{P}}^F_i$. For the secondary membership, $\hat{\alpha}-\text{cut}$ as the cut set operation for $\hat{\hat{P}}^\mu_i$, $\hat{\beta}-\text{cut}$ as the cut set operation for $\hat{\hat{P}}^\eta_i$, and $\hat{\gamma}-\text{cut}$ as the cut set operation for $\hat{\hat{P}}^\nu_i$. If the $\left\langle\alpha,\beta,\gamma,\hat{\alpha},\hat{\beta},\hat{\gamma}\right\rangle-\text{cut}$ operations have been done against $\hat{\hat{P}}^{T,I,F,\mu,\eta,\nu}_i$ and will be reduced to a type-1 neutrosophic control point relation (T1NCPR), $\hat{P}^{T,I,F}_i$ by using $\langle\alpha,\beta,\gamma\rangle-\text{cut}$ operations. While the Equations (23) to (26) are as follows:

$$t=\overline{b}_k=\underline{b}_k=\frac{\sum_{j=1}^{n}x_j\frac{\left(\overline{I}_j+\underline{I}_j\right)}{2}}{\sum_{j=1}^{n}\frac{\left(\overline{I}_j+\underline{I}_j\right)}{2}} \tag{19}$$

$$\underline{L}_i=\underline{L}_{i+1},\qquad \overline{L}_i=\overline{L}_{i+1} \tag{20}$$

$$\underline{b}_i=\frac{\sum_{j=1}^{L_i}x_j\overline{I}_{j,i}+\sum_{j=\underline{L}_{i+1}}^{n}x_j\underline{I}_{j,i}}{\sum_{j=1}^{L_i}\overline{I}_{j,i}+\sum_{j=\overline{L}_{i+1}}^{n}\underline{I}_{j,i}} \tag{21}$$

$$\overline{b}_i = \frac{\sum_{j=1}^{\overline{L}_i} x_j \underline{I}_{j,i} + \sum_{j=\overline{L}_{i+1}}^{n} x_j \overline{I}_{j,i}}{\sum_{j=1}^{\overline{L}_i} \underline{I}_{j,i} + \sum_{j=\overline{L}_{i+1}}^{n} \overline{I}_{j,i}} \tag{22}$$



(a)

(b) (c)

Figure 3. Neutrosophic type reduction for T2NCPR for; (a) Truth; (b) Indeterminacy and (**c**) Falsity memberships

Neutrosophic B-Spline Curve (NB-sC) Approximation Model

Based on the definition of T2NCPR, this section discusses the T2NB-sC approximation model. The T2NB-sC is mentioned in Definition 6 for truth, indeterminacy, and falsity memberships.

Definition 6 (General Type-2 Neutrosophic B-spline Curve)

Let $\hat{\hat{P}} = \left\{\hat{\hat{P}}_i\right\}_{i=1}^{n+1}$ as a set of type-2 neutrosophic control point relations, thus the type-2 neutrosophic B-spline curve is defined as follows:

$$BsC(t) = \sum_{i=1}^{n+1} \hat{\hat{P}}_i N_i^k(t) \tag{23}$$

With

a) $\hat{\hat{P}}_i$ is T2NCPR numbered from 1 to *n*+1.

b) N_i^k is a B-spline basis function that is written as

$$N_i^1(t) = \begin{cases} 1 & \text{if} \quad t_i \le t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

For *k*=1 and

$$N_i^k(t) = \frac{(t - t_i)}{t_{i+k-1} - t_i} N_i^{k-1}(t) + \frac{(t_{i+k} - t)}{t_{i+k} - t_{i+1}} N_{i+1}^{k-1}(t) \tag{25}$$

For *k*>1 and $i = 2, \ldots n+1$.

c) Value for t_i is knot vector elements that satisfy the relationship $t_i \le t_{i+1}$ and parameter *t* different and changing from $t_{\min}$ to $t_{\max}$ along the curve $BsC(t)$.

Next is the type-2 neutrosophic B-spline curve for truth, T, indeterminacy, I, and falsity, F for primary memberships while truth, μ, indeterminacy, η, and falsity, ν for secondary memberships that will be fuzzified on the *z*-axis can be described as follows:

Definition 7 (Type-2 Neutrosophic B-spline Curve for T, I, F, μ, η and ν)

Let $\hat{\hat{P}}_i^{T,I,F}$ and $\hat{\hat{P}}_i^{\mu,\eta,\nu}$ as the T2NCPR for truth, indeterminacy, and falsity memberships where $i = 1, 2, 3, \ldots, n+1$ for primary and secondary memberships, respectively. T2NBsC is defined as $BsC(t)$ with the curve position vector depending on the value of $n+1$.Thus, the T2NB-sC for truth, indeterminacy, and falsity's primary and secondary memberships are as follows:

$$BsC(t) = \left\langle BsC^T(t), BsC^I(t), BsC^F(t), BsC^\mu(t), BsC^\eta(t), BsC^\nu(t) \right\rangle \tag{26}$$

with

$$\sum_{i=1}^{n+1} \hat{\hat{P}}_i N_i^n(t) = \left\langle \begin{array}{l} \sum_{i=1}^{n+1} \hat{\hat{P}}_i^T N_i^n(t), \sum_{i=1}^{n+1} \hat{\hat{P}}_i^I N_i^n(t), \sum_{i=1}^{n+1} \hat{\hat{P}}_i^F N_i^n(t), \\ \sum_{i=1}^{n+1} \hat{\hat{P}}_i^\mu N_i^n(t), \sum_{i=1}^{n+1} \hat{\hat{P}}_i^\eta N_i^n(t), \sum_{i=1}^{n+1} \hat{\hat{P}}_i^\nu N_i^n(t) \end{array} \right\rangle \tag{27}$$

if the T, I, F, μ, η and ν follows the conditions in Equations (13) and (14).

To construct a B-spline model, the knot vector plays a significant influence on the B-spline basis function $N_i^k(t)$ and there are a few types of knot vectors. This study uses an open uniform knot vector for order $k = 4$, $[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 2 \quad 2 \quad 2 \quad 2]$.

Figure 4 shows primary and secondary T2NBsC with neutrosophication on the *z*-axis. From Figure 4, each membership B-spline curve consists of its respective boundary curves. Then, at a certain membership degree value, the T2NBsC model needs to go through cut set operations to find its neutrosophic type reduction and define it as Algorithm 1.

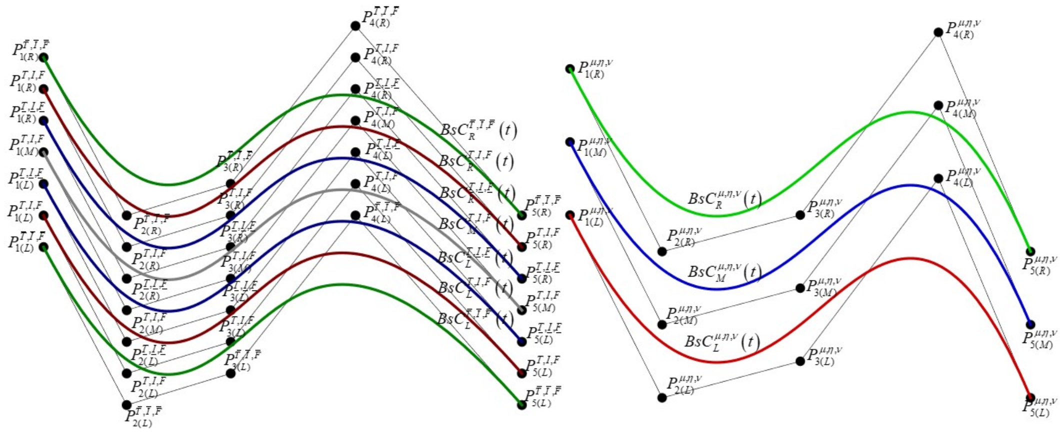


Figure 4. Type-2 neutrosophic B-spline curve for primary and secondary memberships

Neutrosophic B-Spline Curve (NB-sC) Approximation Model

Neutrosophic type-reduction comes next after the neutrosophication method to transform the T2NB-sC, $BsC(t)$ to type-1 neutrosophic B-spline curve (T1NB-sC). This section will produce and demonstrate the T1NB-sC when $\alpha = 0.2$, $\beta = 0.8$ and $\gamma = 1$ by using the interval secondary cut sets, $\left[{}^{\hat{\alpha},\hat{\beta},\hat{\gamma}}P^{\mu,\eta,\nu}_{(L)}, {}^{\hat{\alpha},\hat{\beta},\hat{\gamma}}P^{\mu,\eta,\nu}_{(R)} \right]$, . Based on Algorithm 1 in Section 4 which is neutrosophic type-reduction of T2NCPR, this section will use the algorithm for T2NB-sC. Thus, the algorithm is as follows:

Algorithm 2 (Neutrosophic Type-Reduction of T2NB-sC)

Let $\hat{\alpha}_i = [\hat{\alpha}_1 = 0.2]$, $\hat{\beta}_i = [\hat{\beta}_1 = 0.8]$ and $\hat{\gamma}_i = [\hat{\gamma}_1 = 1]$,

Procedure Improved $\left(\hat{\hat{A}}, \hat{\alpha}, \hat{\beta}, \hat{\gamma} \right)$

For $i = 1$,

Take $\left\langle \hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1 \right\rangle - \text{cuts}$ of secondary membership functions of $\hat{\hat{A}}$;

Let these $\left\langle \hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1 \right\rangle - \text{cuts}$ be $\left[{}^{\hat{\alpha},\hat{\beta},\hat{\gamma}}P^{\mu,\eta,\nu}_{1(L)}, {}^{\hat{\alpha},\hat{\beta},\hat{\gamma}}P^{\mu,\eta,\nu}_{1(R)} \right]$,

Form these $\left\langle \hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1 \right\rangle - \text{cuts}$ as the $\left\langle \alpha, \beta, \gamma \right\rangle - \text{plane}$ for $\hat{P}_1^{T,I,F}$;

If $(i = 1)$

Call Karnik-Mendel and obtain $\left[\underline{b}_1^{T,I,F}, \overline{b}_1^{T,I,F} \right]$, $\underline{L}_1^{T,I,F}, \overline{L}_1^{T,I,F}$ for $\left\langle \alpha_1, \beta_1, \gamma_1 \right\rangle$ using **Equations (23);**

Else

Initialize $\underline{L}_1^{T,I,F}$ and $\overline{L}_1^{T,I,F}$ by **Equation (24)**,

Initialize $\left[\underline{b}_1^{T,I,F}, \overline{b}_1^{T,I,F} \right]$ by **Equations (25)** and **(26)** respectively.

Call Karnik-Mendel and obtain $\left[\underline{b}_1^{T,I,F}, \overline{b}_1^{T,I,F} \right]$, $\underline{L}_1^{T,I,F}, \overline{L}_1^{T,I,F}$ for $\left\langle \alpha_1, \beta_1, \gamma_1 \right\rangle$ using **Equation (25)** and **(26);**

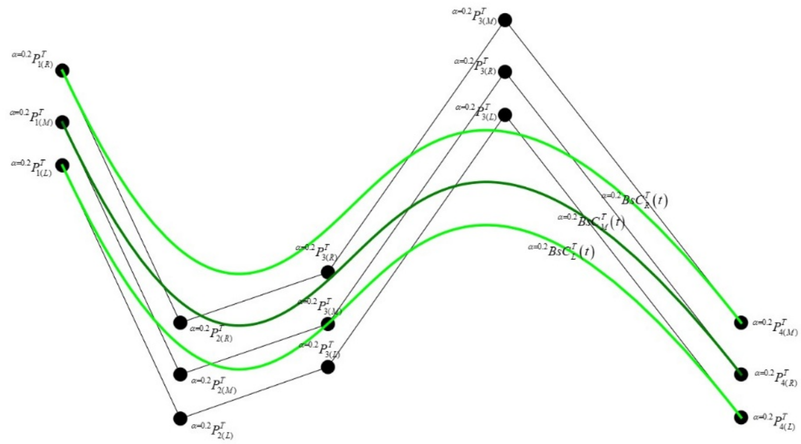
End if;

End for.

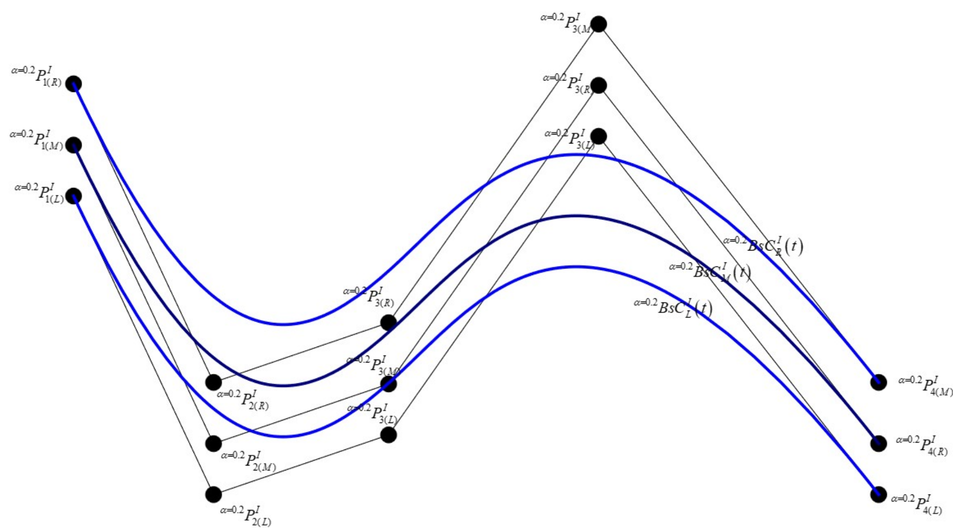
Obtain the type-reduced set $BsC_i^{T,I,F}$ as T1NCPR.

End procedure.

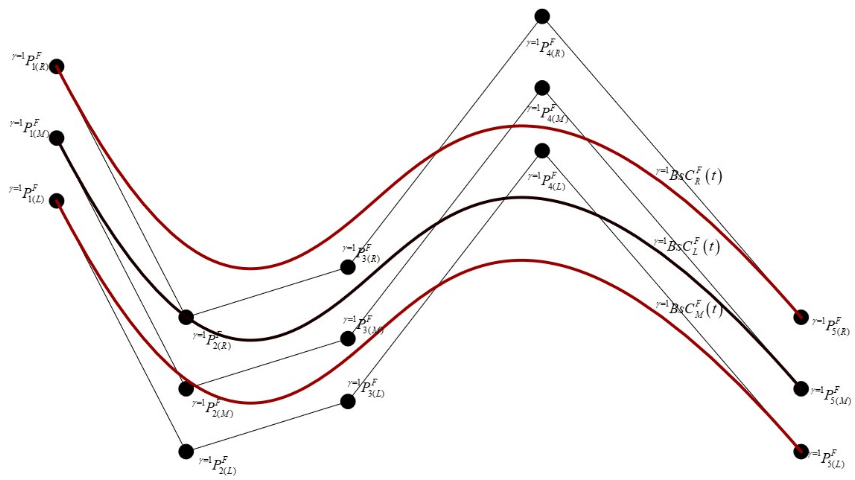
Figure 5 shows the T1NsBC after going through the neutrosophic type-reduction of T2NBsC. Thus, the type-reduced set or T1NBsC for $\alpha = 0.2$, $\beta = 0.8$ and $\gamma = 1$ are illustrated as follows:



(a)



(b)



(c)

Figure 5. Type-1 neutrosophic B-spline curve for; (a) Truth membership when $\alpha = 0.2$; (b) Indeterminacy membership when $\beta = 0.8$ and (c) Falsity membership when $\gamma = 1$

Finally, Figure 5 depicts the curves between left, right, and mean curves for truth, indeterminacy, and falsity memberships to visualize the T1NB-sC approximation for specific alpha, beta, and gamma values. In the graphics, the black curve represents the mean T1NB-sCs for truth, indeterminacy, and falsity memberships, whereas the green, blue, and red curves represent the left and right of T1NB-sC for respective degrees. The $\alpha = 0.2$ and $\beta = 0.8$ values are opposite, as stated in Definition 2, which describes T2NN features. However, the T1NB-sCs for truth and indeterminacy are equivalent since the regions are convex and concave, respectively. For falsehood membership, the difference is only 0.1 at $\gamma = 1$. At the end of this section, an algorithm to generate the neutrosophic type reduction of T2NB-sCs is demonstrated as shown in Figure 6.

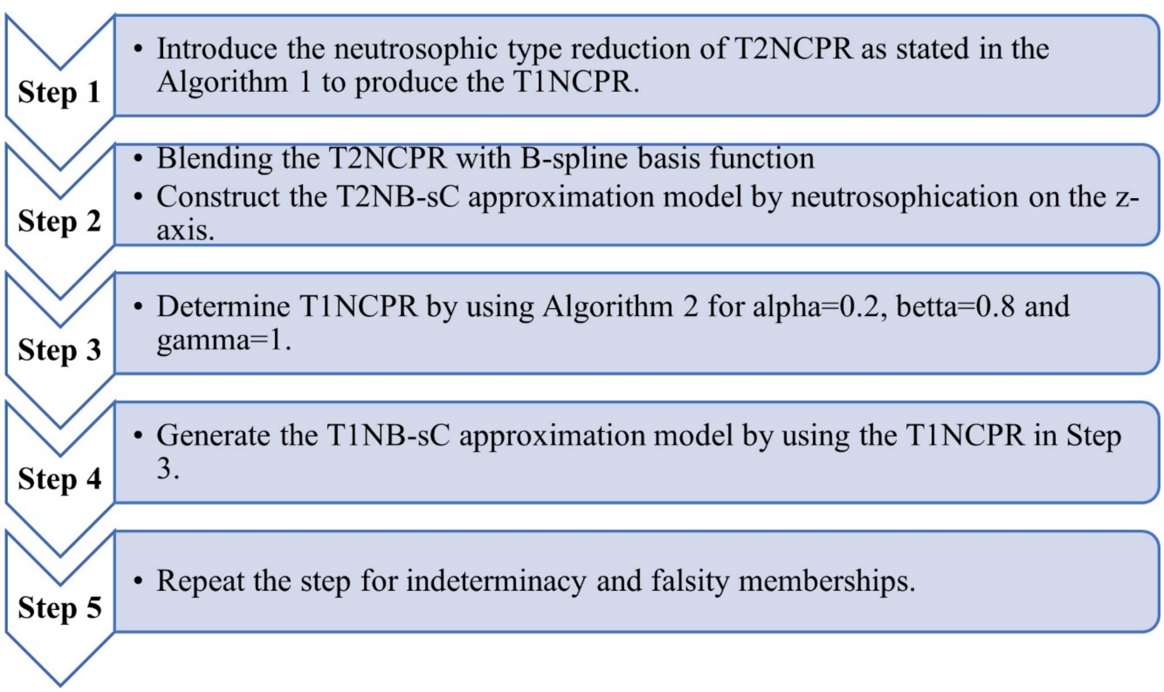


Figure 6. An algorithm for the neutrosophic type reduction process of type-2 neutrosophic B-spline curves approximation models

Neutrosophic B-Spline Curve (NB-sC) Approximation Model

This study proposes a neutrosophic type reduction method for general type-2 neutrosophic models, addressing both primary and secondary memberships of truth, indeterminacy, and falsity using an improved version of the Liu algorithm. By applying cut set operations, the method enables the representation of both primary and secondary membership degrees—truth, indeterminacy, and falsity—based on the type reduction of the T2NCPR. One of the key benefits of this technique is its ability to define α, β, and γ values for the left and right type-1 neutrosophic control point relations (T1NCPR) in the type-1 neutrosophic B-spline curves (T1NB-sCs), regardless of whether these values are identical or distinct, depending on the characteristics of the neutrosophic data.

Future studies could explore alternative basis functions, such as the non-uniform rational B-spline (NURBS) model, to construct neutrosophic curve representations. Additionally, this research can be extended to support surface modeling.

The type reduction process of T2NB-sCs also holds promise for three-dimensional curve modeling applications, such as defining road surface boundaries, delineating closed map perimeters, or calculating surface areas using α, β, and γ level values. Furthermore, the method can be applied in engineering scenarios where uncertainty in data exists, enabling the reconstruction of objects based on their original uncertain properties.

Conflicts of Interest

The author(s) confirm that there are no conflicts of interest associated with the publication of this article.

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