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RESEARCH ARTICLE

Applying Hybrid Fuzzy Conjoint Analysis and Cognitive Maps to Identify Influential Attribute Relationships for Assessment Model Development

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Abstract In order to remedy the attribute model's complexity issue, a more systematic strategy to getting started planning must be undertaken during developing an assessment model. This is necessary to resolve problems and controversies associated with the analysis and interpretation of attribute priority and relationships. The aim is to create a more structured model. Hence, this study proposes the use of hybrid triangular fuzzy conjoint and cognitive maps (TrFCCM) methods as an intelligent procedure for identifying attribute priority and constructing an influential relations map (IRM) during the early stage of assessment model development. The case study demonstrates the successful implementation and suitability of this procedure. The findings indicate that executive function plays a significant role in determining students' mathematics problem-solving ability, followed by attention, working memory, emotion, metacognition, and motivation attributes as alternative assessment factors. Furthermore, the resulting IRM provides insights into the relationship between attributes and enhances understanding of the importance of neuroscience mechanistic in mathematics problem-solving ability. The present research advances the scientific knowledge of how analyses multi-criteria decision-making and human decisions using a triangular fuzzy number-based conjoint and cognitive mapping procedure. It also introduces a more effective procedure for identifying and extracting influential relations among attributes during assessment model development. More impressively, this procedure demonstrates a higher level of application, usability, and performance compared to the state-of-the-art (SOTA) procedure.

Keywords: Fuzzy conjoint method, fuzzy cognitive maps, assessment model, attributes, triangular fuzzy number, problem-solving ability.

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Introduction

Mathematics is at the basis of the modernization of science and technology, in addition to being a crucial component in the field of Science, Technology, Engineering, and Mathematics (STEM) [1]. Mathematics education becomes quite difficult when it is used as a standard for evaluation and chosen as a component in establishing a country's educational achievement, like through international assessments such as the Trend in Science and Mathematics Study (TIMSS) and the Programme for International Student Assessment (PISA) [2]. Students' mathematics problem-solving ability (SMPSA) is one of the vital elements of the evaluation [3,4]. As a result, any problems or circumstances that may hinder the success and outcome of SMPSA must be identified, examined, analysed, and then treated with prudently.

The key problems of the situation are the level of MPSA remains low [4-9]. Based on previous studies, it was found that among the attributes (factors) that lead to SMPSA are such as emotion, readiness, motivation towards mathematics, metacognitive coordination, memory, problem-solving mechanisms

[10-15]. In the context of neuroscience (NS) knowledge, these attributes are the stages of the mechanism of neural connections and brain region networks that occur before, during and after learning [16,17]. This condition is known as neuroscience mechanistic, which is the manifestation or behavioral process of neuroplasticity and neurotransmitters that form a network of activity in any part of the brain according to the orientation of stimuli and information received [18,19]. Neuroscience computing studies found that once students are exposed to mathematics tasks, parts of the brain such as the posterior cingulate, medial frontal cortex, ventral frontal regions, left temporal cortex, and left dorsal will be active and form connections [20-25]. According to [26], the 4 neuroscience mechanistic attributes introduced in the AGES Model namely attention, generation, emotion, and spacing play an important role in determining the success of learning processes such as mathematics. However, according to [27], there is less study that links mathematics problem solving and mechanistic neuroscience attributes.

As a result, the causal factors (attributes) must be investigated to determine the effect and significance of each upon SMPSA. If these attributes are filtered and discovered early on, suitable intervention can be implemented to guarantee that the SMPSA is at its best. Therefore, apart from written tests where pen and paper are the main medium, we need a more intelligent and flexible assessment platform as a backup. So, there is a need for an alternative assessment model that can measure SMPSA based on neuroscience mechanistic attributes as an indicator model. Experts and researchers such as [2,11,28] suggest that to deal with problems in SMPSA. Nevertheless, the development of the evaluation model requires a more systematic initial planning to deal with the attribute model's complexity issue [29-31]. [31,32] present issues and controversies regarding the procedure of analyzing and interpreting attributes in model development. According to them, the error in choosing the analysis procedure causes the attributes to not be justified accurately. If appropriate and accurate procedures are used, identification and influential relations among attributes can be extracted as best as possible so that those attributes can provide a positive impact to users in the future [29].

The identification procedure and analysis of influential relations among attributes is related to multi-criteria decision-making and human judgments [33,34]. As a result, every single data collecting and analysis procedure required the use of a system capable of defining complexity in human perception and thought [35,36]. Fuzzy analytic models and multi-criteria decision-making methods are ideal for acquiring different perspectives or making decisions by assessing levels based on preferences [37,38]. According to [32,35], educators are the parties who often develop measurement and assessment models. Nevertheless, the study by [39] revealed that there are constraints among educators in applying fuzzy analytic models and multi-criteria decision-making methods due to a knowledge and practical gap. In conclusion, to fill gaps and reduce problems in the procedure of identifying and analyzing influential relations among attributes to develop an assessment model. Therefore, this study proposes a procedure known as the Triangular Fuzzy Conjoint Cognitive Maps (TrFCCM) method. The present study offers a more accurate attribute identification technique and an influential relations analysis procedure in making decisions for developing an assessment model. This paper's main contributions include:

i. Demonstrate how the triangular fuzzy number-based conjoint and cognitive map methods can be used to analyse data from expert surveys.
ii. Introducing a more effective attributes identification and influential relations analysis procedure to extract an early overview of the model elements.

In summary, the proposed methodology combines fuzzy conjoint and fuzzy cognitive map methods within fuzzy analytics and multi-criteria decision-making. Firstly, all the attributes are gathered through literature review with guidance and content analysis. Next, the relevance of the attributes will be assessed and ranked using the fuzzy conjoint model, which assigns a degree of similarity to each attribute. Next, the fuzzy cognitive maps are employed to calculate the weightage of each attribute and map the influential relationships between them. The process and potential benefits of the proposed framework are illustrated using a real case study involving experts and teachers in Pasir Gudang District, Johor, Malaysia. The remaining sections of the paper are organized as follows, the next section provides a summary of the literature review on fuzzy triangular number, fuzzy conjoint model, fuzzy cognitive map, and SMPSA assessment model development problem. Next section presents the development of a hybrid TrFCCM method. Case study section describes an empirical case conducted in Pasir Gudang District, Johor, to demonstrate the proposed method. Finally, the next section offers discussion, comparison performance, conclusions and suggestions for future research directions.

Literature Review

Triangular Fuzzy Number and Fuzzy Conjoint Model

The previously, triangular fuzzy number *A* is generally described as ways.

Definition 1. [40] The triplet (a_1, a_2, a_3) can be used to define a triangular fuzzy number *A*. The algorithm $\mu_A(x)$ for memberships function is

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{x - a_3}{a_2 - a_3}, & a_2 \le x \le a_3 \\ 0, & a_3 < x \end{cases}$$

is in position $a_1 \le a_2 \le a_3$, in which the values of a_1 dan a_3 correspond to the lower and upper elements of *A*, whereas a_2 stands for the medium value.

Definition 2. [41] $A * B = \{a_i * b_j,\ a_i \in A, b_j \in B\}$ establishes the arithmetic operations on the triangular fuzzy numbers $A = (a_1, a_2, a_3)$ with $B = (b_1, b_2, b_3)$, when $* = \{+, -, \times, \div\}$. To make it more specific, assuming any pair of triangular fuzzy numbers, $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$, then

For addition operation $(+)$: $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ (1)

For Subtraction operation $(-)$: $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ (2)

For multiplication $(\times)$: $k \times A = (ka_1, ka_2, ka_3), k \in R, k \ge 0$ (3)

$$A \times B = (a_1 b_1, a_2 b_2, a_3 b_3) \tag{4}$$

For division operation $(\div)$: $A^{-1} = (a_1, a_2, a_3)^{-1} \cong \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right)$,

$$a_1 > 0, a_2 > 0, a_3 > 0$$

$$A \div B \cong \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right), a_1 \ge 0, b_1 \ge 0 \tag{5}$$

Definition 3. [42] This equation can be used to determine the similarity degree of *A* and *B*:

$$Sim(A, B) = \frac{1}{1 + d(A, B)} \tag{6}$$

is in position $d(A, B) = |P(A) - P(B)|$, where $P(A) = \frac{a_1 + 4a_2 + a_3}{6}$ and $P(B) = \frac{b_1 + 4b_2 + b_3}{6}$.

Triangular Fuzzy Cognitive Maps

Triangular Fuzzy Cognitive Maps (TrFCM) are more suitable to be implemented with the orientation of the first position of the data being unsupervised [43]. TrFCM can work and perform functions with at least three expert opinions [44]. TrFCM will model any feed item as a collection of attributes and mapping causal relationships between them. This efficiency is very different compared to the conventional FCM model, that is, usually FCM only builds relationships and ON-OFF positions between attributes [43,45]. However, through this Triangular Fuzzy Cognitive Maps, the position and intertwining of the causes of the problem can be extracted more accurately by only using the weighting of the attribute [39].

Remark 1. When the TrFCM nodes are fuzzy sets, they are referred to as fuzzy triangular nodes.
Remark 2. Simple Triangular FCMs are those that have edge weights or causalities from the set {-1, 0, 1}.
Remark 3. A Triangular Fuzzy Cognitive Maps (TrFCM) is a directed graph with concepts like attributes, criteria etc, as nodes and causalities as edges, It represents causal relationships between concepts.
Remark 4. When there is a feedback in an TrFCM, or when the causal relations flow through a cycle in a revolutionary way, the TrFCM is called a dynamical system.
Remark 5. A fixed point is the equilibrium state of a dynamical system that is a unique state vector. Consider a TrFCM with nodes TrC_1, TrC_2, … TrC_n.
Remark 6. If the TrFCM settles down with a state vector repeating in the form $A_1 > A_2 > A_i ... > A_1$ then this equilibrium is called a limit cycle.

Triangular Fuzzy Cognitive Maps (TrFCMs) enhance traditional fuzzy cognitive maps by representing each concept node as a fuzzy triangular node, using triangular fuzzy numbers (TFNs) to reflect uncertainty in concepts such as student understanding. In Simple TrFCMs, the edge weights are limited to discrete values {-1, 0, 1}, which simplify causal modeling: 1 indicates a positive influence, -1 a negative one, and 0 no influence, making it especially practical for qualitative expert input. A TrFCM is generally structured as a directed graph, with nodes representing concepts like criteria or symptoms and fuzzy-weighted edges depicting the strength and direction of causal relationships. When feedback loops or cycles are present, where a concept can indirectly influence itself, the system becomes dynamical, allowing the network to evolve iteratively until it stabilizes. A stable state where concept activations no longer change is called a fixed point, indicating equilibrium in the system's dynamics, whereas if the system repeatedly cycles through a sequence of states, it reaches what is known as a limit cycle, signaling periodic behavior. To enhance clarity, Figure 1 showing the fuzzy triangular nodes, causal edges, network structures, feedback dynamics, fixed point convergence, and limit cycles.

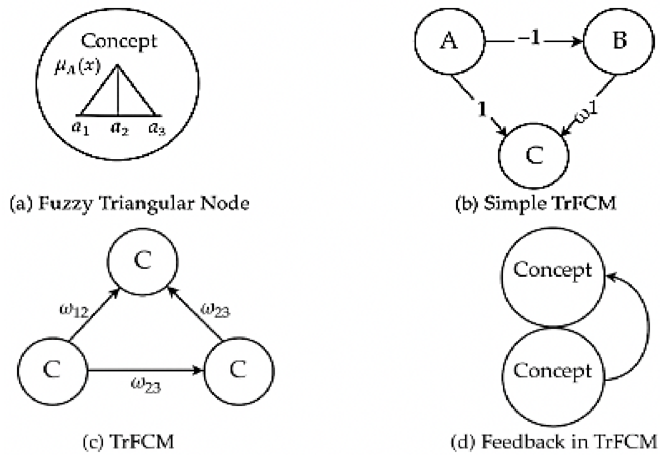

Figure 1. Illustration of TrFCM

Students' Mathematics Problem-Solving Ability and Neuroscience Mechanistic

Neuroscience is a continually developing multidisciplinary field of research and understanding [46]. [47] define neuroscience as the study of how a specific area of the brain changes knowledge, behaviour, mental processes, and learning. Much neuroscience research focuses on the brain's entire performance in the thought process, including the rate, limitations, and potentials of cognition until permanent knowledge, attitudes, values, and behaviours are formed [48]. Mind, Brain, and Education (MBE), and other theories have revealed an understanding of neuroscience's potential in explaining the ability to learn and how students create their learning setting [47,49,50]. The AGES model [26] and the RAD Learning concept [51] analyse how neuroscience promotes student understanding of mathematics. Attention, Generation, Emotion, and Spacing are characteristics of the AGES Model that explain the way individuals cope with every learning circumstance and alter their self and social context [26,51]. Furthermore, the RAD Learning concept, which outlines three neuroscience structures, namely the Reticular activating system (RAS), the Affective filter in the amygdala, and Dopamine, guarantees that learning takes place effectively through self-efficacy as well as interaction with the environment [51].

Recently, the advancement of neuroscience has contributed to the study of one's learning processes, particularly in terms of the functionality of specific areas of the brain involved [50]. According to [52-54], the clash between mathematics education and neuroscience has a significant impact on the ability to solve mathematics problems. Neuroscience research has revealed that mathematics problem-solving is a distinct complicated activity [55,56]. According to researchers, mathematics problem-solving is a mechanistic involving certain areas of the brain that have a role in structuring activities before, during, and after they occur. Moreover, neuroscience mechanistic (NSM) can assist students in successfully solving mathematics problems. Educators can present enjoyable problem-solving tasks based on students' brain development, capacities, memory, and cognitive functions. [14,27] clarify the way executive brain function develops when mathematics content and problem-solving activities target student brain activity. For this reason, assessment of students' mathematics problem-solving ability (SMPSA) is more appropriate if measured based on neuroscience mechanistic attributes [69].

A Hybrid Triangular Fuzzy Conjoint Cognitive Maps (TrFCCM) Method

In this section, a hybrid model based on the Fuzzy Conjoint and the Fuzzy Cognitive Map models is presented to identify and extract the interdependence and influence between attributes in the development of the assessment model. In summary, the proposed model consists of two main stages, the first is to identify the priority and position of attributes using the Triangular fuzzy conjoint method. Next, the second stage is to construct an influential relations map (IRM) between attributes and calculate their influence weights through the Triangular fuzzy cognitive map method. The flowchart of the proposed TrFCCM method hybrid model is shown in Figure 2.

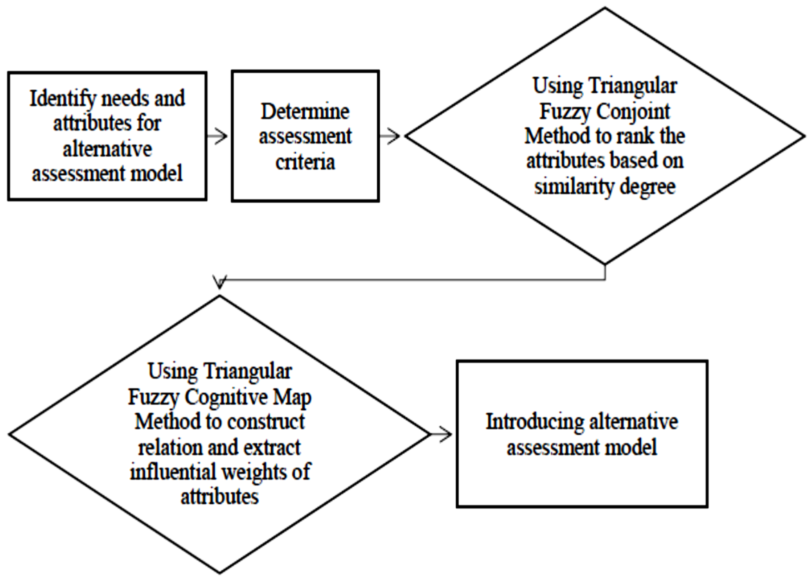

Figure 2. Flowchart of the proposed hybrid TrFCCM model

The Triangular Fuzzy Conjoint Method

Procedure 1: Specify the attribute set that reflects the given input data, $A = \{A_i\}\ (i = 1,2,3 \ldots n)$.

Procedure 2: As specified by TFN, assign suitable predetermined linguistic values to feed scoring, $V_j = \left(b_1^j, b_2^j,\ b_3^j\right)$ with $j = 1,2,3 \ldots k$

Table 1. The way linguistic values in TFN establish the membership function

Linguistic values	Rating	TFNs
Very strongly disagree	1	(0.0, 0.0, 0.1)
Strongly disagree	2	(0.0, 0.1, 0.3)
Disagree	3	(0.1, 0.3, 0.5)
Neutral	4	(0.3, 0.5, 0.7)
Agree	5	(0.5, 0.7, 0.9)
Strongly agree	6	(0.7, 0.9, 1.0)
Very strongly agree	7	(0.9, 1.0, 1.0)

Procedure 3: Compute the number of responds, r_{ij} for each of the linguistic values, $V_j, j = 1,2,3 \ldots k$ matching the attributes A_i.

Procedure 4: Calculate the attribute A_i's weight with linguistic value V_j as follows:

$$w_{ij} = \frac{r_{ij}}{\sum_{j=1}^{k} r_{ij}} \tag{7}$$

Procedure 5: Ascertain the attribute's overall membership function. $\tilde{A}_i = (a_1^i, a_2^i, a_3^i)$ as:

$$\tilde{A}_i = \sum_j^k w_{ij} V_j\,, i = 1,2,3, \ldots n, j = 1,2,3, \ldots k \tag{8}$$

Procedure 6: Establish the degree of similarities to which the aggregated linguistic ratings for the *i*-th attributes $\tilde{A}_i = (a_1^i, a_2^i, a_3^i), i = 1,2,3 \dots n,$ and the linguistic ratings, $V_j = (b_1^j, b_2^j, b_3^j), j = 1,2,3, \dots, k$ by using the formula of similarity measure as:

$$S_{ij}(\tilde{A}_i, V_j) = \frac{1}{1 + d(P(\tilde{A}_i) - P(V_j))}, i = 1,2,3 \dots, n, j = 1,2,3, \dots, k \tag{9}$$

with $P(\tilde{A}_i) = \frac{a_1^i + 4a_2^i + a_3^i}{6}$ and $P(V_j) = \frac{b_1^j + 4b_2^j + b_3^j}{6}$.

The Triangular Fuzzy Cognitive Maps

The procedure in implementing TrFCM are as follows.

Procedure 1: Construct the Triangular Fuzzy Connection Matrix. Employing linguistic values, generate a $n \times n$ fuzzy matrix, that is frequently referred as the connection matrix. The average of each triangular fuzzy number is calculated as $\frac{a_1 + a_2 + a_3}{3}$ to provide a representative crisp value for interpretation and thresholding.

Table 2. The linguistic values

Linguistic values	Weightage of TrFCM	Average value of TrFCM
Very Low (VL)	(0.0, 0.0, 0.25)	0.08
Low (L)	(0.0, 0.25, 0.5)	0.25
Medium (M)	(0.25, 0.5, 0.75)	0.50
High (H)	(0.5, 0.75, 1.0)	0.75
Very High (VH)	(0.75, 1.0, 1.0)	0.92

Procedure 2: Identify the maximum fuzzy weight. From the triangular fuzzy matrix connection Tr(M) determine the maximum fuzzy weight value. This maximum (often based on the average of the TrFCM) will serve as a threshold for concept activation in subsequent steps.

Procedure 3: Initialize and compute state transitions. Let TrC_1, $TrC_2 \cdots TrC_n$ represent the concept nodes of the TrFCM, and let A_1 be the initial state vector, typically $A_1 = (1, 0, 0, \cdots 0)$, where the first concept is activated. Multiply the state vector with the fuzzy connection matrix: $A_1Tr(M) = a_1, a_2, \cdots a_n$. Each resulting value ai is a triangular fuzzy number. Apply a thresholding operation using the maximum fuzzy weight from Procedure 2, denoted by $\rightarrow A1Tr(M)\ Max(weight)$, if a component a_i exceeds the threshold (based on the average value), set it to 1 (activated); otherwise, set it to 0 (deactivated). This results in a new binary state vector A_2.

Procedure 4: Iterate until convergence. Repeat the process: multiply A_2 by *Tr(M)*, apply thresholding, and generate the next state vector A_3, and so on. Continue iterating until the system reaches either: a fixed point, where the state vector remains unchanged across iterations, or a limit cycle, where the state vectors begin to repeat in a cyclic pattern.

This final state reflects the dynamic equilibrium or behavior pattern of the system modeled by the TrFCM.

Case Study

Finding out the priority and position of attributes, to construct an influential relations map (IRM) between attributes and calculate their influence weights, is the primary goal of this research in the step of developing an assessment model to measure Students' Mathematics Problems-solving Ability (SMPSA). This study has involved 42 mathematics teachers, experts in mathematics education and also including neuroscience experts in Pasir Gudang District, Johor, Malaysia. Using quantitative and semi-quantitative approaches, with purposeful simple sampling methods, fuzzy questionnaires and unstructured interviews were administered. Table 3 shows the mapping of attributes used in the first stage (Step 1) to survey mathematics teachers' perceptions of the relevance between neuroscience mechanistic (NSM) and SMPSA, and NSM indicators' aggregating. In the next stage (Step 5), 5 experts were interviewed to determine the level of relationship between attributes, for example the relationship between Emotion (C_1) and Motivation (C_2). The following are the steps in the study, results and discussion of findings.

Table 3. The attributes of the survey towards relevance between NSM and SMPSA and NSM indicators' aggregating

Elements	Attributes	Statement
Relevance between NSM and SMPSA	A_1	SMPSA is determined by the student's thinking process
	A_2	Students who know the process of thinking are students who can solve problems
	A_3	The process of thinking is the Neuroscience practice
	A_4	Neuroscience elements are required in assessing SMPSA
	A_5	Emotions influence students' problem-solving actions
	A_6	Motivation affects students' problem-solving actions
	A_7	Attention of students are important in SMPSA
	A_8	Students' confidence is important in SMPSA
	A_9	Students' satisfaction is important in SMPSA
	A_{10}	Problem definition skills are important in solving mathematics problems
	A_{11}	The ability to visualize problem situations is important in solving mathematics problems
	A_{12}	The ability to abstract problems is important in solving mathematics problems
	A_{13}	The skill of hypothesizing problems is important in solving mathematics problems
	A_{14}	Problem representation skills are important in solving mathematics problems
	A_{15}	Problem modelling skills are important in solving mathematics problems
NSM indicators' aggregating	C_1	Emotion This term refers to either positive or negative self-talk. Affects the student's attention, motivation to study, choice of learning strategies, self-regulation of learning, and their academic achievement.
	C_2	Motivation Refer to individuals' ideas of autonomy and the motives they have for acting in a certain situation. A feeling of willingness, need, want, and compulsion.
	C_3	Attention Refers to human biological systems and complicated cognitive functions that tend to focus on distinguishing features when processing massive amounts of information. Also known as the belief system of humans.
	C_4	Executive function The ability to manipulate objects intellectually, to evaluate, prepare, and strategize. Key components to organizational success, decision-making, and life choices. Other memory systems are provided with cognitive resources.
	C_5	Metacognition Refers to the ability to plan, create goals, and allocate resources before learning, as well as the ability to monitor and reflect on what new things are learnt.
	C_6	Working memory The ability to retain information and recall it later, to process incoming information accurately and rapidly, and to appraise one's own ability to understand something. Include attention control.

Step 1: Extract the responses from the teachers for every attribute A_i and C_i. Those are the outcomes.

Table 4. The frequency in which respondents preferred specific linguistic values

Elements	Attributes	V_1	V_2	V_3	V_4	V_5	V_6	V_7	Total
Relevance between NSM and SMPSA	A_1	0	1	3	7	10	9	7	37
	A_2	0	1	0	6	8	14	8	37
	A_3	0	0	0	14	10	7	6	37
	A_4	0	0	0	9	11	10	7	37
	A_5	0	1	2	7	10	11	6	37
	A_6	0	0	0	6	7	16	8	37
	A_7	0	1	0	7	5	11	13	37
	A_8	0	1	0	4	8	14	10	37
	A_9	0	0	1	8	6	16	6	37
	A_{10}	1	0	0	3	9	14	10	37
	A_{11}	0	0	0	3	10	13	11	37
	A_{12}	0	0	0	3	6	15	13	37
	A_{13}	0	1	1	4	3	19	9	37
	A_{14}	0	0	2	3	9	13	10	37
	A_{15}	0	0	0	4	6	19	8	37
NSM indicators' aggregating	C_1	1	0	0	5	11	6	14	37
	C_2	0	0	1	6	7	10	13	37
	C_3	0	0	0	6	9	6	16	37
	C_4	0	0	1	3	4	14	15	37
	C_5	0	0	2	4	7	9	15	37
	C_6	0	0	2	3	8	13	11	37

Step 2: Use equation (7) to get the weight w_{ij}, indicates the linguistic values V_j associated with attributes A_i and C_i.

Table 5. The weight, w_{ij}.

Elements	Attributes	V_1	V_2	V_3	V_4	V_5	V_6	V_7
Relevance between NSM and SMPSA	A_1	0	0.0270	0.0811	0.1892	0.2703	0.2432	0.1892
	A_2	0	0.0270	0	0.1622	0.2162	0.3784	0.2162
	A_3	0	0	0	0.3784	0.2703	0.1892	0.1622
	A_4	0	0	0	0.2432	0.2973	0.2703	0.1892
	A_5	0	0.0270	0.0541	0.1892	0.2703	0.2973	0.1622
	A_6	0	0	0	0.1622	0.1892	0.4324	0.2162
	A_7	0	0.0270	0	0.1892	0.1351	0.2973	0.3514
	A_8	0	0.0270	0	0.1081	0.2162	0.3784	0.2703
	A_9	0	0	0.0270	0.2162	0.1622	0.4324	0.1622
	A_{10}	0.0270	0	0	0.0811	0.2432	0.3784	0.2703
	A_{11}	0	0	0	0.0811	0.2703	0.3514	0.2973
	A_{12}	0	0	0	0.0811	0.1622	0.4054	0.3514
	A_{13}	0	0.0270	0.0270	0.1081	0.0811	0.5135	0.2432
	A_{14}	0	0	0.0541	0.0811	0.2432	0.3514	0.2703
	A_{15}	0	0	0	0.1081	0.1622	0.5135	0.2162
NSM indicators' aggregating	C_1	0.0270	0	0	0.1351	0.2973	0.1622	0.3784
	C_2	0	0	0.0270	0.1622	0.1892	0.2703	0.3514
	C_3	0	0	0	0.1622	0.2432	0.1622	0.4324
	C_4	0	0	0.0270	0.0811	0.1081	0.3784	0.4054
	C_5	0	0	0.0541	0.1081	0.1892	0.2432	0.4054
	C_6	0	0	0.0541	0.0811	0.2162	0.3514	0.2973

Step 3: This step is to determine the overall membership functions of each attribute A_i and C_i using equation (8).

Table 6. Overall membership functions of attribute A_i and C_i

Elements	Attributes	Overall membership function of attribute A_i and C_i
Relevance between NSM and SMPSA	A_1	(0.408, 0.604, 0.788)
	A_2	(0.512, 0.708, 0.868)
	A_3	(0.396, 0.596, 0.792)
	A_4	(0.468, 0.668, 0.848)
	A_5	(0.432, 0.628, 0.808)
	A_6	(0.548, 0.748, 0.9)
	A_7	(0.528, 0.72, 0.868)
	A_8	(0.544, 0.74, 0.892)
	A_9	(0.5, 0.7, 0.86)
	A_{10}	(0.552, 0.744, 0.892)
	A_{11}	(0.572, 0.772, 0.924)
	A_{12}	(0.612, 0.808, 0.94)
	A_{13}	(0.56, 0.756, 0.892)
	A_{14}	(0.532, 0.732, 0.888)
	A_{15}	(0.588, 0.788, 0.928)
NSM indicators' aggregating	C_1	(0.612, 0.784, 0.912)
	C_2	(0.628, 0.792, 0.9)
	C_3	(0.668, 0.824, 0.924)
	C_4	(0.7, 0.856, 0.936)
	C_5	(0.66, 0.816, 0.908)
	C_6	(0.612, 0.784, 0.9)

Step 4: The last step in the first stage is to determine the similarity degree using equation (9).

Table 7. Similarity degree $S(A_i, V_j)$ for relevance between NSM and SMPSA

A_i	V_1	V_2	V_3	V_4	V_5	V_6	V_7	S_{max}	$V(S_{max})$	Rank
A_1	0.6308	0.6732	0.7680	0.9074	0.9107	0.7804	0.7239	0.9107	V_5	15
A_2	0.5934	0.6308	0.7133	0.8319	0.9980	0.8465	0.7804	0.9980	V_5	1
A_3	0.6334	0.6763	0.7720	0.9130	0.9053	0.7764	0.7205	0.9130	V_4	14
A_4	0.6068	0.6460	0.7328	0.8586	0.9659	0.8206	0.7583	0.9659	V_5	6
A_5	0.6216	0.6628	0.7545	0.8886	0.9305	0.7949	0.7364	0.9305	V_5	11
A_6	0.5803	0.6160	0.6944	0.8065	0.9615	0.8746	0.8043	0.9615	V_5	8
A_7	0.5896	0.6266	0.7079	0.8246	0.9875	0.8542	0.7870	0.9875	V_5	3
A_8	0.5828	0.6188	0.6980	0.8112	0.9684	0.8691	0.7996	0.9684	V_5	5
A_9	0.5964	0.6342	0.7177	0.8380	0.9934	0.8403	0.7752	0.9934	V_5	2
A_{10}	0.5814	0.6173	0.6961	0.8086	0.9646	0.8721	0.8021	0.9646	V_5	7
A_{11}	0.5723	0.6070	0.6830	0.7911	0.9398	0.8934	0.8201	0.9398	V_5	10
A_{12}	0.5616	0.5950	0.6679	0.7708	0.9113	0.9208	0.8431	0.9208	V_6	13
A_{13}	0.5783	0.6137	0.6916	0.8026	0.9560	0.8792	0.8082	0.9560	V_5	9
A_{14}	0.5855	0.6219	0.7019	0.8165	0.9759	0.8631	0.7945	0.9759	V_5	4
A_{15}	0.5678	0.6019	0.6766	0.7825	0.9276	0.9047	0.8296	0.9276	V_5	12

The similarity degree value, as indicated by Table 7 above, falls between 0.9107 and 0.9980. With a rating of V_5 (agree), attribute A_2 has the highest similarity degree value of 0.9980. Attribute A_1 has the lowest similarity degree value, at 0.9107, positioning it into the V_5 rating level. This result determined that the statement "Students who know the process of thinking are students who can solve problems" received the highest agreement. Meaning, the thinking process that is mechanistic neuroscience plays a very important role in determining SMPSA. [6] in a report also submitted that the cognitive appraisal process ensures that students can solve a mathematics problem. In addition, also showing a high value of similarity degree is for attribute A_9 and A_7 with their respective statements related to "satisfaction" and "attention" which are also important in SMPSA. This shows that, SMPSA is inclined to feedback the results obtained after completing the task and also focus on the task. This is a mechanism or regulation of belief, desire, willingness, and motivation related to the metacognitive process mechanism [57].

Table 8. Similarity degree $S(C_i, V_j)$ for NSM indicators'

C_i	V_1	V_2	V_3	V_4	V_5	V_6	V_7	S_{max}	$V(S_{max})$	Rank
C_1	0.5682	0.6024	0.6772	0.7833	0.9288	0.9036	0.8287	0.9288	V_5	4
C_2	0.5663	0.6002	0.6745	0.7796	0.9236	0.9085	0.8329	0.9236	V_5	6
C_3	0.5562	0.5889	0.6602	0.7606	0.8971	0.9357	0.8557	0.9357	V_6	2
C_4	0.5474	0.5792	0.6479	0.7444	0.8746	0.9615	0.8772	0.9615	V_6	1
C_5	0.5591	0.5922	0.6643	0.7661	0.9047	0.9276	0.8489	0.9276	V_6	5
C_6	0.5688	0.6031	0.6781	0.7845	0.9305	0.9021	0.8274	0.9305	V_5	3

Table 8 provides an analysis of the similarity degree, ranking the NSM indicators in ascending order as $C_2 < C_5 < C_1 < C_6 < C_3 < C_4$. The priority positions are as follows, Executive function (C_4), Attention (C_3), Working memory (C_6), Emotion (C_1), Metacognition (C_5), and Motivation (C_2). These results indicate that executive function plays a significant role in determining SMPSA. However, it is important not to overlook other attributes, as these factors have a complex relationship and need to be considered together, especially when assessing students' intellectual abilities more accurately [11,12,14]. Therefore, the analysis proceeds to the second stage, where an influential relations map (IRM) is obtained and the weightage of each attribute is determined using a fuzzy cognitive map, as outlined in the subsequent steps.

Step 5: The next step requires employing linguistic variables linked to the fuzzy cognitive map to generate a fuzzy matrix that's called the connection matrix.

Table 9. Connection matrix of *Tr(M)*

$$Tr(M) = \begin{bmatrix} 0 & VH & M & M & M & M \\ H & 0 & VH & H & M & M \\ H & M & 0 & VH & H & H \\ M & M & M & 0 & VH & H \\ M & M & H & H & 0 & VH \\ M & M & M & H & VH & 0 \end{bmatrix}$$

$Tr(M)$	$Tr(C_1)$	$Tr(C_2)$	$Tr(C_3)$	$Tr(C_4)$	$Tr(C_5)$	$Tr(C_6)$
$Tr(C_1)$	0	(0.75,1,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)
$Tr(C_2)$	(0.5,0.75,1)	0	(0.75,1,1)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)
$Tr(C_3)$	(0.5,0.75,1)	(0.25,0.5,0.75)	0	(0.75,1,1)	(0.5,0.75,1)	(0.5,0.75,1)
$Tr(C_4)$	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	0	(0.75,1,1)	(0.5,0.75,1)
$Tr(C_5)$	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.5,0.75,1)	0	(0.75,1,1)
$Tr(C_6)$	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.75,1,1)	0

Step 6: Considering the linguistic table, ascertain the matrix's the maximum weight.

Table 10. Average weightage of *Tr(M)*

$Tr(M)$	$Tr(C_1)$	$Tr(C_2)$	$Tr(C_3)$	$Tr(C_4)$	$Tr(C_5)$	$Tr(C_6)$
$Tr(C_1)$	0	0.92	0.5	0.5	0.5	0.5
$Tr(C_2)$	0.75	0	0.92	0.75	0.5	0.5
$Tr(C_3)$	0.75	0.5	0	0.92	0.75	0.75
$Tr(C_4)$	0.5	0.5	0.5	0	0.92	0.75
$Tr(C_5)$	0.5	0.5	0.75	0.75	0	0.92
$Tr(C_6)$	0.5	0.5	0.5	0.75	0.92	0

Step 7: Find the limit cycle and identify the triggering pattern.

Let *Tr(C₁)* is *ON* state and other nodes in *OFF* state.

$A^{(1)} = (1,0,0,0,0,0) \rightarrow Tr(C_1)$

$A^{(1)}Tr(C_1)weight = \{(0), (0.75,1,1), (0.25,0.5,0.75), (0.25,0.5,0.75), (0.25,0.5,0.75), (0.25,0.5,0.75)\}$

$A^{(1)}Tr(C_1)average = \{(0), (\mathbf{0.92}), (0.5), (0.5), (0.5), (0.5)\}$

$\rightarrow A^{(1)}Tr(C_1)\max(weight) = (0, 1, 0, 0, 0, 0) = A_1^{(1)} \rightarrow Tr(C_2)$

$A_1^{(1)}Tr(C_2)average = \{(0.69), (0), (\mathbf{0.8464}), (0.69), (0.46), (0.46)\}$

$\rightarrow A_1^{(1)}Tr(C_2)\max(weight) = (0, 0, 1, 0, 0, 0) = A_2^{(1)} \rightarrow Tr(C_3)$

$A_2^{(1)}Tr(C_3)average = \{(0.6348), (0.4232), (0), (\mathbf{0.7787}), (0.6348), (0.6348)\}$

$\rightarrow A_2^{(1)}Tr(C_3)\max(weight) = (0, 0, 0, 1, 0, 0) = A_3^{(1)} \rightarrow Tr(C_4)$

$$A_3^{(1)}Tr(C_4)average = \{(0.3893), (0.3893), (0.3893), (0), (\mathbf{0.7164}), (0.5802)\}$$
$$\rightarrow A_3^{(1)}Tr(C_4)\max(weight) = (0,0,0,0,1,0) = A_4^{(1)} \rightarrow Tr(C_5)$$
$$A_4^{(1)}Tr(C_5)average = \{(0.3582), (0.3582), (0.5373), (0.5373), (0), (\mathbf{0.6591})\}$$
$$\rightarrow A_4^{(1)}Tr(C_5)\max(weight) = (0,0,0,0,0,1) = A_5^{(1)} \rightarrow Tr(C_6)$$
$$A_5^{(1)}Tr(C_6)average = \{(0.3295), (0.3295), (0.3295), (0.4943), (\mathbf{0.6064}), (0)\}$$
$$\rightarrow A_5^{(1)}Tr(C_6)\max(weight) = (0,0,0,0,1,0) = A_6^{(1)} = A_4^{(1)} \rightarrow Tr(C_5)$$

Based on the analysis, the resulting triggering pattern is $C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4 \rightarrow C_5 \rightarrow C_6 \rightarrow C_5$.

Step 8: To the remaining attributes, apply the exact same process.

Table 11. Total weightage of the attributes

Attributes *ON*	$Tr(C_1)$	$Tr(C_2)$	$Tr(C_3)$	$Tr(C_4)$	$Tr(C_5)$	$Tr(C_6)$	Triggering pattern
(1,0,0,0,0,0)	0.3295	0.3295	0.3295	0.4943	0.6064	0	$C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4 \rightarrow C_5 \rightarrow C_6 \rightarrow C_5$
(0,1,0,0,0,0)	0.3582	0.3582	0.3582	0.5373	0.6591	0	$C_2 \rightarrow C_3 \rightarrow C_4 \rightarrow C_5 \rightarrow C_6 \rightarrow C_5$
(0,0,1,0,0,0)	0.3893	0.3893	0.3893	0.584	0.7164	0	$C_3 \rightarrow C_4 \rightarrow C_5 \rightarrow C_6$
(0,0,0,1,0,0)	0.4232	0.4232	0.4232	0.6348	0.7787	0	$C_4 \rightarrow C_5 \rightarrow C_6 \rightarrow C_5$
(0,0,0,0,1,0)	0.4232	0.4232	0.6348	0.6348	0	0.7787	$C_5 \rightarrow C_6 \rightarrow C_5 \rightarrow C_6$
(0,0,0,0,0,1)	0.4232	0.4232	0.4232	0.6348	0.7787	0	$C_6 \rightarrow C_5 \rightarrow C_6 \rightarrow C_5$
Total	2.3466	2.3466	2.5582	3.520	3.5393	0.7787	

A complete version of fuzzy cognitive map is shown in Figure 3.

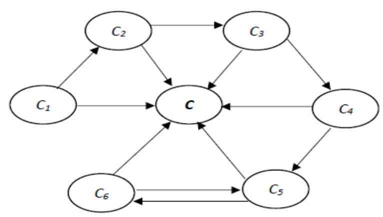

Figure 3. Induced attributes on an influential relations map (IRM)

The analysis of Table 11 displays the total weightage of the attributes and the triggering pattern in the relationship between the attributes. While Figure 3 explains the actual relationship between attributes using a triangular fuzzy cognitive map. Different in the order of ranking obtained in the first stage, where the results of expert consensus explain that attributes C_5 and C_6 which are respectively for Metacognitive and Working memory form a dynamical system. These results reveal and decipher complexities that were previously difficult to translate. This study successfully showed the results of a combination of two methods that are better than the previous study, which only reported the results through one method such as fuzzy number-based conjoint and fuzzy cognitive maps separately, but there are still questions that cannot be explained [37-38, 43, 44, 58-61]. This study also proves the robustness of the fuzzy hybrid method as the same results were obtained by researchers such as [34-36, 45, 62-66]. It can be concluded that, the end of result every time a student is faced with a mathematics task is a synergy of metacognitive abilities, working memory and also influenced by executive function. However, at the beginning it is still influenced by the elements of attention, emotion and motivation. Therefore, the TrFCCM procedure can unravel the attribute complexity and clearly capturing the position and influential relations among the attributes involved.

Performance Comparison

To demonstrate the efficiency and effectiveness of the procedures introduced in this study, performance comparisons were conducted among various methods employing different algorithms. The fuzzy Delphi method (FDM), the fuzzy Decision Making Experiment and Evaluation Laboratory (DEMATEL), and the fuzzy Analytical Hierarchy Process (AHP) had been selected in particular based on their state-of-the-art (SOTA) status and afterwards tested and calibrated.

i. FDM: Compares scores and similarities to determine the level of acceptance and agreement.
ii. Fuzzy DEMATEL: A method for constructing and analyzing relational maps that provides a direct

comparison to evaluate TrFCCM's effectiveness in handling causal relationships and uncertainties.

iii. Fuzzy AHP: Contrasts with TrFCCM to demonstrate the latter's strengths in cognitive mapping, specifically in dealing with fuzziness.

iv. TrFCCM: A proposed method.

There are indeed two stages that comprise this kind of comparison. The first stage aims to assess the accuracy and consistency in producing scores and ranking attributes. In this experiment, the comparison was only performed between TrFCCM and FDM. The second stage involves the production of an influential relations map (IRM) between TrFCCM, fuzzy DEMATEL, and fuzzy AHP. Next, are the detail about criteria, experimental setup, results, and implications of the comparison.

Criteria for Comparison

These methods have been evaluated based on only two criteria, namely structural accuracy, which contains elements such as overall performance, positional consistency, aggregation, interpretability and strength. Second, sensitivity analysis for multiple attributes was also conducted to evaluate the computational efficiency. This metric was chosen because it is important and sufficient to determine the practical applicability of each method in constructing attribute scores, rankings and IRM.

Experiment Setup

The experimental setup for comparing the performance of the four methods involves several key steps. A dataset consisting of 21 attributes was collected through the administration of a questionnaire with the aim of obtaining expert opinions. The attributes are then divided into two parts based on the elements that have been set as in Table 3. Pre-processing steps including filtering and data cleaning are being performed to guarantee the reliability and consistency of the input data. Every method had been employed for analysing the dataset. The first application is to determine the score and then based on the score, each attribute is ranked. For this process, TrFCCM and FDM procedures are applied. Meanwhile, in forming an influential relations map (IRM), the performance of TrFCCM was tested together with fuzzy DEMATEL and fuzzy AHP methods. The performance of each method was evaluated using the defined criteria. Multiple runs were conducted to assess accuracy and sensitivity, ensuring the robustness of the results.

Performance Metrics and Results

The following table and figure summarize the performance of each method. For the first level of comparison, the results are as shown in Table 12.

Table 12. Attributes scores and ranking comparison

Elements	Attributes	FDM		TrFCCM	
		Fuzzy score	Ranking	Similarity	Ranking
Relevance between NSM and SMPSA	A_1	0.600	14	0.9107	15
	A_2	0.696	10	0.9980	1
	A_3	0.595	15	0.9130	14
	A_4	0.661	12	0.9659	6
	A_5	0.623	13	0.9305	11
	A_6	0.732	5	0.9615	8
	A_7	0.705	9	0.9875	3
	A_8	0.725	7	0.9684	5
	A_9	0.687	11	0.9934	2
	A_{10}	0.729	6	0.9646	7
	A_{11}	0.756	3	0.9398	10
	A_{12}	0.787	1	0.9208	13
	A_{13}	0.736	4	0.9560	9
	A_{14}	0.717	8	0.9759	4
	A_{15}	0.768	2	0.9276	12
NSM indicators' aggregating	C_1	0.769	5	0.9288	4
	C_2	0.773	4	0.9236	6
	C_3	0.805	2	0.9357	2
	C_4	0.831	1	0.9615	1
	C_5	0.795	3	0.9276	5
	C_6	0.765	6	0.9305	3

Table 12 presents the results of the FDM and TrFCCM analysis. It is evident that attribute A_{12} has the highest fuzzy score of 0.787 in the FDM analysis, ranking it in first place. On the other hand, attribute A_3 has the lowest score of 0.595, placing it fifteenth. In the context of the TrFCCM analysis, which is based on FCM, attribute A_2 achieves the highest similarity score of 0.9980 (refer to Tables 7 and 8), also ranking it in first place. This indicates the strong agreement among experts and highlights the importance of this attribute. Conversely, attribute A_1 has the lowest score of 0.9107, placing it fifteenth. These results demonstrate a significant disparity in attribute ranking between the two methods. For instance, attributes A_2 and A_{12} are prominent in the TrFCCM analysis, but in the FDM analysis, their rankings differ significantly. This suggests that TrFCCM establishes strong relationships between attributes and compels experts to consider them during evaluations. In contrast, FDM relies solely on expert consensus for each attribute without considering relationships between attributes. This can be observed in the results for attribute C_4, where both methods obtain high scores (0.831 for FDM and 0.9615 for TrFCCM). This implies that attribute C_4 holds significant importance and influence in decision-making. Furthermore, the application of the method reveals that the ultimate goal determines its relevance. TrFCCM provides a more comprehensive analysis of attribute interactions, while FDM offers meaningful consensus information and facilitates decision-making for stakeholders in the given context.

In summary, FDM is an effective method to quickly reach consensus among experts and can be especially useful in scenarios where expert opinions differ. TrFCCM is more suitable for exploring complex relationships between attributes, and has potential for more specific insights and provides a comprehensive understanding of a problem. To compare the performance of the proposed method (TrFCCM) with Fuzzy DEMATEL and Fuzzy AHP in terms of influential indicators and their order, the step continues by normalizing the score for each method to the same scale to facilitate the comparison process. Then a re-evaluation of the order of indicators for each method and visualizing the map is done. In this process, min-max normalization to scale it between 0 and 1 is used [68]. The results are shown in Table 13 below.

Table 13. Indicator, order and normalized score

Methods	Influential indicator, (C_1-C_6)	Order	Normalized score (C_1-C_6)	Rank (C_1-C_6)
Fuzzy DEMATEL	(15.9784, 16.9772, 18.1004, 18.2431, 18.8257, 17.8190)	$C_1 > C_2 > C_6 > C_3 > C_4 > C_5$	(0.000, 0.372, 0.745, 0.796, 1.000, 0.635)	[6, 5, 3, 2, 1, 4]
Fuzzy AHP	(0.101, 0.122, 0.164, 0.18, 0.211, 0.221)	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6$	(0.000, 0.175, 0.525, 0.658, 0.917, 1.000)	[6, 5, 4, 3, 2, 1]
TrFCCM	(2.3466, 2.3466, 2.5582, 3.520, 3.5393, 0.7787)	$C_6 > C_1 > C_2 > C_3 > C_4 > C_5$	(0.552, 0.552, 0.630, 1.000, 1.000, 0.000)	[4, 5, 3, 2, 1, 6]

When examining the results of fuzzy DEMATEL analysis within the context of the order, it becomes evident that C_1 is the attribute that initiates the process, followed by C_2 and C_6. Similarly, the fuzzy AHP analysis also identifies C_1 as the first attribute, followed by C_2 and C_3. However, in contrast to the TrFCCM analysis, which positions C_6 as the initiator, followed by C_1 and C_2. To facilitate a more focused comparison, creating a visual representation of the normalized scores proves to be highly effective in emphasizing the interpretability and practical insights provided by TrFCCM and other methods.

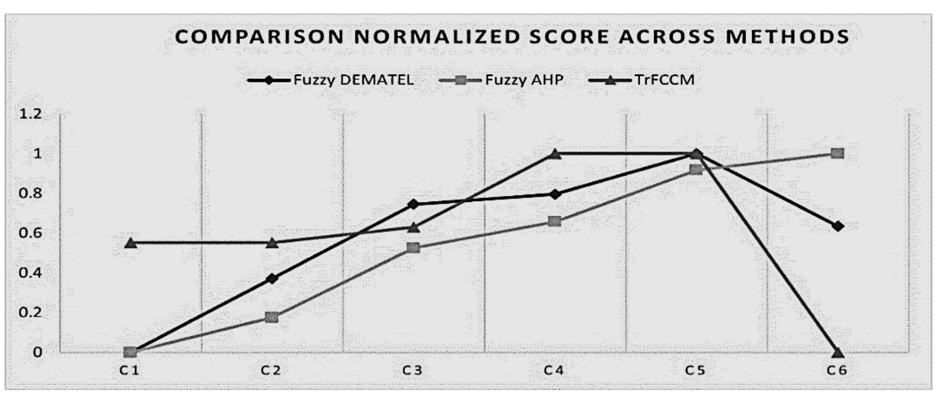

Figure 4. Visualization of influential indicator

The Fuzzy DEMATEL analysis displays a wider range of scores, with the highest score belonging to attribute C_5. Similarly, the AHP Fuzzy analysis shows a range of scores that are the same, with the highest score assigned to C_6. On the other hand, the TrFCCM analysis reveals a different pattern, with high scores for C_1, C_2, C_3, and C_4, and the lowest scores for attribute C_6. This analysis suggests that TrFCCM provides a more unique perspective, confirming that C_1, C_2, and C_3 are equally influential attributes. This viewpoint is more appropriate and aligns better with real-world conditions. It implies that various factors can coexist, each with a comparable level of influence. The normalized scores for TrFCCM also indicate a more balanced distribution of attribute influence, suggesting greater robustness and consistency in its application. Therefore, the findings suggest that TrFCCM offers a clearer analysis, emphasizing the simultaneous ranking of attribute importance. This makes TrFCCM more suitable as a practical decision-making procedure.

To delve deeper into the analysis, a sensitivity analysis is conducted. A slight change amounting to approximately 5% is introduced to the original scores and rankings. The scores and rankings are then recalculated, and the sensitivity is determined by evaluating the resulting impact. Specifically, this sensitivity analysis focuses on multiple attributes, specifically A_1-A_5. The table below presents the results of this experiment.

Table 14. Sensitivity comparison

Attributes	FDM		TrFCCM	
	Ranking change (Original – After = Change)	Sensitivity (%)	Ranking change (Original – After = Change)	Sensitivity (%)
A_1	14 > 13 (1)	7.14	15 > 14 (1)	6.67
A_2	10 > 8 (2)	20.00	1 > 1 (0)	0.00
A_3	15 > 14 (1)	6.67	14 > 15 (1)	7.14
A_4	12 > 11 (1)	8.33	6 > 6 (0)	0.00
A_5	13 > 12 (1)	7.69	11 > 10 (1)	9.09

Observations were made and obtained for both FDM and TrFCCM. In the case of FDM, the sensitivity percentages for attributes ranged from 6.67% to 20.00%. On the other hand, for TrFCCM, the sensitivity percentages for attributes ranged from 0.00% to 9.09%. The lower sensitivity percentages for TrFCCM indicate that small changes in input values have less impact on the rankings compared to FDM. This suggests that TrFCCM is more stable and robust in handling variations in expert inputs, making it superior to FDM in terms of sensitivity.

The findings illustrate that TrFCCM preserves a high level of computational efficiency despite outperforming the other methods with regard to of accuracy and interpretability. The visual representation and the analysis of normalized scores demonstrate that TrFCCM provides a more balanced and insightful approach to constructing Influential Relational Maps (IRMs). This enhances its superiority over Fuzzy DEMATEL and Fuzzy AHP. These findings suggest that TrFCCM has significant potential for future applications in educational research and beyond.

Discussion and Future Directions

Role of Executive Function and Influence of Attention, Working Memory, Emotion, Metacognition, and Motivation in Mathematics Problem-Solving

Findings from the TrFCCM reveal a significant impact of executive function on students' mathematical problem-solving abilities. Executive function encompasses cognitive processes like planning, cognitive flexibility, and inhibitory control, which are crucial in tackling complex math problems. This aligns with existing research that highlights the role of executive function in academic performance, particularly in mathematics [14,27,50,52-54]. Moreover, attention, working memory, emotions, metacognition, and motivation are identified as key factors influencing mathematical problem-solving abilities [69]. Attention is crucial for maintaining focus and concentration during problem-solving tasks. Students with higher levels of attention can better ensure and prepare their cognitive resources and states. Working memory also plays a role in holding and manipulating information during problem-solving. Strong working memory capacity and orientation enable students to handle multiple steps and complex operations. Emotions influence students' engagement and persistence in problem-solving tasks. Positive emotions can enhance motivation and cognitive performance, while negative emotions can hinder thought processes.

Metacognition involves self-awareness and cognitive regulation. Students with strong metacognitive skills can plan, monitor, and evaluate their problem-solving approaches. Additionally, motivation drives the effort and persistence required to solve challenging problems. Intrinsic motivation, in particular, is associated with a deeper appreciation of mathematical tasks.

The study's results also provide educators with insights from IRM that can guide more targeted and focused interventions, allowing them to strengthen key attributes such as executive function and working memory to enhance students' mathematical problem-solving abilities. This finding can also be integrated into curriculum design, where emphasis can be placed on developing cognitive and emotional skills alongside mathematical knowledge. Furthermore, this study bridges the gap between neuroscience research and educational practice. The results demonstrate how understanding cognitive and emotional processes can improve assessment and intervention in learning, in line with similar research reports [69]. As a result, policy makers can use these findings to develop educational policies that support the holistic development of students, including curricula that integrate cognitive, emotional, and motivational factors.

Development of Influential Relations Map (IRM)

The IRM, designed using the TrFCCM method, visually and analytically represents the relationship between influential attributes in students' mathematical learning ability, particularly in problem-solving. This map offers several perspectives:

i. Attribute interrelatedness: The IRM illustrates how attributes are interrelated and mutually influence each other. For instance, executive functioning can directly impact attention and working memory, which in turn affect problem-solving abilities.
ii. Neuroscience mechanistic understanding: The IRM enhances our understanding of the neuroscience mechanisms underlying mathematical problem solving. It demonstrates how cognitive and emotional processes interact to shape student performance.

In summary, the TrFCCM method allows for the development of a more comprehensive and accurate evaluation model that can contribute ideas to minimize issues in the complex interaction of multiple attributes. This approach provides a systematic procedure for early-stage planning in the development of an evaluation model, ensuring that critical attribute positions and relationships are identified prior to their integration with other attributes.

Methodological Contributions

This study introduces a new hybrid approach that combines triangular fuzzy conjoint analysis with cognitive maps. This method effectively addresses the complexity in analyzing attribute relationships in the development of assessment models by handling uncertainty. The use of triangular fuzzy numbers allows for modeling uncertainty and inaccuracy in experts judgement. Another benefit is that it describes the relationship between the agreed attributes. Cognitive maps provide a clear and interpretable visualization of the relationships between attributes, facilitating better understanding and communication of their respective influences.

Limitations and Future Research

First, there are constraints in terms of sample size. The case study was conducted with a specific sample selection, which may limit the generalizability of the study results. Therefore, future research should involve larger and more diverse samples to confirm and extend the results. The second constraint is related to the filtering technique. Although the TrFCCM method is effective, further improvements can enhance its applicability and ease of use. Future studies could explore the integration of advanced computational techniques and software tools. In addition, long-term effects are also part of the constraints. Longitudinal studies are needed to investigate the long-term effects of interventions based on identified attribute relationships. Such studies will provide a deeper insight into the development of problem-solving abilities over time. Future research could also apply the TrFCCM method to other educational domains and subjects to explore its versatility and effectiveness in different contexts. Combining insights from multiple disciplines, such as psychology, neuroscience, and education, can further enhance understanding and the development of assessment models.

Conclusions

As a conclusion, this study effectively proposed a procedure to identify the priority of attributes and able to calculate influence weights as well as to construct an influential relations map (IRM) between attributes, in the step of developing a model. The case study proves that this procedure can be implemented and suitable to be used to identify the priority of attributes. As well as being able to construct an influential relations map among attributes in the initial process of developing an assessment model to measure students' mathematics problems-solving ability. Metacognitive, working memory, and executive function are conceptually interconnected and significantly impact the process of solving mathematics problems. Factors such as attention, motivation, and emotions all interact to ensure that students' cognition and behaviour are at their most effective in determining the perfection of solving mathematics problems. The performance comparison results also explain that this procedure has its own superiority and displays a different perspective, more unique and more consistent. As a step in the future, this procedure can be developed to other problems or can be applied in different fields. Further research is needed, especially to ensure that this procedure is relevant to address the attribute model's complexity issue.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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