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Generalization of the Graph of Fuzzy Topographic Topological Mapping

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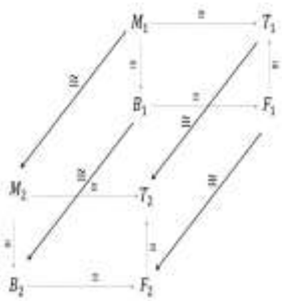

ABSTRACT

In 1999, Tahir *et. al*. developed the Fuzzy Topographic Topological Mapping to determine the location of epileptic foci in epilepsy disorder patients. The model which consists of topological and fuzzy structure is composed into three mathematical algorithms. There are two FTTMs up to now, namely FTTM version 1 and FTTM version 2. FTTM version 1 is homeomorphic to FTTM version 2. The homeomorphism of FTTM can be presented using graphs. In this paper, we generalized the graph of FTTM. In other words, we will prove that, if there exist k versions of FTTM, the new elements will produce a graph of degree $8k^3+8k-24$.

Keywords: FTTM, Sequence of FTTM, Elements Order, Graph of FTTM

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1.2 1. INTRODUCTION

1.3

Epilepsy affects about 1% of the world population [1, 2]. It is one of the most prevalent neurological disorders [3, 4], and a group of related defects characterized by a repeated seizures [5, 6]. Epilepsy happens as a result of abnormal electrical activity in the brain [6, 1]. Brain cells are connected by sending electrical signals in an organized pattern. In epilepsy these electrical signals become abnormal [1, 6], therefore they produce seizures. These signals can be within a certain part or be spread in the brain. Seizures are classified as focal [7] (when seizures appear just in one area of the brain) [8, 9], or generalized [7] (Seizures that appear in all areas of the brain) [8, 10]. Epilepsies are mainly caused by genetic factors [11, 8], while the others are gained / etiological epilepsies [11, 1]. There are many factors involved in the appearance of epileptic seizures, including ischemic injury, tumors, and trauma [12]. Epilepsy can be a complication of many immunological and inflammatory diseases affecting the central nervous system.

In 1999, Tahir *et. al* [13], developed the Fuzzy Topological Mapping (FTTM) technique, in which involved the introduction of the MEG / EEG signals to a fuzzified space, with the aim of identifying the location of the epileptic foci, i.e. the point at which the seizure emanates.

FTTM version 1 (Fig.1) was developed to present a 3-D view of unbounded signal current source [14, 15]

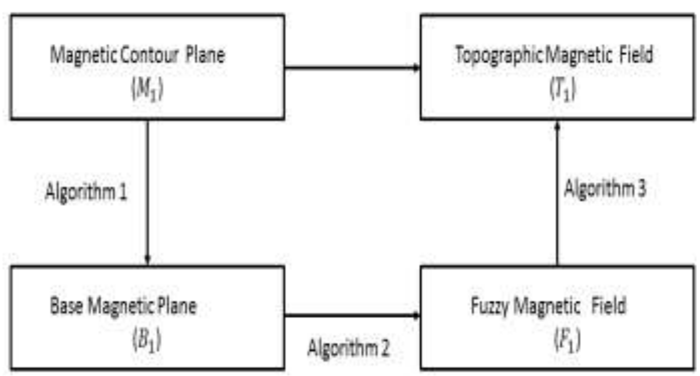

Fig. 1 FTTM 1

Besides that, *FTTM* version 2 (Fig.2) can process image data of magnetic field.

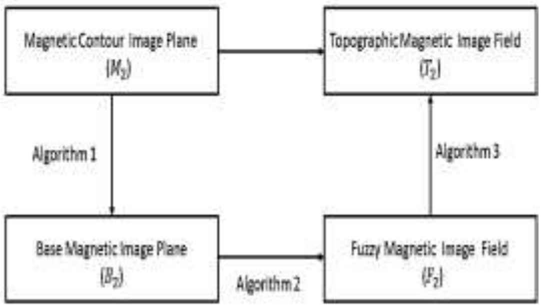

Fig. 2 FTTM2

FTTM version 1 as well as *FTTM* version 2 is specially designed to have equivalent topological structures between its components [16]. In other words, there are homeomorphisms between each element of *FTTM* version 1 and *FTTM* version 2 (Figure 3) [16, 17].

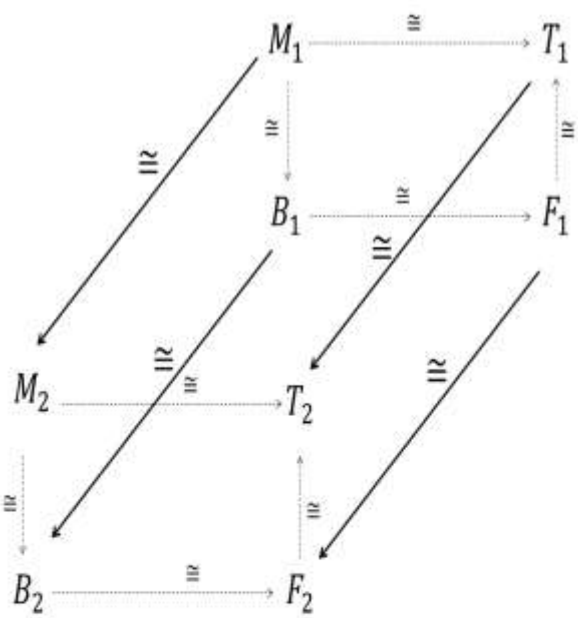

Figure.3 Homeomorphism between FTTM 1 and FTTM 2

2. MATERIALS AND METHOD

In this paper our main focus will be on the graph of the new elements of order two, three and four. Firstly, we will present that if there exist *k* elements of *FTTM*, the new elements of order two will produce a graph of degree $24\mathrm{k}^2 - 16\mathrm{k} - 8$. In addition, we will prove that, the new elements of order three will produce a graph of degree $8\mathrm{k}^3 - 24\mathrm{k}^2 + 24\mathrm{k} - 8$, and the new elements of order four will produce a graph of degree $8\mathrm{k} - 8$. Finally, the degree of the graph of the new elements of *FTTM* will be computed by summing up the degree of the graphs of the new elements of order two, three and four.

1.4 2.1 Graph of Finite Sequence of Fuzzy Topographic Topological Mapping of Order 2

Definition 1: **[**18]

Let $FTTM_i = (M_i, B_i, F_i, T_i)$ be *FTTM* version *i* such that M_i, B_i, F_i and T_i are topological spaces with $M_i \cong B_i \cong F_i \cong T_i$. The sequence of k *FTTM* is

$$FTTM^k = \{FTTM_1, FTTM_2, FTTM_3, \cdots, FTTM_k\}$$

Such that

$$M_i \cong M_{i+1}, B_i \cong B_{i+1}, F_i \cong F_{i+1}, T_i \cong T_{i+1}$$

Definition 2: [19]

Let $FTTM^k = \{FTTM_1, FTTM_2, FTTM_3, \cdots, FTTM_k\}$

be a sequence of k $FTTM$. then

1. The cube that produced from a combination of *i* versions of the sequence of $FTTM$ is said to be a cube of order *i* and $i = 2,3,4$.
2. $C_{i,j}FTTM^k$ represents the cube of order two that can be produced from the combination of $FTTM_i$ and $FTTM_j$ in $FTTM^k$, where $1 \le i \le j \le k$
3. $\left|C_{i,j}FTTM^k\right|_{1\le i\le j\le k}$ represents the number of cubes of order two that can be produced from the combination of $FTTM_i$ and $FTTM_j$ in $FTTM^k$, such that $1 \le i \le j \le k$

Definition 3: [20]

2. The new element is said to be an element of order *i* if its components appear in exactly *i* versions of $FTTM^k$.
3. $\alpha_a FTTM^k$ represents the set of all elements of order *a* that can be generated by $FTTM^k$.

Theorem 1: [19]

$$|\alpha_a FTTM^k| = 7(k^2 - k)$$

Theorem 2: [19]

The new elements of order two will produce a graph of degree $(24k^2 - 16k - 8)$

3.2 Graph of Finite Sequence of Fuzzy Topographic Topological Mapping of Order 3

3.3

Definition 4:

1. $C_{i,j,l}FTTM^k$ represents the cube of order three that can be produced from the combination of $FTTM_i$,

$FTTM_j$ and $FTTM_\mathrm{i}$ in $FTTM^k$, where $1 \leq i < j < l \leq k$

2. $\left|C_{i,j,l}FTTM^k\right|_{1\leq i<j<l\leq k}$ represents the number of cubes of order three that can be presented from the combination of $FTTM_i$, $FTTM_j$ and $FTTM_l$ in $FTTM^k$ such that $1 \leq i < j < l \leq k$

Lemma 1:

$$\left|C_{i,j,l}FTTM^k\right|_{1\leq i<j<l\leq k} = \frac{k(k-1)(k-2)}{6}$$

Proof:

The number of ways to choose three versions from a sequence of k version is $\binom{k}{3}$. Thus

$$\left|C_{i,j,l}FTTM^k\right|_{1\leq i<j<l\leq k} = \binom{k}{3}$$

$$= \frac{k!}{3!\,(k-1)!}$$

$$= \frac{k(k-1)(k-2)}{6}$$

□

Lemma 2:

Any $C_{i,j,l}FTTM^k$ will generate 36 new elements of order 3.

Proof:

The new elements can be generated by simple construction as given below:

$$(M_i,B_i,F_j,T_l),(M_i,B_i,F_l,T_j),(M_j,B_j,F_i,T_l),$$

$$(M_j,B_j,F_l,T_i),(M_l,B_l,F_i,T_j),(M_l,B_l,F_j,T_i),$$

$$(M_i,B_j,F_j,T_l),(M_l,B_j,F_j,T_i),(M_j,B_i,F_i,T_l),$$

$$(M_l,B_i,F_i,T_j),(M_i,B_l,F_l,T_j),(M_j,B_l,F_l,T_i),$$

$$(M_i,B_j,F_l,T_i),(M_i,B_l,F_j,T_i),(M_j,B_i,F_l,T_j),$$

$$(M_j,B_l,F_i,T_j),(M_l,B_i,F_j,T_l),(M_l,B_j,F_i,T_l),$$

$$(M_j,B_l,F_i,T_i),(M_l,B_j,F_i,T_i),(M_i,B_l,F_j,T_j),$$

$$(M_l,B_i,F_j,T_j),(M_i,B_j,F_l,T_l),(M_j,B_i,F_l,T_l),$$

$$(M_i,B_j,F_i,T_l),(M_j,B_i,F_j,T_l),(M_i,B_l,F_i,T_j),$$

$$(M_j,B_l,F_j,T_i),(M_i,B_j,F_l,T_j),(M_j,B_i,F_l,T_i),$$

$$(M_l,B_j,F_l,T_i),(M_i,B_l,F_i,T_j),(M_l,B_i,F_l,T_j),$$

$$(M_i,B_j,F_i,T_l),(M_l,B_j,F_i,T_j),(M_i,B_l,F_j,T_l).$$

All of the above elements are of order three. □

Theorem 3:

$|\alpha_3 FTTM^k| = 6k^3$-18^2+$12k$

Proof:

From Lemma 1 and Lemma 2

$$|\alpha_3 FTTM^k| = 36\left|C_{i,j,l}FTTM^k\right|_{1\leq i<j<l\leq k}$$

$$= 36 \times \frac{\left(k(k-1)(k-2)\right)}{6}$$

$$= 6(k^2-1)(k-2)$$

$$= 6(k^3-3k^2+2k)$$

$$= \quad 6k^3-18k^2+12k$$

□

Lemma 3:

Any element of order three will produce a graph of degree 2 or 0.

Proof:

From Lemma 2, we know that $FTTM_i$, $FTTM_j$ and $FTTM_l$ will generate 36 new elements of order three; 24 elements will produce a graph of degree 2; which are

$$(M_i,B_i,F_j,T_l),(M_i,B_i,F_l,T_j),(M_i,B_j,F_j,T_l),$$

$$(M_i,B_j,F_l,T_i),(M_i,B_j,F_l,T_l),(M_i,B_l,F_j,T_i),$$

$$(M_i,B_l,F_j,T_j),(M_i,B_l,F_l,T_j),(M_j,B_j,F_i,T_l),$$

$$(M_j,B_j,F_l,T_i),(M_j,B_i,F_i,T_l),(M_j,B_i,F_l,T_j),$$

$$(M_j,B_i,F_l,T_l),(M_j,B_l,F_i,T_j),(M_j,B_l,F_i,T_i),$$

$$(M_j,B_l,F_l,T_i),(M_l,B_l,F_i,T_j),(M_l,B_l,F_j,T_i),$$

$$(M_l,B_i,F_i,T_j),(M_l,B_i,F_j,T_l),(M_l,B_i,F_j,T_j),$$

$$(M_l,B_j,F_i,T_l),(M_l,B_j,F_i,T_i),(M_l,B_j,F_j,T_i)$$

while 12 elements will produce a graph of degree zero, because all its vertices are isolated; which are

$$(M_i,B_j,F_i,T_l),(M_i,B_j,F_l,T_j),(M_i,B_l,F_i,T_j),$$

$$(M_i,B_l,F_j,T_l),(M_j,B_i,F_j,T_l),(M_j,B_i,F_l,T_i),$$

$(M_j, B_l, F_j, T_i), (M_j, B_l, F_i, T_l), (M_l, B_i, F_l, T_j),$

$(M_l, B_i, F_j, T_i), (M_l, B_j, F_l, T_i), (M_l, B_j, F_i, T_j)$ □

Theorem 4:

The new elements of order three will produce a graph of degree $8k^2 - 24k + 16$.

Proof:

From Lemma 1, Lemma 2 and Lemma 3 we can obtain the degree of $\alpha_3 FTTM^k$ as follows.

$$deg(\alpha_3 FTTM^k) =$$

$$\frac{k(k-1)(k-2)}{6} \times (24) \times (2) + H$$

$$= 8(k^2 - k)(k-2) + H$$

$$= 8(k^3 - 3k^2 + 2k) + H$$

$$= 8(k^3 - 24k^2 + 16k) + H \quad (1)$$

where H is the number of edges that have to be added to the graph due to the fact that $FTTM_i$ and $FTTM_{i+1}$ are homeomorphic component wise, therefore

$$H = 2(4) + 8(k-2)$$

$$= 8 + 8k - 16$$

$$= 8k - 8 \quad (2)$$

Substitute (2) in (1) we can have

$$deg(\alpha_3 FTTM^k) = 8k^3 - 24k^2 + 16k + 8k - 8$$

$$= 8k^3 - 24k^2 + 24k - 8$$ □

2.3 Graph of Finite Sequence of Fuzzy Topographic Topological Mapping of Order 4

3.4

Definition 5:

1. $C_{i,j,l,m} FTTM^k$ represents the cube of order four that can be produced from the combination of $FTTM_i$, $FTTM_j$, $FTTM_l$ and $FTTM_m$ in $FTTM^k$, where $1 \leq i < j < l < m \leq k$
2. $\left|C_{i,j,l,m} FTTM^k\right|_{1 \leq i < j < l < m \leq k}$ represents the number of cubes of order four that can be presented from the combination of $FTTM_i$, $FTTM_j$, $FTTM_l$ and $FTTM_m$ in $FTTM^k$, such that $1 \leq i < j < l < m \leq k$

Lemma 4:

$$\left|C_{i,j,l,m} FTTM^k\right|_{1 \leq i < j < l < m \leq k} = \frac{k(k-1)(k-2)(k-3)}{24}$$

Proof:

The number of ways to choose four versions from a sequence of k version is $\binom{k}{4}$. Thus

$$\left|C_{i,j,l,m} FTTM^k\right|_{1 \leq i < j < l < m \leq k} = \binom{k}{4}$$

$$= \frac{k!}{4!\,(k-4)!}$$

$$= \frac{k(k-1)(k-2)(k-3)}{24}$$

Lemma 5:

Any $C_{i,j,l,m} FTTM^k$ will generate 24 new elements of order 4.

Proof:

The new elements can be generated by simple construction, as given below.

$(M_i, B_j, F_l, T_m), (M_i, B_j, F_m, T_l), (M_i, B_l, F_j, T_m),$

$(M_i, B_l, F_m, T_j), (M_i, B_m, F_j, T_l), (M_i, B_m, F_l, T_j),$

$(M_j, B_i, F_l, T_m), (M_j, B_i, F_l, T_m), (M_j, B_l, F_i, T_m),$

$(M_j, B_l, F_m, T_i), (M_j, B_m, F_i, T_l), (M_j, B_m, F_l, T_i),$

$(M_l, B_j, F_i, T_m), (M_l, B_j, F_m, T_i), (M_l, B_i, F_j, T_m),$

$(M_l, B_i, F_m, T_j), (M_l, B_m, F_j, T_i), (M_l, B_m, F_i, T_j),$

$(M_m, B_i, F_l, T_j), (M_m, B_j, F_i, T_l), (M_m, B_l, F_j, T_i),$

$(M_m, B_l, F_i, T_j), (M_m, B_i, F_j, T_l), (M_m, B_i, F_l, T_j),$

All of the above elements are of order four. □

Theorem 5:

$$|(\alpha_4 FTTM^k)| = k(k-1)(k-2)(k-3)$$

Proof:

From Lemma 4 and Lemma 5 we can find that

$$|(\alpha_4 FTTM^k)| = 24 \left| C_{i,j,l,m} FTTM^k \right|_{1 \le i<j<l<m\le k}$$

$$= 24 \frac{k(k-1)(k-2)(k-3)}{24}$$

$$= k(k-1)(k-2)(k-3) \qquad \square$$

Lemma 6:

Any element of order four will produce a graph of degree zero.

Proof:

From Lemma 5 we know that $FTTM_i$, $FTTM_j$, $FTTM_l$ and $FTTM_m$ will generate 24 new elements of order four; but all of them will produce a graph of degree zero, because all its vertices are isolated as given below.

$$(M_i, B_j, F_l, T_m), (M_i, B_j, F_m, T_l), (M_i, B_l, F_j, T_m),$$
$$(M_i, B_l, F_m, T_j), (M_i, B_m, F_j, T_l), (M_i, B_m, F_l, T_j),$$
$$(M_j, B_i, F_l, T_m), (M_j, B_i, F_l, T_m), (M_j, B_l, F_i, T_m),$$
$$(M_j, B_l, F_m, T_i), (M_j, B_m, F_i, T_l), (M_j, B_m, F_l, T_i),$$
$$(M_l, B_j, F_i, T_m), (M_l, B_j, F_m, T_i), (M_l, B_i, F_j, T_m),$$
$$(M_l, B_i, F_m, T_j), (M_l, B_m, F_j, T_i), (M_l, B_m, F_i, T_j),$$
$$(M_m, B_i, F_l, T_j), (M_m, B_j, F_i, T_l), (M_m, B_l, F_j, T_i),$$
$$(M_m, B_l, F_i, T_j), (M_m, B_i, F_j, T_l), (M_m, B_i, F_l, T_j) \qquad \square$$

Theorem 6:

The new elements of order four will produce a graph of degree 8k – 8.

Proof:

From Lemma 4, Lemma 5 and Lemma 6 we can obtain the degree of $(\alpha_4 \mathrm{FTTM}^k)$ as follows.

$$deg(\alpha_4 FTTM^k) = \big(k(k-1)(k-2)(k-3)\big) \times (24) \times (0) + H$$

$$= H$$

where $H = 8k - 8$ (from Equation 2). Thus

$$(\alpha_4 FTTM^k) = 8k - 8 \qquad \square$$

2.4 Graph of Finite Sequence of Fuzzy Topographic Topological Mapping

3.5

Finally, we can compute the degree of the graph of finite sequences of FTTM.

Theorem 7:

Let $deg(FTTM^k)$ to represent the degree of the graph of finite sequences of *FTTM.* Then

$$deg(FTTM^k) = 8k^3 + 16k - 24$$

Proof:

From Theorem 2, Theorem 4, and Theorem 5 we can find that

$$deg(FTTM^k) = (24k^2 - 16k - 8)$$
$$+ (8k^3 - 24k^2 + 24k - 8) + (8k - 8)$$
$$= 8k^3 + 16k - 24$$

$\square$

3. RESULTS AND DISCUSSION

In this paper, it is presented that if there exist k elements of *FTTM*, the new elements of order two will produce a graph of degree $(24k^2 - 16k - 8)$. In addition to the above result, it is also shown that the new elements of order three will produce a graph of degree $(8\mathrm{k}^2 - 24\mathrm{k} + 16)$. while the new elements of order four will produce a graph of degree $(8k - 8)$. Finally, the degree of the graph of the new elements of *FTTM* was computed by summing up the degree of the graphs of the new elements of order two, three and four.

4. CONCLUSION

The result of this paper generalized the graph of *FTTM* . Furthermore, categorization of the new elements was done by finding the degree of the graph where it was found that some of the new elements are producing graphs of degree zero signifying that they have isolated components.

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