

## Flow and heat transfer in MHD visco-elastic fluid with ohmic heating

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### GRAPHICAL ABSTRACT

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An investigation is made of the motion of a visco-elastic, MHD free convective flow and mass transfer past an infinite vertical plate. The effects of ohmic heating and viscous dissipation are taken into account. For solving the non-dimensional governing equations of motion perturbation technique has been used and the important properties of the overall structure of the fluid motion are studied. The effect of various parameters of the velocity field, concentration field and temperature distribution are discussed with the help of graphical illustration.

*Keywords:* visco-elastic, rivlin-ericksen, chemical effect, thermo-diffusion, magnetic field, nusselt number, sherwood number.

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## 1. INTRODUCTION:

The study of MHD visco-elastic fluids is considerably useful in chemical and process industries as they exhibit both the memory and elastic effects. However, the study in this direction is limited due to complicated rheological equations involved in describing the dynamical behavior of these fluids. Unlike Newtonian fluids the visco-elastic fluids behave differently under the friction imposed on them.

Chamber and Young (1958) have studied on the diffusion of a chemically reactive species in a Laminar Boundary layer flow. Das *et al* (1999) have explained the mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. The effect of chemical reaction on an unsteady MHD flow through an impulsively started semi-infinite vertical plate has discussed by Muthucumaraswami *et al.* (2008). Alam *et al* (2007) have investigated numerically the effect of chemical reaction on an unsteady hydromagnetic free convection and mass transfer flow past an impulsively started infinite inclined porous plate in presence of heat generation or absorption. The transient natural convection flow of an incompressible viscous fluid past an impulsively started semi-infinite isothermal vertical plate with mass diffusion by taking into account a homogeneous chemical reaction of first order was discussed by Muthucumaraswami and Ganesan (2002).

The Soret effect has been found to be useful as the Soret effect is utilized for isotope separation, in a mixture of

Heat and mass transfer on flow past a vertical plate have been discussed by several authors; viz. Somers (1956), Soundalgekar and Ganesan (1981) and Lin and Wu (1995) in various ways include various physical aspects. Applications of Magnetohydrodynamic flows found in solar physics, meteorology, cosmic fluid dynamics, geophysics, astrophysics and in the motion of earth's core. MHD free convection flows have important applications in aeronautics, chemical engineering and electronics and in the field of stellar and planetary magnetospheres. Gases of light and medium of very light molecular weight. A few studies on Soret effect by taking various aspects of the flow have been initiated by Gebhart and Pera (1971) and Georgantopoulos (1979). Kafoussias and Williams (1995) have analyzed the effect of thermal-diffusion and diffusion-thermo on mixed free –forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Kafoussias (1992), Islam and Alam (2007) have contributed significantly to this field of study.

But all the above authors have considered Newtonian fluids in their fields of study. In the present paper, we are interested to discuss a flow problem associated with second-grade fluid which follows from generalized Rivlin-Ericksen's fluid where normal stress differences are taken into account. Some authors etc Sanyal and Dasgupta (2003), Mokhtar *et al* (2007), Ghosh and Sana (2008,2009), Choudhury and Dey (2010), Choudhury and Das (2014), Choudhury and Dhar (2014) etc. have contributed their efforts in this field.

The effects of visco-elasticity, Soret effect and chemical reaction in MHD free convective flow and mass transfer past an infinite vertical plate with ohmic heating effect in presence of viscous dissipation has been discussed in this study.

The constitutive equation for the second-grade fluid is taken in the form:

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \tag{1}$$

where  $\sigma$  is the stress tensor,  $A_n$  are the kinematic Rivlin-Ericksen tensors;  $\mu_1, \mu_2, \mu_3$  are the material coefficients

describing the viscosity, elasticity and cross-viscosity respectively. From thermodynamic consideration it is noticed that the material coefficients  $\mu_1$  and  $\mu_3$  are positive and,  $\mu_2$  is negative [Coleman and Markovitz [1964]]. The equation (1) was derived by Coleman and Noll [1960] from that of simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

It is reported that solution of poly-isobutylene in Cetane at 30°C simulate a second-order fluid and the material constants for the solutions of various concentrations have been determined by Markovitz.

## 2. MATHEMATICAL FORMULATION

The free convective flow of an incompressible and electrically conducting visco-elastic fluid, where that  $\bar{x}$ -axis is taken along the plate in upward direction and  $\bar{y}$ -axis normal to it. A transverse constant magnetic field is applied *i.e.* in the direction of  $\bar{y}$ -axis.

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \Rightarrow \bar{v} = -v_0 (v_0 > 0)$$

Where  $v_0$  a constant of integration and negative sign indicates that suction is in towards the plate.

Since the motion is two-dimensional and length of the plate is large therefore all physical variables are independent of  $\bar{x}$ . Let  $\bar{u}$  and  $\bar{v}$  be the components of velocity in  $\bar{x}$  and  $\bar{y}$  directions respectively, taken along and perpendicular to the plate.

The governing equations of motion are

$$\tag{2}$$

$$\bar{v} \left( \frac{d\bar{u}}{d\bar{y}} \right) = v_1 \left( \frac{d^2\bar{u}}{d\bar{y}^2} \right) + v_2 \left( \bar{v} \frac{d^3\bar{u}}{d\bar{y}^3} \right) + g\beta(\bar{T} - T_\infty) + g\bar{\beta}(\bar{C} - C_\infty) - \left( \frac{\sigma B_0^2}{\rho} \right) \bar{u} \tag{3}$$

$$\bar{v} \left( \frac{d\bar{T}}{d\bar{y}} \right) = \left( \frac{k}{\rho C_p} \right) \left( \frac{d^2\bar{T}}{d\bar{y}^2} \right) + v_1 \left( \frac{d\bar{u}}{d\bar{y}} \right)^2 + \left( \frac{v_2}{C_p} \right) \left( \bar{v} \left( \frac{d\bar{u}}{d\bar{y}} \right) \left( \frac{d^2\bar{u}}{d\bar{y}^2} \right) \right) + \left( \frac{\sigma B_0^2}{\rho C_p} \right) \bar{u}^2 \tag{4}$$

$$\bar{v} \left( \frac{d\bar{C}}{d\bar{y}} \right) = D \left( \frac{d^2\bar{C}}{d\bar{y}^2} \right) + D_1 \left( \frac{d^2\bar{T}}{d\bar{y}^2} \right) - K_1(\bar{C} - C_\infty) \tag{5}$$

The boundary conditions are

$$\bar{y} = 0: \bar{u} = 0; \bar{T} = T_w; \bar{C} = C_w$$

$$\bar{y} \rightarrow \infty: \bar{u} \rightarrow 0; \bar{T} \rightarrow T_\infty; \bar{C} \rightarrow C_\infty \tag{6}$$

where  $g$  is the acceleration due to gravity,  $\bar{T}$  is the temperature of the fluid,  $T_w$  is the temperature near the plate,  $T_\infty$  is the temperature far away from the plate,  $Pr$  is the Prandtl number,  $\bar{C}$  is the species concentration of the

fluid,  $C_w$  is the concentration near the plate,  $C_\infty$  is the concentration far away from the plate,  $K_1$  is the chemical reaction constant,  $\beta$  is the coefficient of thermal expansion,  $\bar{\beta}$  is the coefficient of mass expansion,  $\sigma$  is the magnetic permeability of the fluid,  $\rho$  is the fluid density,  $B_0$  is the magnetic field coefficient,  $D$  is the chemical molecular diffusivity,  $D_1$  is the coefficient of thermal diffusivity,  $v_1$  is the kinematic viscosity of the fluid,  $v_2$  is the visco-elasticity.

Introducing following non-dimensional parameters

$$y = \frac{\bar{y}v_0}{v_1}, u = \frac{\bar{u}}{v_0}, Pr = \frac{v_1\rho C_p}{k}, \theta = \frac{\bar{T}-T_\infty}{T_w-T_\infty}, \phi = \frac{\bar{C}-C_\infty}{C_w-C_\infty},$$

$$Gr = \frac{v_1 g \beta (T_w - T_\infty)}{v_0^3}, Gm = \frac{v_1 \bar{\beta} g (C_w - C_\infty)}{v_0^3}, Ec = \frac{v_0^2}{C_p (T_w - T_\infty)},$$

$$M^2 = \frac{\sigma v_1 B_0^2}{v_0^2 \rho}, \nu_1 = \frac{\mu_1}{\rho}, Sc = \frac{v_1}{D}, So = \frac{D_1 (T_w - T_\infty)}{v_1 (C_w - C_\infty)}, K = \frac{K_1 v_1}{v_0^2} \tag{7}$$

where Gr is the Grashof number for heat transfer, Gm is the Grashof number for mass transfer, So is the Soret number, Ec is the Eckert number, K is the chemical reaction parameter, k is the thermal conductivity, Sc is the Schmidt

number,  $\theta$  is the dimensionless temperature,  $\phi$  is the dimensionless concentration.

Into the equations (3)-(5), we get

$$\left(\frac{d^2u}{dy^2}\right) + \left(\frac{du}{dy}\right) - \alpha_1 \left(\frac{d^3u}{dy^3}\right) - M^2u = -Gr\theta - Gm\phi \tag{8}$$

$$\left(\frac{d^2\theta}{dy^2}\right) + Pr \left(\frac{d\theta}{dy}\right) + PrEc \left(\frac{du}{dy}\right)^2 + \alpha_1 PrEc \left(\left(\frac{du}{dy}\right) \left(\frac{d^2u}{dy^2}\right)\right) + PrEcM^2u^2 = 0 \tag{9}$$

$$\left(\frac{d^2\phi}{dy^2}\right) + Sc \left(\frac{d\phi}{dy}\right) + ScSo \left(\frac{d^2\theta}{dy^2}\right) - KSc\phi = 0 \tag{10}$$

where  $\alpha_1 = \frac{\nu_2 v_0^2}{v_1^2}$  is the visco-elastic parameter. The

modified boundary conditions are

$$y = 0: u = 0; \theta = 1; \phi = 1 \qquad y \rightarrow \infty: u \rightarrow 0; \theta \rightarrow 0; \phi \rightarrow 0 \tag{11}$$

### 3. METHOD OF SOLUTION

To solve the nonlinear coupled differential equations (8)-(10), we assume the solution of the following form:

$$u(y) = u_0(y) + Ec u_1(y) + O(Ec^2)$$

$$\theta(y) = \theta_0(y) + Ec \theta_1(y) + O(Ec^2)$$

$$\phi(y) = \phi_0(y) + Ec \phi_1(y) + O(Ec^2) \tag{12}$$

Substituting (12) into the equations (8)-(10) and equating the coefficient of like powers of Ec, we have

**Zerth-Order equations:**

$$\left(\frac{d^2u_0}{dy^2}\right) + \left(\frac{du_0}{dy}\right) - \alpha_1 \left(\frac{d^3u_0}{d^3y}\right) - M^2u_0 = -Gr\theta_0 - Gm\phi_0 \tag{13}$$

$$\left(\frac{d^2\theta_0}{dy^2}\right) + Pr \left(\frac{d\theta_0}{dy}\right) = 0 \tag{14}$$

$$\left(\frac{d^2\phi_0}{dy^2}\right) + Sc \left(\frac{d\phi_0}{dy}\right) + ScSo \left(\frac{d^2\theta_0}{dy^2}\right) - KSc\phi_0 = 0 \tag{15}$$

**First-Order equations:**

$$\left(\frac{d^2u_1}{dy^2}\right) + \left(\frac{du_1}{dy}\right) - \alpha_1 \left(\frac{d^3u_1}{dy^3}\right) - M^2u_1 = -Gr\theta_1 - Gm\phi_1 \tag{16}$$

$$\left(\frac{d^2\theta_1}{dy^2}\right) + Pr \left(\frac{d\theta_1}{dy}\right) + \alpha_1 Pr \left(\frac{du_0}{dy}\right) \left(\frac{d^2u_0}{dy^2}\right) + Pr \left(\frac{du_0}{dy}\right)^2 + PrM^2u_0^2 = 0 \tag{17}$$

$$\left(\frac{d^2\phi_1}{dy^2}\right) + Sc \left(\frac{d\phi_1}{dy}\right) + ScSo \left(\frac{d^2\theta_1}{dy^2}\right) - KSc\phi_1 = 0 \tag{18}$$

and the relevant boundary conditions are

$$\begin{aligned} y = 0: u_0 = 0; u_1 = 0; \theta_0 = 1; \theta_1 = 0; \phi_0 = 1; \phi_1 = 0 \\ y \rightarrow \infty: u_0 \rightarrow 0; u_1 \rightarrow 0; \theta_0 \rightarrow 0; \theta_1 \rightarrow 0; \phi_0 \rightarrow 0; \phi_1 \rightarrow 0 \end{aligned} \tag{19}$$

Solving (14) and (15) with the help of (19), we get

$$\theta_0 = \exp(-Pry) \tag{20}$$

$$\phi_0 = A_1 \exp(-Pry) + A_2 \exp(-r_1y) \tag{21}$$

To solve the equations (13) and (16), we use multiparameter perturbation scheme following Nowinski and Ismail (1965) as  $\alpha_1 \ll 1$  for small rate of shear as follows:

$$\begin{aligned} u_0 &= u_{00} + \alpha_1 u_{01} \\ u_1 &= u_{10} + \alpha_1 u_{11} \end{aligned} \tag{22}$$

Substituting (22) into (13) and (16) and comparing the like terms we obtain the following equations

**Zerth-Order equations:**

$$\frac{d^2u_{00}}{dy^2} + \frac{du_{00}}{dy} - M^2u_{00} = -Gr\theta_0 - Gm\phi_0 \tag{23}$$

$$\frac{d^2u_{10}}{dy^2} + \frac{du_{10}}{dy} - M^2u_{10} = -Gr\theta_1 - Gm\phi_1 \tag{24}$$

**First-Order equations:**

$$\frac{d^2u_{01}}{dy^2} + \frac{du_{01}}{dy} - M^2u_{01} = \frac{d^3u_{00}}{dy^3} \tag{25}$$

$$\frac{d^2u_{11}}{dy^2} + \frac{du_{11}}{dy} - M^2u_{11} = \frac{d^3u_{10}}{dy^3} \tag{26}$$

subject to the boundary conditions:

$$\begin{aligned} y = 0: u_{00} = 0; u_{01} = 0; u_{10} = 0; u_{11} = 0 \\ y \rightarrow \infty: u_{00} \rightarrow 0; u_{01} \rightarrow 0; u_{10} \rightarrow 0; u_{11} \rightarrow 0 \end{aligned} \tag{27}$$

Solving (17)- (18) and (23)-(26) under the boundary condition(19) and (27) respectively, we have

$$u_{00} = A_3 \exp(-Pr y) + A_4 \exp(-r_1 y) + A_5 \exp(-r_2 y) \tag{28}$$

$$u_{01} = A_6 \exp(-Pr y) + A_7 \exp(-r_1 y) + A_8 y \exp(-r_2 y) + A_9 \exp(-r_2 y) \tag{29}$$

$$u_0 = A_{10} \exp(-r_2 y) + A_{11} \exp(-Pr y) + A_{12} \exp(-r_1 y) + A_{13} y \exp(-r_2 y) \tag{30}$$

$$\begin{aligned} \theta_1 = & A_{14} \exp(-2Pr y) + A_{15} \exp(-(r_1 + Pr)y) + A_{16} \exp(-2r_1 y) + A_{17} \exp(-2r_2 y) \\ & + A_{18} \exp(-(r_1 + r_2)y) + A_{19} y \exp(-2r_2 y) + A_{20} y \exp(-(r_2 + Pr)y) + A_{21} \exp(-(r_2 + Pr)y) \\ & + A_{22} y \exp(-(r_1 + r_2)y) + A_{23} y^2 \exp(-2r_2 y) \\ & + A_{24} \exp(-Pr y) \end{aligned} \tag{31}$$

$$\begin{aligned} \phi_1 = & A_{25} \exp(-Pr y) + A_{26} \exp(-2Pr y) + A_{27} \exp(-(r_1 + Pr)y) + A_{28} \exp(-2r_1 y) + A_{29} \exp(-2r_2 y) \\ & + A_{30} \exp(-(r_1 + r_2)y) + A_{31} y \exp(-2r_2 y) \\ & + A_{32} \exp(-(r_2 + Pr)y) + A_{33} y \exp(-(r_1 + r_2)y) + A_{34} y \exp(-(r_2 + Pr)y) \\ & + A_{35} y^2 \exp(-2r_2 y) + A_{36} \exp(-r_1 y) \end{aligned} \tag{32}$$

$$\begin{aligned} u_{10} = & A_{37} \exp(-Pr y) + A_{38} \exp(-2Pr y) + A_{39} \exp(-(r_1 + Pr)y) + A_{40} \exp(-2r_1 y) \\ & + A_{41} \exp(-r_1 y) + A_{42} \exp(-2r_2 y) + A_{43} \exp(-(r_1 + r_2)y) + A_{44} y \exp(-2r_2 y) \end{aligned}$$

$$\begin{aligned}
 &+A_{45}y\exp(-(r_2 + Pr)y) + A_{47}y\exp(-(r_1 + r_2)y) + A_{46}y\exp(-(r_2 + Pr)y) + A_{48}y^2\exp(-2r_2y) \\
 &+ A_{49}\exp(-r_2y) \tag{33}
 \end{aligned}$$

$$\begin{aligned}
 u_{11} = &A_{50}y\exp(-r_2y) + A_{51}\exp(-Pr y) + A_{52}\exp(-2Pr y) + A_{53}\exp(-(r_1 + Pr)y) \\
 &+ A_{54}\exp(-2r_1y) + A_{55}\exp(-r_1y) + A_{56}\exp(-(r_1 + r_2)y) + A_{57}y\exp(2r_2y) + A_{58}\exp(-2r_2y) \\
 &+ A_{59}\exp(-(r_2 + Pr)y) + A_{60}y\exp(-(r_2 + Pr)y) + A_{61}y\exp(-(r_1 + r_2)y) + A_{62}y^2\exp(-2r_2y) \\
 &+ A_{63}\exp(-r_2y) \tag{34}
 \end{aligned}$$

$$\begin{aligned}
 u_1 = &A_{64}\exp(-r_2y) + A_{65}\exp(-Pr y) + A_{66}\exp(-2r_2y) + A_{67}\exp(-2Pr y) + A_{68}\exp(-(r_1 + Pr)y) + A_{69}\exp(-2r_1y) \\
 &+ A_{70}\exp(-r_1y) + A_{71}\exp(-(r_1 + r_2)y) + A_{72}y\exp(-2r_2y) + A_{73}y\exp(-(r_2 + Pr)y) \\
 &+ A_{74}\exp(-(r_2 + Pr)y) + A_{75}y\exp(-(r_1 + r_2)y) \\
 &+ A_{76}y^2\exp(-2r_2y) + A_{77}y\exp(r_2y) \tag{35}
 \end{aligned}$$

$$\begin{aligned}
 u = &A_{78}\exp(-2r_2y) + A_{79}\exp(-Pr y) + A_{80}\exp(-r_1y) + A_{81}y\exp(-r_2y) + A_{82}\exp(-2r_2y) + A_{83}\exp(-2Pr y) \\
 &+ A_{84}\exp(-(r_1 + Pr)y) + A_{85}\exp(-2r_1y) + A_{86}\exp(-(r_1 + r_2)y) + A_{87}y\exp(-2r_2y) \\
 &+ A_{88}y\exp(-(r_2 + Pr)y) + A_{89}\exp(-(r_2 + Pr)y) + A_{90}y\exp(-(r_1 + r_2)y) + A_{91}y^2\exp(-2r_2y) \\
 &+ A_{92}y\exp(-2r_2y) \tag{36}
 \end{aligned}$$

$$\begin{aligned}
 \theta = &\exp(-Pr y) + Ec(A_{14}\exp(-2Pr y) + A_{15}\exp(-(r_1 + Pr)y) + A_{16}\exp(-2r_1y) + A_{17}\exp(-2r_2y) \\
 &+ A_{18}\exp(-(r_1 + r_2)y) + A_{19}y\exp(-2r_2y) + A_{20}y\exp(-(r_2 + Pr)y) + A_{21}\exp(-(r_2 + Pr)y) \\
 &+ A_{22}y\exp(-(r_1 + r_2)y) + A_{23}y^2\exp(-2r_2y) \\
 &+ A_{24}\exp(-Pr y) ) \tag{37}
 \end{aligned}$$

$$\begin{aligned}
 \phi = &A_1\exp(-Pr y) + A_2\exp(-r_1y) + Ec(A_{25}\exp(-Pr y) + A_{26}\exp(-2Pr y) + A_{27}\exp(-(r_1 + Pr)y) + A_{28}\exp(-2r_1y) \\
 &+ A_{29}\exp(-2r_2y) + A_{30}\exp(-(r_1 + r_2)y) + A_{31}y\exp(-2r_2y) \\
 &+ A_{32}\exp(-(r_2 + Pr)y) + A_{33}y\exp(-(r_1 + r_2)y) + A_{34}y\exp(-(r_2 + Pr)y) \\
 &+ A_{35}y^2\exp(-2r_2y) + A_{36}\exp(-r_1y) ) \tag{38}
 \end{aligned}$$

The non-dimensional form of skin friction at the surface is given by

$$\sigma = \left(\frac{\partial u}{\partial y}\right)_{y=0} - \alpha_1(\partial^2 u / \partial y^2)_{y=0} = A_{93} - \alpha_1 A_{94} \tag{39}$$

The rate of heat transfer in terms of Nusselt number (Nu) at the plate is given by

$$Nu = \left(\frac{d\theta}{dy}\right)_{y=0} = -Pr + Ec \begin{pmatrix} -PrA_{24} - 2r_2A_{17} - 2PrA_{14} - (r_1 + Pr)A_{15} - 2r_1A_{16} \\ -(r_1 + r_2)A_{18} + A_{19} + A_{20} - (r_1 + Pr)A_{21} + A_{22} \end{pmatrix} \tag{40}$$

The rate of mass transfer in terms of Sherwood number (Sh) at the plate is given by

$$Sh = \left(\frac{d\phi}{dy}\right)_{y=0} = (-PrA_1 - r_1A_2) + Ec(-PrA_{25} - 2PrA_{26} - (r_1 + Pr)A_{27} - 2r_1A_{28} - 2r_2A_{29} - (r_1 + r_2)A_{30} + A_{31} - (r_2 + Pr)A_{32} + A_{33} + A_{34} - r_1A_{36}) \quad (41)$$

#### 4. RESULT AND DISCUSSION

This study brings out the effects of various flow parameters on MHD free convection visco-elastic flow and mass transfer with ohmic heating in presence of viscous dissipation. The effect of visco-elasticity is exhibited through the non-dimensional parameter  $\alpha_1$ . The Newtonian fluid flow is characterized by considering  $\alpha_1=0$  throughout the problem.

To know the physics of the problem the graphical illustrations of the velocity, temperature and concentration and skin friction against  $y$  are presented for different values of Magnetic parameter (M), Chemical reaction parameter, Grashof number for heat transfer (Gr), Grashof number for mass transfer (Gm) and Soret number (So) with the fixed values of the Eckert number  $Ec = 0.03$ , Schmidt number  $Sc = 0.6$  and Prandtl number  $Pr = 5$ .

Figures 1-6 depict the velocity  $u$  against  $y$ . It is observed that the fluid velocity accelerates near the plate and slows down uniformly away from the plate for both Newtonian and non-Newtonian cases. The growing trend of absolute values of visco-elastic parameter  $|\alpha_1|$  ( $\alpha_1=0, -0.05, -0.1$ ) increases the fluid velocity in comparison with Newtonian fluid.

From the figures 1 and 2, we notice that with reduction of magnetic parameter  $M$  the fluid velocity increases. This is happened because transverse magnetic field generates the Lorentz force and it has a tendency to retard the fluid motion.

The variation of temperature has been demonstrated by the figures 7-12. The diminishing trend of temperature is revealed in both Newtonian and non-Newtonian cases with different values of the physical parameter viz. Magnetic parameter, chemical reaction parameter, Grashof number for heat transfer, Grashof number for mass transfer and Soret number. The rising trend of absolute values of visco-elastic parameter  $|\alpha_1|$  ( $\alpha_1=0, -0.05, -0.1$ ) decreases the temperature in comparison with Newtonian fluid.

Figures 13-18 present the variation of species concentration in the flow domain. These figures depict that concentration accelerates near the plate and decelerates away from the plate for different values of the physical parameters. The increasing trend of absolute values of visco-elastic parameter  $|\alpha_1|$  ( $\alpha_1=0, -0.05, -0.1$ ) enhances concentration in comparison with Newtonian fluid.

From the practical point of view it is very important to know the skin friction and consequently, the viscous drag in non-dimensional form.

Figures 19-23 exhibit the variation of skin friction  $\sigma$  against magnetic parameter (M), Chemical reaction parameter (K), Grashof number for heat transfer (Gr), Grashof number for mass transfer (Gm) and Soret number (So). It is noticed that the skin friction  $\sigma$  decreases with the increasing absolute values of visco-elastic parameter  $|\alpha_1|$  ( $\alpha_1=0, -0.05, -0.1$ ). It is also seen that the skin friction decreases with the increasing values of magnetic parameter  $M$  and chemical reaction parameter  $K$  and reverse trend is obtained in case of Grashof number for heat transfer (Gr), Grashof number for mass transfer and Soret number (So).

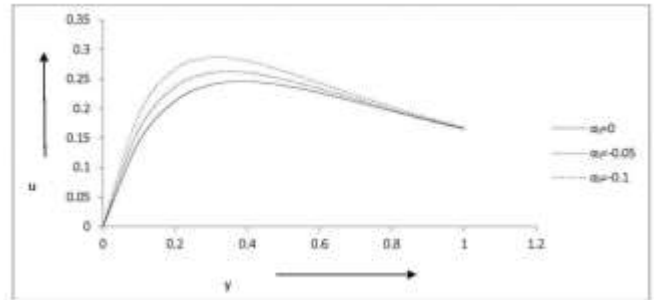


Fig.1 Fluid velocity  $u$  against  $y$  for  $M=5, K = 0.5, Gr = 5, Pr = 5, Gm = 6, So = 1$

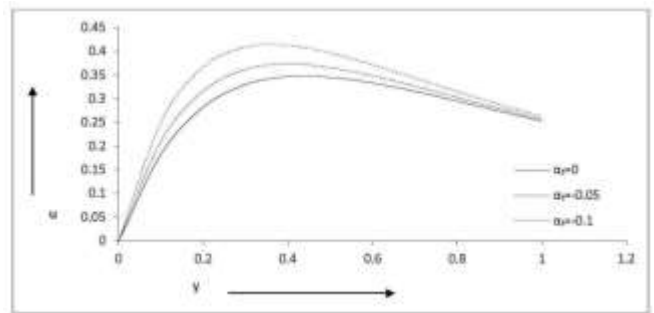
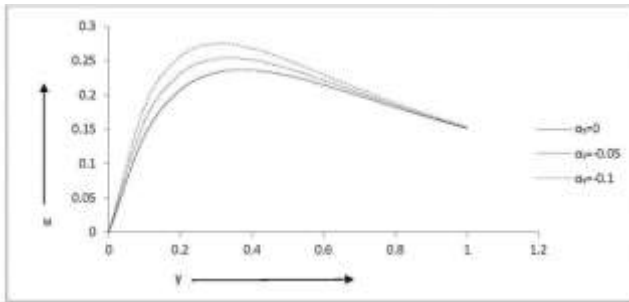
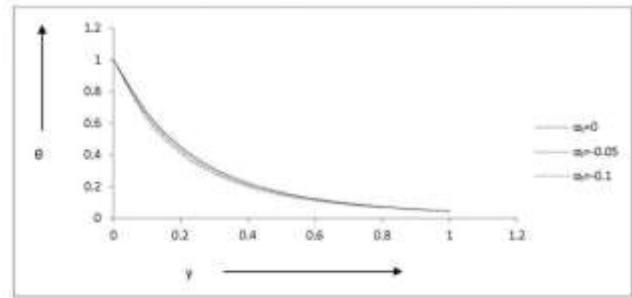


Fig.2 Fluid velocity  $u$  against  $y$  for  $M=4, K=0.5, Gr=5, Pr=5, Gm=6, So=1$



**Fig.3** Fluid velocity  $u$  against  $y$  for  $M=5$ ,  $K=0.7$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$



**Fig.7** Variation of temperature  $\theta$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$

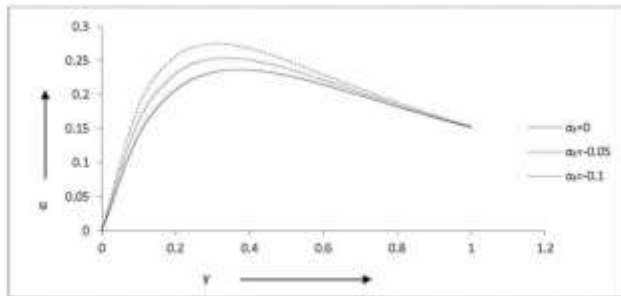
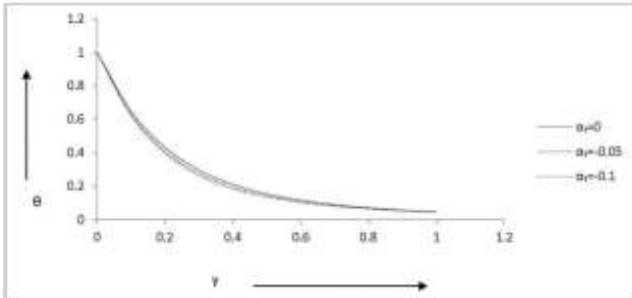
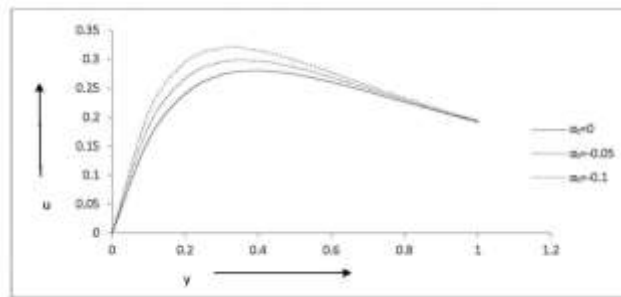


Figure 4: Fluid velocity  $u$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=4$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$

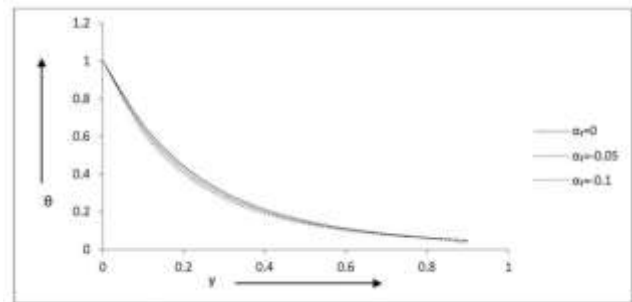
**Fig.4** Fluid velocity  $u$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=4$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$



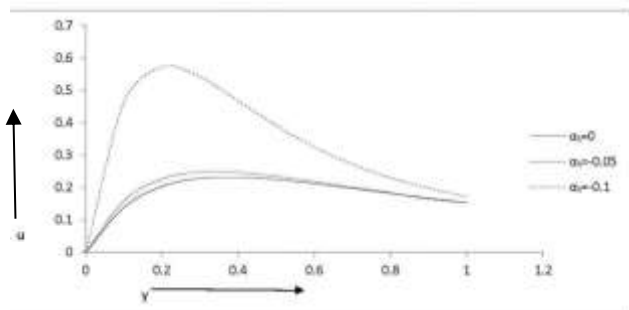
**Fig.8** Variation of temperature  $\theta$  against  $y$  for  $M=4$ ,  $K=0.5$ ,  $Gr=6$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$



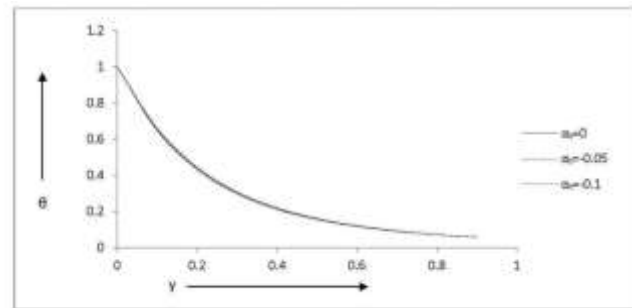
**Fig.5** Fluid velocity  $u$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=7$ ,  $So=1$



**Fig.9** Variation of temperature  $\theta$  against  $y$  for  $M=5$ ,  $K=0.7$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$

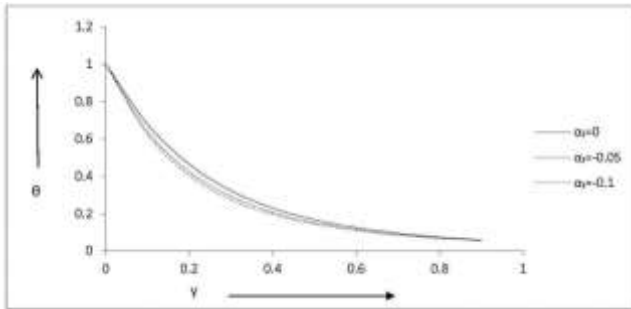


**Fig.6** Fluid velocity  $u$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=0.8$

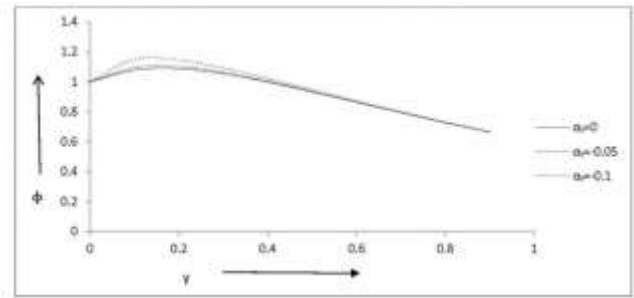


**Fig.10** Variation of temperature  $\theta$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=4$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$

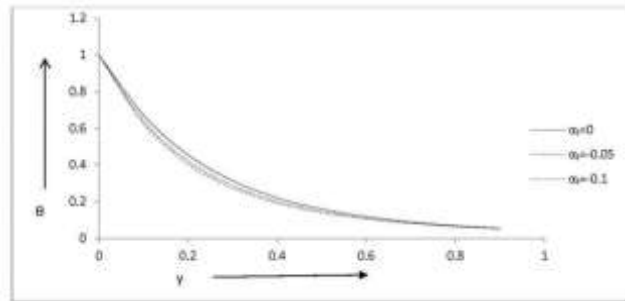




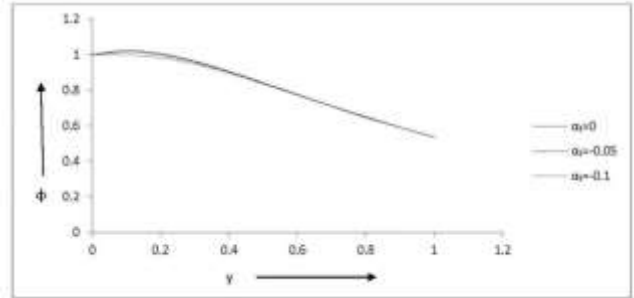
**Fig.11** Variation of temperature  $\theta$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=7$ ,  $So=1$



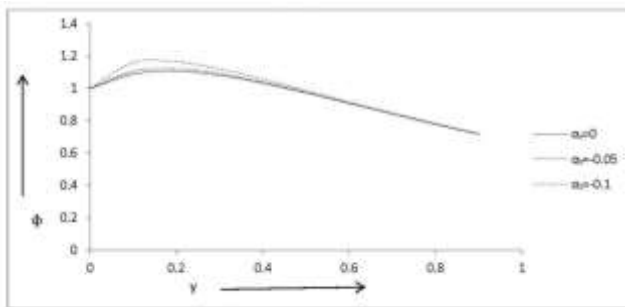
**Fig.15** Variation of concentration  $\phi$  against  $y$  for  $M=5$ ,  $K=0.7$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$ .



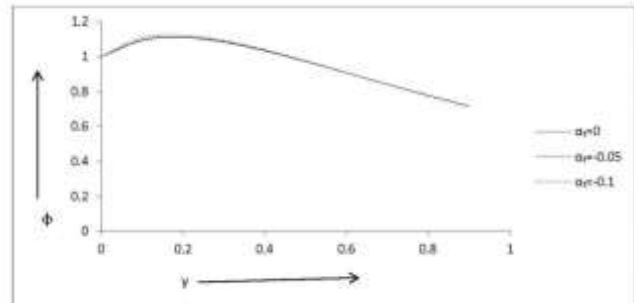
**Fig.12** Variation of temperature  $\theta$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=0.8$



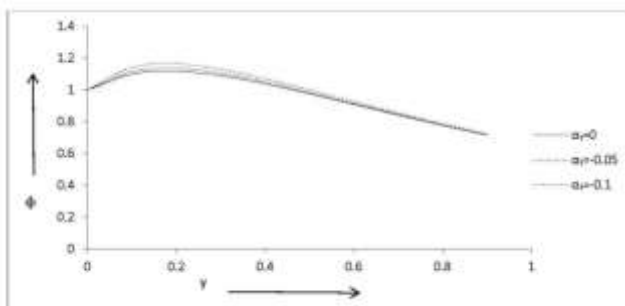
**Fig.16** Variation of concentration  $\phi$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=4$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$ .



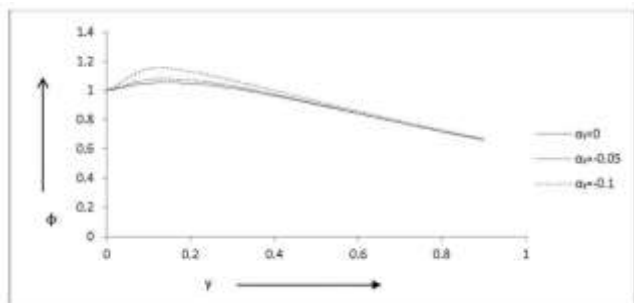
**Fig.13** Variation of concentration  $\phi$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$ .



**Fig.17** Variation of concentration  $\phi$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=7$ ,  $So=1$ .



**Fig.14** Variation of concentration  $\phi$  against  $y$  for  $M=4$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=1$ .



**Fig.18** Variation of concentration  $\phi$  against  $y$  for  $M=5$ ,  $K=0.5$ ,  $Gr=5$ ,  $Pr=5$ ,  $Gm=6$ ,  $So=0.8$ .

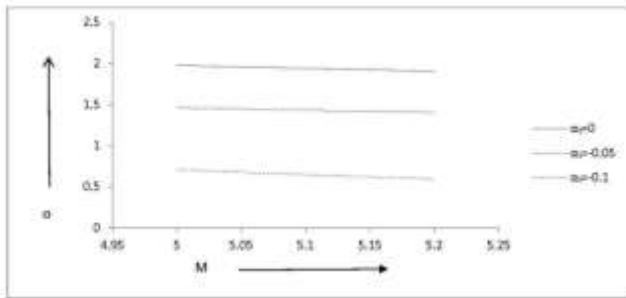


Fig.19 Skin friction  $\sigma$  against Magnetic parameter M

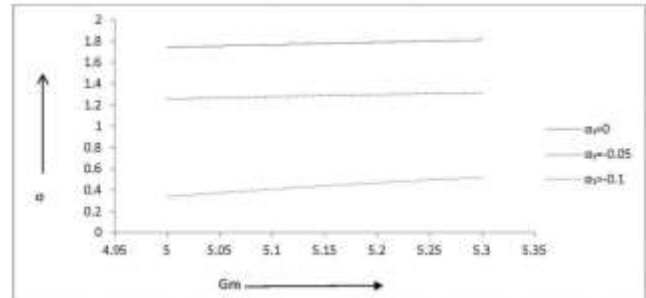


Fig.22 Skin friction  $\sigma$  against Grashof number for mass transfer Gm

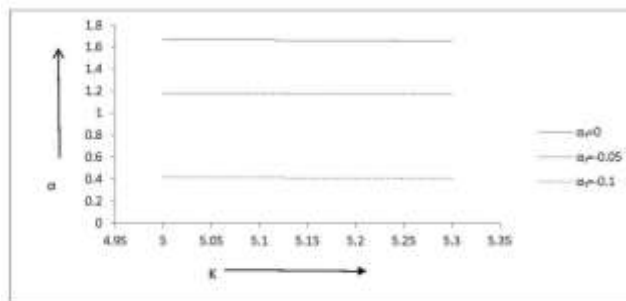


Fig. 20 Skin friction  $\sigma$  against Chemical reaction parameter K

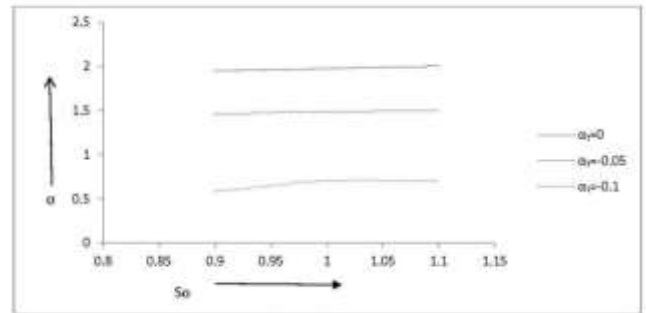


Fig.23 Skin friction  $\sigma$  against Soret number So.

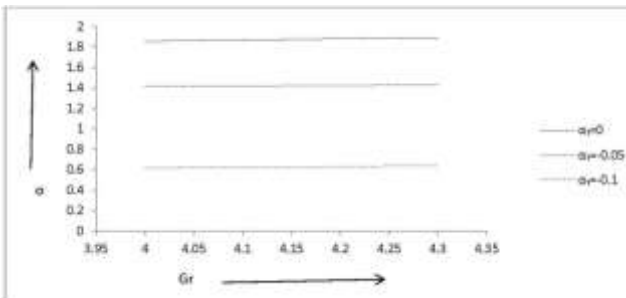


Fig.21 Skin friction  $\sigma$  against Grashof number for heat transfer Gr

## 5. CONCLUSION

The present investigation leads to the following conclusions:

1. The fluid velocity is significantly affected by the visco-elastic parameter along with other flow parameters at all points of the fluid flow region.

2. The modification of the absolute values of visco-elastic parameter diminish the fluid temperature.
3. The variation of concentration distribution is appreciably affected by visco-elastic parameter in combination with other flow parameters.
4. Skin friction plays a vital role in the flow characteristics.

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