MJFAS
Malaysian Journal of Fundamental and Applied Sciences

Kamari *et al.* | Malaysian Journal of Fundamental and Applied Sciences, Vol. 21 (2025) 1678-1696

RESEARCH ARTICLE

Pythagorean Neutrosophic Method Based on the Removal Effects of Criteria (PNMEREC): An Innovative Approach for Establishing Objective Weights in Multi-Criteria Decision-Making Challenges

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Received: 23 May 2024
Accepted: 15 Jan. 2025

Abstract In the realm of multi-criteria decision-making (MCDM), the significance of criteria weights cannot be overstated. As a result, researchers have innovated and introduced various approaches aimed at precisely determining these weightings. This paper proposes a novel methodology that combines the Pythagorean neutrosophic set (PNS) with Method Based on the Removal Effects of Criteria (MEREC). This integrated method, PNMEREC seeks to offer a thorough and dependable method for evaluating criteria and establishing weightage in MCDM scenarios. PNS provides a more detailed way of handling uncertainty than traditional fuzzy sets or intuitionistic fuzzy sets by considering truth, indeterminacy, and falsity memberships values for each element, allowing for a wider range of uncertainties to be captured. This paper also introduces 5-point, 9-point, and 11-point PNS linguistic variables that can be utilized to represent evaluations from experts. The newly established linguistic variable scales enable decision-makers to express their criteria with clearer and heightened precision in PNS settings according to their preferences. A comparative analysis is conducted by comparing PNMEREC result with PN-Entropy and PN-Statistical Variance procedure to investigate the performance of the proposed method. The result of comparative analysis indicates that the weights produced by the PNMEREC method exhibit a high degree of reliability and stability, as demonstrated by the significant Pearson correlation coefficient values. Hence, the PNMEREC methodology possesses the capacity to thoroughly capture the determination of criterion weight via a more thorough and nuanced analysis. A sensitivity analysis is also conducted to compare the performance of the PNMEREC method using two distance techniques. The results reveal that the choice of distance technique impacts weight distribution and prioritization. Specifically, PN-Hamming emphasizes sharper distinctions, while PN-Euclidean offers a more balanced allocation.

Keywords: decision-making, criteria weights; objective weight; MCDM; Pythagorean neutrosophic set.

Introduction

Multi-criteria decision-making (MCDM) is an evolving discipline that equips decision-makers with the methodologies and tools to address and resolve cases in which multiple and conflicting criteria are involved. In various real-world scenarios, decisions are seldom straightforward, often involving multiple objectives or constraints that may compete or complement each other. MCDM aims to facilitate the process of decision-making by systematically analyzing, evaluating, and ranking alternative options based on their performance across multiple criteria or attributes. Multi-objective decision-making (MODM) and multi-attribute decision-making (MADM) are two critical concepts in the field of MCDM [1]. MODM focuses on situations where decision-makers need to optimize several conflicting objectives simultaneously. Unlike traditional decision-making models that aim to find a single optimal solution, MODM acknowledges that real-world decisions often involve multiple objectives that cannot be optimized simultaneously without trade-offs. MADM, on the other hand, focuses on situations where decision-makers need to evaluate and rank alternatives based on multiple attributes or criteria. Both methods are essential for assisting decision-makers in addressing intricate decision-making challenges across different fields [2-5].

Another important aspect of MCDM is the weights of the criteria. Assigning weights to criteria in MCDM allows decision-makers to express the relative importance of each criterion in achieving the desired outcome [6]. Multiple weighting methods have been proposed in literature and utilized to address various MCDM problems, including the point allocation method, the direct rating method, Analytic Hierarchy Process (AHP), Entropy method, Mean Weight, standard deviation, Statistical Variance procedure, Criteria Importance Through Inter-criteria Correlation (CRITIC) and more [7-11]. Keshavarz-Ghorabaee *et al*. [12] introduced the Method Based on the Removal Effects of Criteria (MEREC), for evaluating criteria weights. The MEREC method assigns greater weight to a criterion if its exclusion significantly impacts the overall performance of alternatives. This approach not only involves weighting each criterion but also assists decision-makers in potentially eliminating certain criteria from the decision-making process. By examining how the performance of an alternative changes with the removal of criteria, this perspective introduces a novel way of determining criterion weights. Essentially, causality serves as the foundational principle of this method.

In modern MCDM processes, accurately determining the weights of criteria or alternatives poses a significant challenge due to the constraints of traditional weighting methods such as fuzzy sets or intuitionistic fuzzy sets. These methods often fall short in fully capturing the uncertainty, indeterminacy, and imprecision inherent in real-world information. This can result in biased or suboptimal decisions, especially in complex environments. The main problems include inadequate representation of uncertainty, limited flexibility and precision, difficulties in managing incomplete and inconsistent information, and a lack of robustness against variability, all of which compromise the reliability of decision outcomes.

Despite the extensive use of MEREC with various uncertainty models like fuzzy sets and single-valued neutrosophic sets (SVNS), a significant gap exists in integrating MEREC with PNS. PNS extends SVNS by offering greater flexibility and precision in handling uncertainty and indeterminacy, which are crucial in complex decision-making scenarios. Existing studies have not addressed the challenges of adapting MEREC to the higher dimensionality and unique relationships of PNS. This gap highlights the need for a robust framework combining PNS and MEREC to enhance accuracy and reliability in decision-making, bridging theoretical advancements with practical applications.

This study improves upon the previous MEREC weighting method by incorporating Pythagorean neutrosophic set (PNS) into the framework. Jansi *et al*. [13] introduced the concept of PNS which is an extension of neutrosophic sets as a mathematical tool used to deal with uncertainty and vagueness in data. In a PNS set, each element has three values: truth-membership (τ), indeterminacy-membership (ξ), and falsity-membership (η). These values represent the degree to which an element belongs to the set, is indeterminate, or does not belong to the set, respectively. Thus, PNS provides a robust framework for decision-making in uncertain and vague environments. By explicitly modelling indeterminacy and considering both truth and falsity memberships, PNS enables decision-makers to account for incomplete or conflicting information more effectively, leading to more informed and reliable decisions [14-16].

Typically, decision-making involves the utilization of human language, often referred to as linguistic variables. These linguistic variables essentially encapsulate the words or terms employed in language. Consequently, adopting this linguistic variable approach offers decision-makers a convenient means to articulate their evaluations [17-19]. For this paper, we will introduce three new scales specifically tailored for PNS environment which are 5-point, 9-point and 11-point linguistic variables for experts to express their evaluation in a decision-making scenario. The newly introduced linguistic variable expands the spectrum of values for membership functions by incorporating additional parameters into the PNS. This broadens the scope of considered values, effectively addressing uncertainty and indeterminacy inherent in decision-making scenarios.

The main contributions of this study are (i) It introduces new scales of PNS linguistic variables in terms of 5-point, 9-point, and 11-point scales. (ii) It introduced PNMEREC, an integrated weighting technique that provides a proficient and effective framework for addressing vagueness, uncertainty, inconsistency, and indeterminacy in practical decision-making scenarios. (iii) It introduces two new techniques to compute distance in the PNS setting, namely the PN-Hamming distance technique and the PN-Euclidean distance technique. (iv) It evaluates and compares the performance of our proposed method with two other criteria weight computation methods. We believe that by utilizing the proposed method, decision-makers can gain enhanced clarity and confidence in assigning weights to criteria, thereby increasing the accuracy and reliability of MCDM outcomes.

The remaining sections of this paper are structured in the following manner: Section 2 discusses relevant literature for this study. Section 3 provides a detailed explanation of the proposed approach. Section 4 showcases the application of the proposed methodology through an illustrative example centered on the selection of Halal suppliers within MCDM scenarios. Finally, Section 5 serves as the concluding remarks.

Relevant Literature

MCDM Method

Multi-criteria decision-making (MCDM) encompasses a comprehensive framework within decision analysis, focusing on systematic approaches to resolve scenarios with conflicting criteria. Essentially, the methodologies and models employed by MCDM strive to provide decision-makers with systematic tools to navigate the complexities inherent in decision-making processes that involve multiple objectives and constraints [20-21]. Numerous well-known Multiple Criteria Decision Making (MCDM) methodologies have been widely employed across various research domains. These include the Weighted Sum Model (WSM), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), VIekriterijumsko KOmpromisno Rangiranje (VIKOR), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), Elimination Et Choix Traduisant la Réalité (ELECTRE), Complex Proportional Assessment (COPRAS), Evaluation based on Distance from Average Solution (EDAS), Analytic Hierarchy Process (AHP), Best Worst Method (BWM) and more. These methods have garnered significant attention and utilization by researchers in diverse academic fields [22-30].

Lately, there have been several intriguing studies on the MCDM method. Hezam *et al*. [31] uses neutrosophic MCDM method to determine the priority groups for COVID-19 Vaccine. With the surge in COVID-19 cases, fair vaccine distribution is imperative. To address this challenge effectively, governments must establish priority groups for vaccine allocation. In this study, they propose a framework comprising four primary criteria—age, health status, gender, and occupation—and fifteen sub-criteria. A neutrosophic Analytic Hierarchy Process (AHP) is employed to evaluate these criteria and subsequently rank the COVID-19 vaccine alternatives using a neutrosophic TOPSIS method. The findings suggest that healthcare personnel, individuals with high-risk health conditions, the elderly, essential workers, pregnant and lactating mothers should be prioritized for vaccination. Furthermore, their analysis underscores the importance of matching the most suitable vaccine to the needs of patients and healthcare workers, thus optimizing vaccine distribution strategies.

Veeramani [32] addressed the selection of the best supplier in the clothing sector by using a hybrid Interval-Valued Neutrosophic MCDM approach. The Interval-Valued Neutrosophic Analytic Hierarchy Process (IVNAHP) and the Interval-Valued Neutrosophic Technique for Order Preference Similarity to Ideal Solution (IVNTOPSIS) approaches are applied for supplier selection. Questionnaires are distributed to textile company experts, with the first questionnaire used to evaluate criteria weights based on IVNAHP, and the second to rank companies based on IVNTOPSIS. Through the analysis, they effectively showcased how the IVNMCDM model can tackle supplier selection challenges in the apparel industry. They confirmed the reliability of the model through comprehensive comparison analysis and by using real-world data, confirming its usefulness in practical scenarios.

Anwar *et al*. [33] employed a neutrosophic MCDM approach to evaluate performance and recommendation of best players in sports league. They propose a neutrosophic TOPSIS method for assessing and recommending the top batsman and bowler in a case study of Indian Premier League (IPL) 2021. Their study gathered player data from reliable online sources for the IPL 2021 and the data was transformed into SNVS format to address vagueness, uncertainty, and inconsistency in the information. The player rankings are computed using neutrosophic TOPSIS with two different methods for calculating criterion weights. The obtained rankings are then evaluated and compared using Kendal Tau (τ). The resulting τ values are 0.83 for bowling rankings and 0.72 for batting rankings, demonstrating the efficiency and effectiveness of the proposed method.

These research endeavors serve as compelling evidence showcasing how MCDM methodologies prove their practical utility and efficacy across various real-world scenarios and academic fields. MCDM approaches offer valuable insights and solutions that address complex decision-making challenges in diverse contexts, underscoring their versatility and relevance in numerous disciplines [34].

Subjective Weighting Method

Real-world problems often rely on multiple criteria rather than a single criterion, which are typically tangible and challenging to quantify. In such cases, experts or decision-makers use their experience to provide relative preferences. However, this can lead to confusion and increased uncertainty. Subjectivity in multi-criteria decision analysis can result in unpredictable negative outcomes [35].

Preferences from decision-makers can be gathered using two methods: direct weighting and pair-wise comparison. In the direct weighing method, decision-makers assign numerical values to describe the weights of various attributes. These techniques include the swing method, Delphi method, direct rating, trade-off, point allocation, ranking method, Nominal Group Technique (NGT), and the Simple Multi-attribute Rating Technique (SMART). These weights are usually assigned on a scale from ten upwards [6, 10].

In the pair-wise comparison method, decision-makers examine various criteria in pairs, assessing their significant differences. This method involves a decision-making process where each criterion is meticulously compared against others, establishing preferences for each criterion pair. Saaty [36] introduced a numerical scale ranging from 1 to 9, aimed at transforming qualitative data into a quantitative format. This scale facilitates the determination of preference values between criteria. This method is straightforward, easy to compute, and effective for qualitative decision-making factors. Methods in this category include the Analytical Hierarchy process (AHP), Analytic Network process (ANP), Best–Worst method (BWM), Digital Logic approach (DL), Modified Digital Logic approach (MDL), and weighted least square method [35].

Objective Weighting Method

Objective weighting methods assign numerical values to different criteria or objectives within a decision-making process. These values reflect the relative importance of each criterion in achieving the overall goal. These methods come in various forms, from simple scoring systems to complex mathematical algorithms. They help streamline decision-making by ensuring that the most critical factors receive appropriate weightage [37]. There are various objective weighting methods that have been developed, such as the Entropy method, CRiteria Importance Through Inter-criteria Correlation (CRITIC), mean weight, standard deviation, statistical variance procedure, CILOS, IDOCRIW, MEREC and their modifications [11-12, 38-40].

This section provides a brief overview of the related research on the applications and advancements of objective weighting methods. The focus is on three methods: Entropy, Statistical Variance procedure, and MEREC, as these are utilized in the computational analysis of this study.

In a recent research study, Yücenur *et al*. [41] evaluates medical waste disposal techniques, which are critical due to their harmful effects on the environment and human health. A MCDM model was developed and solved using a two-stage methodology. In the first stage, the Entropy method was used to prioritize types of medical waste. In the second stage, disposal techniques were assessed using the WASPAS and EDAS methods. Based on the Entropy method, it was discovered that the most significant types of medical waste produced by healthcare facilities are Radioactive waste, followed by Amalgam waste and Chemical waste. The importance weights for these are 0.1751, 0.1478, and 0.1408 respectively. Based on the WASPAS method, Mechanical operations were rated as the best way to dispose of medical waste in the model, with a score of 0.7761. Irridation methods and Sterilization techniques came next. According to the EDAS method, Mechanical operations were also found to be the most suitable disposal method, scoring 0.9328. The consistency of results from both methods confirms the reliability of the proposed model.

Şahin [42] utilized a comprehensive decision-making approach to select logistics center locations in Turkey. Choosing the right location for these centers is crucial for optimizing trade and economy. It incorporates four weighting methods: Entropy, standard deviation, statistical variance, and MEREC. Six ranking methods are utilized: EDAS, MARCOS, MAUT, ROV, TOPSIS, and WASPAS; Spearman's correlation coefficient; and Borda methods. The rationale behind this study is to avoid potential inaccuracies that may arise from relying solely on one method. By combining multiple methods, the decision-making process becomes more reliable. The integrated results highlight the importance of total imports as a key criterion for location selection. The ensemble ranking identifies Istanbul, Ankara, Izmir, Kocaeli, and Konya as the top five alternatives. Overall, the study emphasizes the necessity of a robust decision-making model to ensure dependable outcomes, given the varying rankings produced by different methods.

Mastilo *et al*. [43] conducted an analysis on the financial indicators of the banking sector in Bosnia and Herzegovina using the MEREC and MARCOS methodologies. It aims to rank banks based on financial data from 2022. MEREC determines the importance of financial indicators and their weights, while MARCOS ranks banks accordingly. Raiffeisen Bank emerges as the most efficient and financially favorable. This research also identifies exemplary banks that can guide others in enhancing their performance. The limitations of this study include reliance on available data and predefined methodologies, neglecting external factors. To improve, additional indicators and comparative analyses with other countries' banking sectors are necessary.

Sönmez & Toktaş [44] performed a study on the new generation supplier selection in the medical devices industry. The medical device industry faces struggle in meeting customer demands, leading to longer delivery times, partly due to ineffective supplier selection and strict compliance with medical device regulations. To address this, a novel approach utilizing the Combined Compromise Solution (CoCoSo) method was proposed for supplier ranking, and the MEREC method was employed to assess the weight of supplier criteria. The final assessment identified the number 1 supplier as the preferred choice, notably for holding a quality certification for 13 years and offering a two-week supply period, indicating a favorable option. These findings aligned with the assessments of company managers and engineers. This study fills a gap in research on supplier selection in the medical device sector, offering insights that can benefit industry practice.

These research efforts serve as convincing proof that the advancements of criterion weightage methods in MCDM are revolutionizing how complex decisions are made across various sectors. By enhancing accuracy, transparency, adaptability, and integration with the latest technologies, these methods are ensuring that decision-making processes are more robust, inclusive, and aligned with contemporary challenges. As MCDM continues to evolve, the relevance of these evolving weightage methods will only grow, supporting more informed and sustainable decision-making in an increasingly complex world.

Methodology

Preliminaries

This segment introduces the basic definitions related to PNS.

Definition 1. [45]
Let X be a universe or non-empty set. A Pythagorean neutrosophic set with τ and η are dependent neutrosophic components that is given by:

$$A = \{(x, \tau_A(x), \xi_A(x), \eta_A(x)) | x \in X\} \tag{1}$$

Where τ_A represent the degree of membership, ξ_A represent the degree of indeterminacy and η_A represent the degree of non-membership respectively such that $\tau, \xi, \eta \in [0,1]$ and satisfying the following conditions:

$$\tau_A(x) + \eta_A(x) \leq 1 \tag{2}$$

$$0 \leq (\tau_A(x))^2 + (\eta_A(x))^2 \leq 1 \tag{3}$$

$$0 \leq (\tau_A(x))^2 + (\xi_A(x))^2 + (\eta_A(x))^2 \leq 2 \tag{4}$$

Definition 2. [46]
Let $x_1 = (\tau_{x_1}, \xi_{x_1}, \eta_{x_1})$, $x_2 = (\tau_{x_2}, \xi_{x_2}, \eta_{x_2})$ and $x = (\tau_x, \xi_x, \eta_x)$ are any two PNSs, the following operational rules applies as follows:

i. $$x_1 \oplus x_2 = \left(\sqrt{{\tau_{x_1}}^2 + {\tau_{x_2}}^2 - {\tau_{x_1}}^2 {\tau_{x_2}}^2}, \xi_{x_1}\xi_{x_2}, \eta_{x_1}\eta_{x_2}\right) \tag{5}$$

ii. $$x_1 \otimes x_2 = \left(\tau_{x_1}\tau_{x_2}, \xi_{x_1} + \xi_{x_2} - \xi_{x_1}\xi_{x_2}, \sqrt{{\eta_{x_1}}^2 + {\eta_{x_2}}^2 - {\eta_{x_1}}^2 {\eta_{x_2}}^2}\right) \tag{6}$$

iii. $$\mu x = \left(\sqrt{1-(1-\tau_x{}^2)^\mu}, \xi_x{}^\mu, \eta_x{}^\mu\right) \text{ where } \mu \in \Re \text{ and } \mu \geq 0 \quad (7)$$

iv. $$x^\mu = \left(\tau_x{}^\mu, 1-(1-\xi_x)^\mu, \sqrt{1-(1-\eta_x{}^2)^\mu}\right) \text{ where } \mu \in \Re \text{ and } \mu \geq 0. \quad (8)$$

Proposed Method

This section delves into the establishment of the PNMEREC methodology, showcasing the comprehensive structure of the suggested approach as depicted in Figure 1.

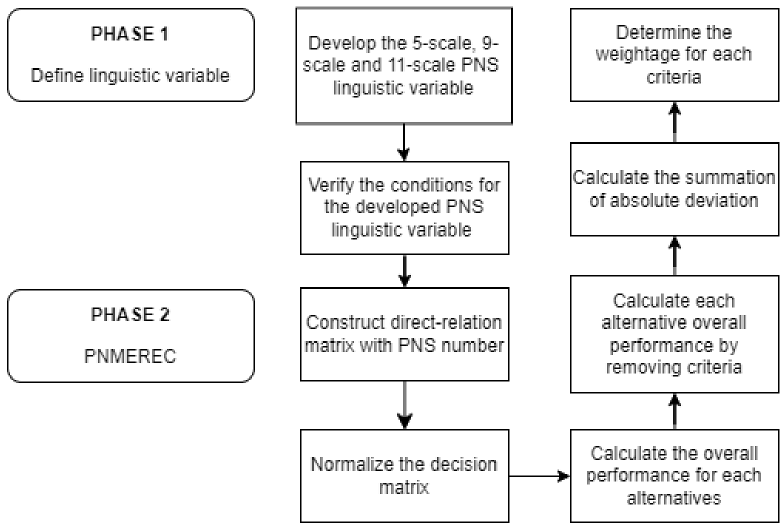

Figure 1. The Proposed PNMEREC methodology

This research presents a novel linguistic scale within the PNS framework and introduces an innovative approach namely PNMEREC by integrating alterations to the initial MEREC technique while upholding its original principles. Comprising eight steps, the proposed method adheres to MEREC's fundamental concept. The primary distinction lies in formulating the PNS linguistic variable and then applying them to the MEREC methodology. Employing the new PNS linguistic scale provides a comprehensive and improved representation of decision-makers' criteria selection. Furthermore, the implementation of the PNMEREC methodology empowers us to adapt the utilization of MEREC methodology within the context of PNS environment.

New Linguistic Variable Development

The linguistic variable developed in this research is formulated within the framework of PNS, while adhering to the conditions outlined in Definition 1. Numerous linguistic variable scales have been employed by previous researchers. For instance, Al-Quran *et al*. [17] established a 5-point scale within the framework of Interval Neutrosophic Vague Sets (INVS), while Abdullah & Goh [18] utilized a 7-point linguistic scale for Pythagorean fuzzy sets (PFS). Biswas *et al*. [19] employed both 5-point and 9-point scales in Single-valued neutrosophic set (SVNS). Additionally, Ismail *et al*. [47] introduced a 7-point linguistic variable scale within the PNS, as depicted in Table 1. Consequently, for this study, we will introduce 5-point, 9-point and 11-point scale linguistic variables within the context of PNS.

Table 1. 7-point scale Pythagorean neutrosophic linguistic variable

Score	Linguistic Variable	Rating Scale in PNS
1	No Effect	$\langle 0.10, 0.80, 0.90\rangle$
2	Low Effect	$\langle 0.20, 0.70, 0.80\rangle$
3	Medium Low Effect	$\langle 0.35, 0.60, 0.60\rangle$
4	Medium Effect	$\langle 0.50, 0.40, 0.45\rangle$
5	Medium High Effect	$\langle 0.65, 0.30, 0.25\rangle$
6	High Effect	$\langle 0.80, 0.20, 0.15\rangle$
7	Very High Effect	$\langle 0.90, 0.10, 0.10\rangle$

The main purpose behind the establishment of these new linguistic variables is to provide more options and flexibility for the decision-makers to express their opinions in the case of MCDM. With the addition of these new scales, decision-makers can choose to adapt either 5-point, 7-point, 9-point or 11-point scale in their criterion evaluation. Table 2 shows the 5-point linguistic variable for PNS:

Table 2. The new 5-point Pythagorean neutrosophic linguistic variable

Score	Linguistic Variable	Rating Scale in PNS
1	Very Low Effect	(0.10,0.85,0.90)
2	Low Effect	(0.30,0.65,0.70)
3	Medium Effect	(0.50,0.45,0.45)
4	High Effect	(0.70,0.25,0.20)
5	Very High Effect	(0.90,0.10,0.05)

Five linguistic terms are established for this new linguistic variable. These terms correspond to pair-wise comparison scales where the scores of 1, 2, 3, 4, and 5 represent “Very low effect”, “Low effect”, “Medium effect”, “High effect”, and “Very high effect” respectively. Table 3 shows the 9-point linguistic variable for PNS:

Table 3. The new 9-point Pythagorean neutrosophic linguistic variable

Score	Linguistic Variable	Rating Scale in PNS
1	Extremely Low Effect	(0.05,0.90,0.95)
2	Very Low Effect	(0.10,0.85,0.90)
3	Low Effect	(0.20,0.80,0.75)
4	Medium Low Effect	(0.35,0.65,0.60)
5	Medium Effect	(0.50,0.50,0.45)
6	Medium High Effect	(0.65,0.35,0.30)
7	High Effect	(0.80,0.25,0.20)
8	Very High Effect	(0.90,0.15,0.10)
9	Extremely High Effect	(0.95,0.05,0.05)

The 9-point scale PNS linguistic variable comprises of nine linguistic terms. These terms align with pairwise comparison scales ranging from 1 to 9, denoting “Extremely low effect”, “Very low effect”, “Low effect”, “Medium low effect”, “Medium effect”, “Medium high effect”, “High effect”, “Very high effect”, and “Extremely high effect” respectively. Table 4 shows the 11-point linguistic variable for PNS:

Table 4. The new 11-point Pythagorean neutrosophic linguistic variable

Score	Linguistic Variable	Rating Scale in PNS
1	Negligible Effect	(0.05,0.90,0.95)
2	Extremely Low Effect	(0.10,0.80,0.85)
3	Very Low Effect	(0.20,0.70,0.75)
4	Low Effect	(0.30,0.60,0.65)
5	Medium Low Effect	(0.40,0.50,0.55)
6	Medium Effect	(0.50,0.45,0.45)
7	Medium High Effect	(0.60,0.40,0.35)
8	High Effect	(0.70,0.30,0.25)
9	Very High Effect	(0.80,0.20,0.15)
10	Extremely High Effect	(0.90,0.15,0.10)
11	Extreme Effect	(0.95,0.05,0.05)

The last PNS linguistic variable, structured on a 11-point scale, consists of eleven linguistic terms. Each of these terms aligns with pair-wise comparison scales spanning from 1 to 11 signifying “Negligible effect”, “Extremely low effect”, “Very low effect”, “Low effect”, “Medium Low effect”, “Medium effect”, “Medium High effect”, “High effect”, “Very High effect”, “Extremely High effect” and “Extreme effect” respectively. After establishing these new linguistic variables, it is essential to verify that these PNS numbers meet the conditions of PNS specified in Equation (2), (3) and (4). Table 5, Table 6, and Table 7 show the verification of the PNS numbers which fulfilled the conditions for a Pythagorean neutrosophic set.

Table 5. Verification for 5-point scale PNS linguistic variable

τ	ξ	η	$\tau+\eta$	$\tau^2+\eta^2$	$\tau^2+\xi^2+\eta^2$
0.10	0.85	0.90	1.00	0.82	1.54
0.30	0.65	0.70	1.00	0.58	1.00
0.50	0.45	0.45	0.95	0.45	0.66
0.70	0.25	0.20	0.90	0.53	0.59
0.90	0.10	0.05	0.95	0.81	0.82

Table 6. Verification for 9-point scale PNS linguistic variable

τ	ξ	η	$\tau+\eta$	$\tau^2+\eta^2$	$\tau^2+\xi^2+\eta^2$
0.05	0.90	0.95	1.00	0.91	1.72
0.10	0.85	0.90	1.00	0.82	1.54
0.20	0.80	0.75	0.95	0.60	1.24
0.35	0.65	0.60	0.95	0.48	0.91
0.50	0.50	0.45	0.95	0.45	0.70
0.65	0.35	0.3	0.95	0.51	0.64
0.80	0.25	0.20	1.00	0.68	0.74
0.90	0.15	0.10	1.00	0.82	0.84
0.95	0.05	0.05	1.00	0.91	0.91

Table 7. Verification for 11-point scale PNS linguistic variable

τ	ξ	η	$\tau+\eta$	$\tau^2+\eta^2$	$\tau^2+\xi^2+\eta^2$
0.05	0.90	0.95	1.00	0.91	1.72
0.10	0.80	0.85	0.95	0.73	1.37
0.20	0.70	0.75	0.95	0.60	1.09
0.30	0.60	0.65	0.95	0.51	0.87
0.40	0.50	0.55	0.95	0.46	0.71
0.50	0.45	0.45	0.95	0.45	0.66
0.60	0.40	0.35	0.95	0.48	0.64
0.70	0.30	0.25	0.95	0.55	0.64
0.80	0.20	0.15	0.95	0.66	0.70
0.90	0.15	0.10	1.00	0.82	0.84
0.95	0.05	0.05	1.00	0.91	0.91

The PNMEREC Procedures

The proposed approach, named PNMEREC, integrates the concepts of Pythagorean neutrosophic set (PNS) with the Method Based on the Removal Effects of Criteria (MEREC) to enhance decision-making within fuzzy environments. PNMEREC strives to offer a thorough and reliable methodology for evaluating criteria and establishing weights in scenarios involving MCDM.

In this study, all the proposed methods have been modified according to the characteristics of the PNS. A modified PNS linear normalization is employed in this methodology. Additionally, the logarithmic function used to determine the alternatives' overall performance is also modified according to the PNS framework. Lastly, we also introduced two enhanced distance techniques to calculate the summation of absolute deviations which are PN-Hamming distance technique and PN-Euclidean distance technique. The proposed steps of PNMEREC are as follows:

Step 1.
Construct the decision matrix consisting of scores that illustrate the ratings or values of each alternative with respect to every criterion. These scores are determined by the decision-makers (DM) based on their opinions for each criterion. The elements within this matrix are represented as x_{ij}, and each of these elements must be greater than zero $(x_{ij} > 0)$. Afterward, each of these scores are converted into PNS numbers in the form of $x_{ij}^k = \langle \tau_{ij}^k,\ \xi_{ij}^k,\ \eta_{ij}^k \rangle$. The decision-makers can choose whether to use 5, 7, 9 or 11 scale rating. Suppose that there are n alternatives and m criteria, and the structure of the decision matrix takes the following form:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{im} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nj} & \cdots & x_{nm} \end{bmatrix} \tag{9}$$

Step 2.
Normalize the decision matrix (N). Normalization is a process of transforming data to a common standard, stripping away factors like optimization orientation such as benefit and cost criteria, the unit of measurement, and the range of variation. By normalizing, data gets reshaped to a consistent standard where in this case, the normalized value is scaled uniformly between 0 and 1 [48-50]. We employ simple linear normalization in PNS form for this approach, similar to the original MEREC method. The elements of the normalized matrix are denoted by n_{ij}^k which has the form of $n_{ij}^k = \langle \tau_{ij}^k,\ \xi_{ij}^k,\ \eta_{ij}^k \rangle$.

$$n_{ij} = \begin{cases} \left(\dfrac{\min \tau_{ij}^k}{\tau_{ij}^k}, \dfrac{\min \xi_{ij}^k}{\xi_{ij}^k}, \dfrac{\min \eta_{ij}^k}{\eta_{ij}^k} \right), & for\ benefit\ criteria \\ \left(\dfrac{\tau_{ij}^k}{\max \tau_{ij}^k}, \dfrac{\xi_{ij}^k}{\max \xi_{ij}^k}, \dfrac{\eta_{ij}^k}{\max \eta_{ij}^k} \right), & for\ cost\ criteria \end{cases} \tag{10}$$

Step 3.
Evaluate the overall performance of the alternatives (S_i). In this step, a modified logarithmic function is used to determine the alternatives' overall performance. The modified equation integrates the logarithmic function in the form of PNS. The calculation utilizes the following equation:

$$S_i = \left(\ln\left(1 + \left(\frac{1}{m} \sum_j \left| \ln(\tau_{ij}^k) \right| \right) \right), \ln\left(1 + \left(\frac{1}{m} \sum_j \left| \ln(\xi_{ij}^k) \right| \right) \right), \ln\left(1 + \left(\frac{1}{m} \sum_j \left| \ln(\eta_{ij}^k) \right| \right) \right) \right) \tag{11}$$

Step 4.
Evaluate the alternatives' performance (S'_{ij}) by removing each criterion. For this step, the same logarithmic function from the prior step is used. The key difference between this step and step 3 is that the performance of each alternative is determined by removing each criterion respectively. S'_{ij} denotes the performance of ith alternative after the removal of jth criterion. The following equation is utilized for the calculation:

$$S'_{ij} = \left(\ln\left(1 + \left(\frac{1}{m} \sum_{k,k\neq j} \left| \ln(\tau_{ik}^k) \right| \right) \right), \ln\left(1 + \left(\frac{1}{m} \sum_{k,k\neq j} \left| \ln(\xi_{ik}^k) \right| \right) \right), \ln\left(1 + \left(\frac{1}{m} \sum_{k,k\neq j} \left| \ln(\eta_{ik}^k) \right| \right) \right) \right) \tag{12}$$

Step 5.
Calculate the summation of absolute deviations. There are numerous distance techniques that can be utilized to calculate the summation of absolute deviations. For this study, we introduced two distance techniques to calculate the removal effect of the jth criterion which are PN-Hamming distance technique and PN-Euclidean distance technique. The normalized equation for each respective distance technique is also formulated as below:

i. The PN-Hamming distance

$$D_j = \frac{1}{3}\sum_{i=1}^{n}\left(\left|\tau'_{ij} - \tau_i\right| + \left|\xi'_{ij} - \xi_i\right| + \left|\eta'_{ij} - \eta_i\right|\right) \tag{13}$$

ii. The normalized PN-Hamming distance

$$D_j = \frac{1}{3n}\sum_{i=1}^{n}\left(\left|\tau'_{ij} - \tau_i\right| + \left|\xi'_{ij} - \xi_i\right| + \left|\eta'_{ij} - \eta_i\right|\right) \tag{14}$$

iii. The PN-Euclidean distance

$$D_j = \left[\frac{1}{3}\sum_{i=1}^{n}\left(\left(\tau'_{ij} - \tau_i\right)^2 + \left(\xi'_{ij} - \xi_i\right)^2 + \left(\eta'_{ij} - \eta_i\right)^2\right)\right]^{\frac{1}{2}} \tag{15}$$

iv. The normalized PN-Euclidean distance

$$D_j = \left[\frac{1}{3n}\sum_{i=1}^{n}\left(\left(\tau'_{ij} - \tau_i\right)^2 + \left(\xi'_{ij} - \xi_i\right)^2 + \left(\eta'_{ij} - \eta_i\right)^2\right)\right]^{\frac{1}{2}} \tag{16}$$

Step 6.
Determine the final weights for each criterion. This step will determine the objective weight for each criterion by using the distance value, D_j from Step 5. The following equation is used to calculate w_j which denotes the weight of the jth criterion:

$$w_j = \frac{D_j}{\sum_k D_k} \tag{17}$$

Results and Discussion

In this segment, we introduce three subsections. The initial sub-section employs a straightforward example to systematically demonstrate the utilization of PNMEREC. The subsequent sub-section conducts a comparative analysis to manifest the validity and consistency of PNMEREC results in comparison to existing methods for determining objective criteria weights. The third sub-section performs a sensitivity analysis of the PNMEREC method using both distance techniques.

Illustrative Example

In this sub-section, we employ a simple decision matrix to demonstrate how PNMEREC can be utilized for determining criteria weights. For this purpose, we will be using 9-point scale PNS linguistic variable.

Step 1.
A company seeks to select a Halal supplier for their business. They are considering five distinct suppliers for this purpose. Each of these suppliers is going to be evaluated based on two benefit criteria which are (C_1) Quality of Products and (C_2) Product Variety. For the cost criteria, we have (C_3) Cost and Pricing and (C_4) Shipping and Delivery Costs. Table 8 displays the components of this decision matrix.

Table 8. The decision matrix of the illustrative example

	C1	C2	C3	C4
S1	7	5	1	4
S2	7	6	3	1
S3	8	5	2	4
S4	8	9	1	5
S5	6	8	2	3

Afterward, these values in the decision matrix are converted into PNS numbers according to Table 3, which utilizes a 9-point scale PNS linguistic variable. Table 9 shows the decision matrix consisting of scales in PNS numbers.

Table 9. The decision matrix in PNS numbers

	C1	C2	C3	C4
S1	(0.80,0.25,0.20)	(0.50,0.50,0.45)	(0.05,0.90,0.95)	(0.35,0.65,0.60)
S2	(0.80,0.25,0.20)	(0.65,0.35,0.30)	(0.20,0.80,0.75)	(0.05,0.90,0.95)
S3	(0.90,0.15,0.10)	(0.50,0.50,0.45)	(0.10,0.85,0.90)	(0.35,0.65,0.60)
S4	(0.90,0.15,0.10)	(0.95,0.05,0.05)	(0.05,0.90,0.95)	(0.50,0.50,0.45)
S5	(0.65,0.35,0.30)	(0.90,0.15,0.10)	(0.10,0.85,0.90)	(0.20,0.80,0.75)

Step 2.
The normalized decision matrix is obtained by using Equation (10). Table 10 shows the normalized matrix. The following illustrates the process of calculating the normalization value for Supplier 1 (S_1) in relation to Criterion (C_1).

$$n_{11} = \left(\frac{0.65}{0.80}, \frac{0.15}{0.25}, \frac{0.10}{0.20}\right) = (0.81, 0.60, 0.50)$$

Table 10. The normalized decision matrix of the illustrative example

	C1	C2	C3	C4
S1	(0.81,0.60,0.50)	(1.00,0.10,0.11)	(0.25,1.00,1.00)	(0.70,0.72,0.63)
S2	(0.81,0.60,0.50)	(0.77,0.14,0.17)	(1.00,0.89,0.79)	(0.10,1.00,1.00)
S3	(0.72,1.00,1.00)	(1.00,0.10,0.11)	(0.50,0.94,0.95)	(0.70,0.72,0.63)
S4	(0.72,1.00,1.00)	(0.53,1.00,1.00)	(0.25,1.00,1.00)	(1.00,0.56,0.47)
S5	(1.00,0.43,0.33)	(0.56,0.33,0.50)	(0.50,0.94,0.95)	(0.40,0.89,0.79)

Step 3.
In this step, the overall performance of the alternatives is calculated by using Equation (11). Table 11 displays the overall performance of the alternatives. The following is the example of computing the overall performance for Supplier 1 (S_1).

$$S_1 = \left(\ln\left(1 + \left(\frac{1}{4}(|\ln(0.81)| + |\ln(1.00)| + |\ln(0.25)| + |\ln(0.70)|)\right)\right), \ln\left(1 + \left(\frac{1}{4}(|\ln(0.60)| + |\ln(0.10)| + |\ln(1.00)| + |\ln(0.72)|)\right)\right), \ln\left(1 + \left(\frac{1}{4}(|\ln(0.50)| + |\ln(0.11)| + |\ln(1.00)| + |\ln(0.63)|)\right)\right)\right) = (0.40, 0.58, 0.61)$$

Table 11. The overall performance of the alternatives

S1	(0.40,0.58,0.61)
S2	(0.53,0.50,0.52)
S3	(0.30,0.51,0.52)
S4	(0.46,0.14,0.17)
S5	(0.44,0.43,0.42)

Step 4.
For this step, we will use Equation (12) to calculate each alternatives' performance (S'_{ij}) by disregarding each respective criterion. The alternatives' performance value is presented in Table 12. The illustration of Supplier 1's (S_1) performance calculation, by excluding the criterion (C_1), is shown as follows:

$$S'_{11} = \left(\ln\left(1+\left(\frac{1}{4}(|\ln(1.00)|+|\ln(0.25)|+|\ln(0.70)|)\right)\right), \ln\left(1+\left(\frac{1}{4}(|\ln(0.10)|+|\ln(1.00)|+|\ln(0.72)|)\right)\right), \ln\left(1+\left(\frac{1}{4}(|\ln(0.11)|+|\ln(1.00)|+|\ln(0.63)|)\right)\right)\right) = (0.36,0.51,0.51)$$

Table 12. The values of (S'_{ij})

	C1	C2	C3	C4
S1	(0.36,0.51,0.51)	(0.40,0.19,0.25)	(0.13,0.58,0.61)	(0.34,0.53,0.54)
S2	(0.50,0.42,0.41)	(0.49,0.15,0.21)	(0.53,0.48,0.48)	(0.11,0.50,0.52)
S3	(0.23,0.51,0.52)	(0.30,0.09,0.12)	(0.16,0.51,0.51)	(0.23,0.46,0.45)
S4	(0.41,0.14,0.17)	(0.36,0.14,0.17)	(0.22,0.14,0.17)	(0.46,0.00,0.00)
S5	(0.44,0.28,0.22)	(0.34,0.23,0.30)	(0.32,0.42,0.41)	(0.28,0.41,0.38)

Step 5.
The removal effect for each criterion is calculated by using the normalized distance technique formula as presented in Equation (14) and Equation (16). These values are depicted in Table 13. For this demonstration, we will compute the distance by using the normalized PN-Hamming distance.

$$D_1 = \frac{1}{3(5)}[(|0.36-0.40|+|0.51-0.58|+|0.51-0.61|)+(|0.50-0.53|+|0.42-0.50|+|0.41-0.52|) + (|0.23-0.30|+|0.51-0.51|+|0.52-0.52|) + (|0.41-0.46|+|0.14-0.14|+|0.17-0.17|) + (|0.44-0.44|+|0.28-0.43|+|0.22-0.42|)] = 0.06$$

Table 13. The normalized PN-Hamming distance value

D1	0.0595
D2	0.1859
D3	0.0571
D4	0.0870

Step 6.
The weightages for each criterion are determined by analyzing how their removal affects the performance of the alternatives. We employ Equation (17) to calculate these weights, demonstrated by the following steps for weight of criterion (C_1):

$$w_1 = \frac{0.0595}{0.0595+0.1859+0.0571+0.0870} = 0.1528$$

Table 14. The weight for each criterion

W1	0.1528
W2	0.4772
W3	0.1466
W4	0.2234

Comparative Analysis

This section aims to evaluate and compare the performance of different methods in determining criteria weights. We chose two objective weighting methods and conducted the analysis by using the same decision matrix as in Table 8. The total weightage for all criterion sums up to 1 for every method. The results obtained for PN-Entropy and PN-Statistical Variance are then compared with weights computed by the PNMEREC method.

Table 15. The weights of criterion for three methods

	PN-Entropy	PN-Statistical Variance	PNMEREC
W1	0.0879	0.0882	0.1528
W2	0.5739	0.5700	0.4772
W3	0.0321	0.0320	0.1466
W4	0.3061	0.3098	0.2234

Table 16. The correlation coefficient of the comparative analysis

	PN-Entropy	PN-Statistical Variance
Correlation coefficient, r	0.9616	0.9585

Table 15 displays the comparison of criterion weights computed by each method. Figure 2 and Figure 3 also give the representation of these weights. The result shows that criterion (C_2) holds the highest weight across all methods, while criterion (C_3) has the lowest weight for every method. Criterion (C_4) earned the second position, with Criterion (C_1) following in third. Table 16 presents the respective Pearson correlation coefficients (r) that reflect the relationship between PNMEREC results and the other methods.

If the correlation coefficient between two variables exceeds 0.4, we can infer a moderate relationship between them. A correlation coefficient surpassing 0.6 indicates a significant relationship between the variables [51]. By referring to the r values in Table 16, it is evident that the criteria weights computed by PNMEREC method exhibit a robust correlation with weights derived from PN-Entropy and PN-Statistical Variance. This suggests that PNMEREC method possesses the potential to serve as a viable alternative method for discerning criteria weights, given its subtle deviations from the weights derived through other methodologies.

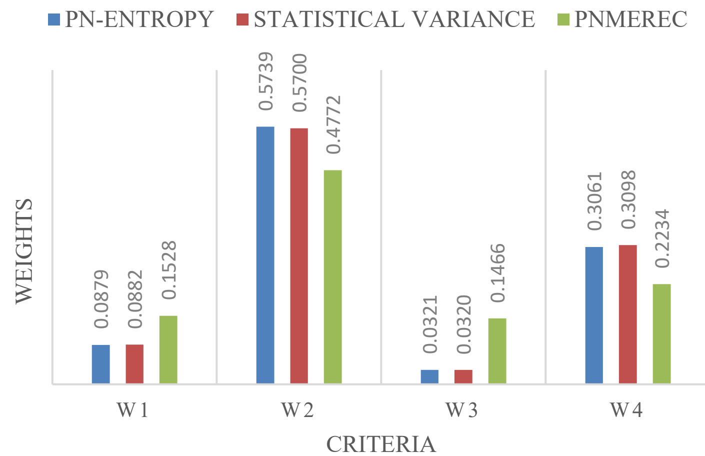

Figure 2. The weights for each method

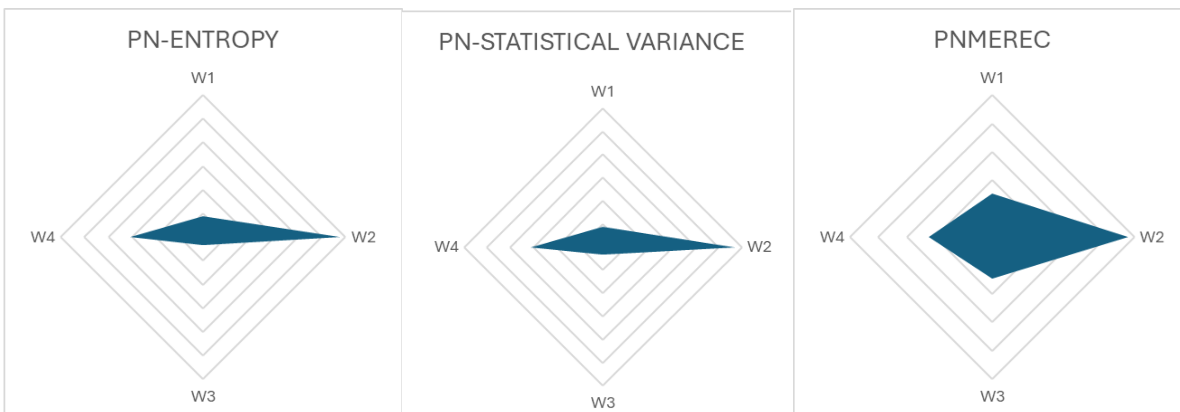

Figure 3. The weights composition for each method

Figure 2 and 3 demonstrate that while PNMEREC yields comparable weightage outcomes to the other two methods, it also displays distinct characteristics that set it apart from them. Those notable differences indicate the strength of this study. PNMEREC's utilization of the Pythagorean neutrosophic number enables a deeper analysis of the interconnections between criteria, highlighting its relevance. Therefore, the PNMEREC methodology has the capability to comprehensively capture the determination of criterion weight through a more intricate and detailed analysis.

To further demonstrate the viability of PNMEREC methodology, we conducted PNMEREC by utilizing 5-point and 11-point PNS linguistic variable and compared it with the other methods. Table 17 and Table 18 show the result of criteria weights determination by using 5-point PNS linguistic variable and 11-point PNS linguistic variable respectively.

Table 17. The weights of criterion using 5-point PNS linguistic variable

	PN-Entropy	PN-Statistical Variance	PNMEREC
W1	0.1606	0.1580	0.1452
W2	0.3266	0.3223	0.3707
W3	0.3217	0.3371	0.3171
W4	0.1911	0.1826	0.1670
Correlation coefficient, r	0.9827	0.9650	

Table 18. The weights of criterion using 11-point PNS linguistic variable

	PN-Entropy	PN-Statistical Variance	PNMEREC
W1	0.2122	0.2082	0.3198
W2	0.5392	0.5426	0.4340
W3	0.1212	0.1207	0.1076
W4	0.1274	0.1284	0.1387
Correlation coefficient, r	0.9045	0.8996	

From the result shown in Table 17 and 18, we can see that PNMEREC method shows homogeneous outcomes throughout the utilization of all three different scales of PNS linguistic variables. Hence, the application of the PNMEREC methodology possesses the capability to yield robust and credible criteria weights essential for MCDM scenarios.

Sensitivity Analysis

Conducting sensitivity analysis is essential to assess the reliability and robustness of decision-making models. In this research, we performed sensitivity analysis on the PNMEREC method by employing two different distance techniques: PN-Hamming distance and PN-Euclidean distance. The comparison of results obtained through these measures allowed us to evaluate the PNMEREC method's responsiveness to variations. This investigation aimed to explore how variations in methodological approaches affect the determination of criteria weights and shape the outcomes of the decision-making framework. Table 19 provides a comparison of criterion weights determined using the PN-Hamming distance and PN-Euclidean distance techniques. Figure 4 also give the representation of these weights.

Table 19. The weights of criterion computed using each distance technique

	PN-Hamming	PN-Euclidean
W1	0.1528	0.1466
W2	0.4772	0.4325
W3	0.1466	0.1848
W4	0.2234	0.2360

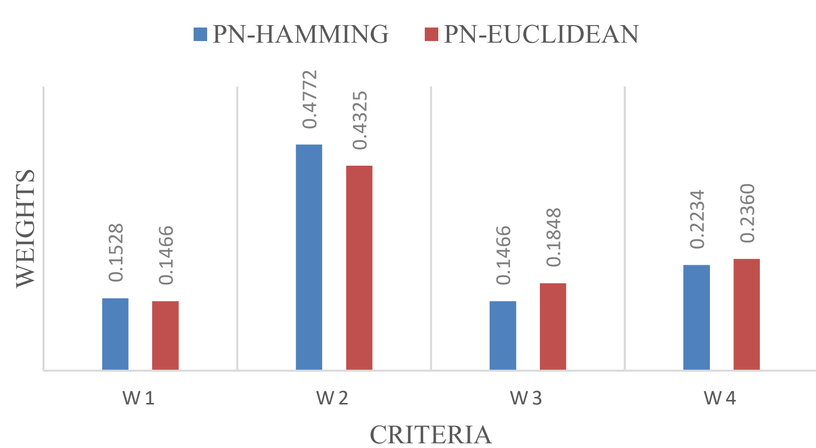

Figure 4. The criterion weights for each distance technique

Table 19 and Figure 4 reveal that using PN-Hamming and PN-Euclidean distance techniques in the same method produces differing weight distributions, highlighting the impact of the chosen technique on decision-making outcomes. Both PN-Hamming and PN-Euclidean emphasize criterion (C_2) as the dominant criterion, though PN-Hamming assigns it a slightly higher weight. PN-Hamming yields more concentrated weights, particularly accentuating criterion (C_2), while PN-Euclidean distributes the weights more smoothly, assigning slightly higher importance to criterion (C_3) and criterion (C_4) compared to PN-Hamming. Both techniques agree on the significance of criterion (C_2) but differ in how they distribute weights among lesser criteria like criterion (C_1) and criterion (C_3). This demonstrates that the choice of distance technique influences prioritization, with PN-Hamming reinforcing differences more distinctly and PN-Euclidean promoting a more balanced transition across criteria.

Conclusions

This paper introduces new PNS linguistic variables using 5-point, 9-point, and 11-point scales. The lack of linguistic variables in the PNS framework can make it difficult for decision-makers in articulating their opinions and assessments in their desired preference. With these new scales, decision-makers have more flexibility in choosing the PNS scale that best suits their research needs. The approach involves defining linguistic terms, creating membership functions, and validating the new linguistic variables with PNS conditions. Larger PNS scales, like the 9-point and 11-point scales, allow for more detailed differences and a better understanding of respondents' views. However, for those who prefer simpler scales, the 5-point scale is also available.

This paper introduces the PNMEREC methodology for determining objective criteria weights in MCDM. In such problems, decision-makers evaluate alternatives based on multiple criteria, and weighting helps express the relative importance of each. The original MEREC method assesses alternative performance by removing criteria. We show how integrating PNS with the MEREC framework enhances the methodology, enabling decision-makers to capture and analyze the significance and interdependencies of criteria effectively. We provide an illustrative example of the proposed method and compare it to PN-Entropy and PN-Statistical Variance. The correlation coefficient values demonstrate that the PNMEREC method yields reliable criterion weights. A sensitivity analysis further compares the PNMEREC method using PN-Hamming and PN-Euclidean distance techniques. This comparison reveals the technical sensitivity of weight calculations, PN-Hamming highlighting distinctions among criteria more sharply, while PN-Euclidean promotes a smoother and more balanced weight distribution. These findings emphasize the importance of choosing the right distance measure to align with decision-making goals and the desired focus on variability. By using PNS, the decision matrix is represented through three types of membership values: truth-membership, indeterminacy-membership, and falsity-membership. This allows for a more detailed representation of uncertainty, especially in situations where multiple factors and criteria must be considered. The proposed method provides decision-makers with clearer insights, improving the accuracy and reliability of MCDM outcomes.

As evidenced by the result of comparative analysis in this study, PNMEREC methodology has proven to be a valuable computation tool for criteria weights in MCDM where traditional approaches fall short. Even though we introduced a new approach to criteria weighting in our proposed method, the outcomes remained consistent with those achieved by existing objective weighting methods. The PNMEREC method is a versatile tool applicable across various industries, including manufacturing, healthcare, supply chain management, transportation, and logistics, enabling robust decision-making in complex scenarios. Moving forward, further research and development in this area can delve into decision making applications, integration with other weighting methods, real-world implementation, robustness, and sensitivity analysis, as well as conducting comparative studies. Additionally, future research could focus on integrating MEREC with other objective and subjective weighting methods, such as Entropy, CRITIC, Integrated Determination of Objective Criteria Weights (IDOCRIW), Adaptive Criteria Weighting (ACW), SWARA, AHP and other methods [52-54]. Exploring other normalization techniques and distance measures within this PNMEREC framework could enhance its capabilities. Collaborations with machine learning algorithms can also contribute to enhanced decision-making. Overall, this innovative approach not only enhances the theoretical foundations of decision science but also holds significant practical implications for diverse applications in multiple fields, ultimately empowering decision-makers to make more informed and effective choices in complex decision environments.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

Acknowledgement

We would like to acknowledge the Ministry of Higher Education Malaysia for their sponsorship of the Fundamental Research Grant Scheme (Project Code: FRGS/1/2023/STG06/UITM/02/5). This financial support has been crucial in advancing our research efforts, and we are grateful for their assistance.

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