

# Integrating Fuzzy-Based Evaluation Method to Analyse Attributes and Parameters for the Assessment Model Development

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**Abstract** In this paper, an assessment model was developed based on the proposed integrated fuzzy-based evaluation method on students' mathematics learning ability. This model classifies six main attributes that structure the overall evaluation model into several parameters. The weightings of these main attributes and parameters were collected through fuzzy questionnaires among teachers and experts based on triangular fuzzy conjoint and fuzzy Delphi methodology. This highlighted integration contributes to a more reasonable and effective procedure for developing a structured and dynamic assessment model. It can reduce the problem of the measurement results obtained straying from the structure of the developed model due to procedural errors in identifying and analyzing the attributes and parameters of the model when it was developed. In addition, the presented case application also provides an analysis protocol that is simpler and easier compared to other complicated and complex approaches to developing assessment models.

**Keywords:** Triangular fuzzy conjoint, Fuzzy Delphi method, assessment model, attributes and parameters, fuzzy-based evaluation method.

## Introduction

In the educational environment, the teaching and learning process conceptually consists of three interrelated components, teaching, learning and assessment [1]. The assessment conducted by educators is based on the main objective of monitoring the process, seeing progress, and analysing student learning results continuously [2]. Evaluation is also carried out to improve the quality of education in the context of development or strengthen the curriculum used [3]. On this basis, various assessment models, tools and instruments are used either in general or specific to the aspect to be evaluated. The effectiveness and quality of an evaluation process and model depend on how it is developed, implemented, and adapted to the learning process [4].

The assessment of mathematics learning ability is also increasingly important based on the translation of this ability in international competency assessment strategies such as Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) [5]. Some efforts and evaluations should have been performed, particularly by utilising cutting-edge technology and recent discoveries in conducting the mathematics learning ability evaluation. Experts and academics have suggested that mathematics learning ability assessment challenges can be handled using new knowledge discoveries such as neuroscience as attributes and measuring parameters [5,6,7]. According to theory, compact assessment models require high sensitivity, precision, and accuracy, as well as the ability to forecast and evaluate [8]. However, when combining measurements with multiple, confusing, mutually influential, and divergent criteria, the assessment model must be both alternative and adaptable [9]. The models' flexibility is determined by the organisational structure of relevant and well-supported theories [10].

After evaluating 30 assessment models, Stanton *et al.* [11] stated that one of the main issues in model development is related to the relationship between the structure of the model and the type of results

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produced. Leigh *et al.* [12] also confirmed that no single assessment model can measure all competencies and it is necessary to implement evaluation in combination with other equivalent models as a complementary effort. This problem and gap are the result of confusion at the beginning of developing the assessment model. If model attributes and parameters are not identified and analyzed prudently, it will weaken the structure of the developed model. This will lead to errors and inaccurate final results [13,14]. In addition, the combination of models to measure another phenomenon is also quite impossible because the original objective of each model is specific to only its phenomenon and it is also specially developed for certain phenomena only [12]. So, in this context, the development stage of the assessment model requires a more comprehensive, detailed and analytical process of identification and analysis of attributes and parameters.

Based on the present state of mathematics learning, the form of assessment employed was similar to a monthly or mid-year test, where it simply assessed students' cognitive or understanding capacity, but previous studies have shown that many more aspects influence mathematics learning ability [5,6,7]. The assessment does not properly assess all areas of students' mathematics learning ability. As a result, an approach of assessment that can predict the general attributes concerned as well as identify the level of students' mathematics learning capacity must be proposed. For the gaps discussed, the structure of the model that will be developed needs to be strengthened to meet the objectives and produce the desired outcome. Therefore, the application of the fuzzy evaluation method needs to be integrated into the process of identification and analysis of model attributes and parameters so that the developed students' mathematics learning ability assessment model is more effective and suitable for the measurement situation.

In the model development phase, fuzzy Delphi is often the choice of model developers, based on the integrity of this method in dealing with several things such as saving time, more flexible expert determination, and analytical efficiency. Researchers [15-20], have applied the fuzzy Delphi method in their design research and model development, which is the phase of determining items and parameters. However, the process of choosing, determining and justifying attributes in the early stages of most researchers only depends on the literature review and the views of a few experts. How do the researchers determine the priority, importance and weighting of any attributes if they do not carry out any analysis first? Is it sufficient to only carry out decision-making personally and not the collective work of experts or individuals who will be fully involved as users of the model? Roberson *et al.* [14] also discussed this issue which is related to the neglect of data analysis and initial interpretation before the model is developed. Sarala and Kavitha [2] asserted that the development of an effective model is based on the justification process of accurate attributes and parameters. So, the attribute justification process is needed, and it is related to evaluation, judgments, and decision-making.

In short, there is a need to develop an assessment model of students' mathematics learning ability by integrating the fuzzy-based evaluation method. The hybrid triangular fuzzy conjoint model and fuzzy Delphi methods are suitable for use to identify attributes and analyze parameters of the evaluation model so that the development process is more systematic and produces structured and effective assessment models and measurement tools. Therefore, this study aims to integrate triangular fuzzy conjoint and fuzzy Delphi methods to identify and analyze attributes and parameters for the students' mathematics learning ability assessment model development. Specifically, this paper demonstrates how an integrated literature review outcome, triangular fuzzy conjoint model, and fuzzy Delphi method can be used as attributes identification and analysis of parameters for assessment model development. An assessment model for measuring students' mathematics learning ability is proposed, and the related importance weights of the attributes and item parameters are calculated. The application case results can also be used for the consultation and guidance for practical assessment model development in the future. This paper is organized as follows. Students' mathematics learning ability and assessment model are reviewed in Literature review section. The integrated fuzzy-based evaluation methods are presented in Methodology section. The proposed methodology is applied to a real case in Application case section. Results, discussions and conclusions are provided in last sections.

## Literature Review

### Mathematics Learning Ability and Assessment Model Development

According to Schoenfeld [21], learning mathematics is a cognitive process where metacognitive skills are tested and is an implication of neurocognitive to mathematics problems. Learning mathematics acts as an assessment instrument and the basis for problem-solving skills and also improves thinking ability [22]. Problem solving that is applied in learning mathematics, is a directed cognitive process based on

the following four definitions, occurs in the cognitive system, the use of cognitive processes for cognitive performance, is guided by the problem-solving process, and needs to be personal where it depends on the individual's knowledge and skills [23]. Many attributes that affect the strengths or contribute to the weaknesses of students' performance in mathematics learning need to be measured [24,25]. Some researchers list emotion, readiness, motivation, metacognitive coordination, memory system, and mathematical problem-solving mechanisms as factors involved [7, 26-32].

Mathematics learning is in the process of receiving the effects of change and transformation of assessment strategies. Mathematics learning and the assessment system need to be adapted so that they are more contemporary and in line with the latest technological changes. Therefore, a more accurate and effective assessment model needs to be adapted to the current learning environment. An assessment model is a basic guide or procedure for assessing a phenomenon based on related theories. In an educational environment, the role of assessment is to improve the effectiveness of learning and produce an effective assessment strategy [3]. Measurements included in assessment strategies are formative assessment, summative assessment, and continuous assessment [33]. It should be emphasized that the assessment made by educators, especially, should be comprehensive and continuous. This means that the assessment carried out should cover all aspects of competence based on accurate strategies and techniques, and can monitor student progress holistically [1]. This is in line with model development research that is rapidly happening and is focused on efforts to produce new knowledge either in design, new development, or improved or modified models [34].

The development of assessment models is a necessity and a complement to the evaluation strategy. Various initiatives in developing assessment models, including the integration of fuzzy evaluation methods or soft computing techniques. This is because it involves various data problems, data orientation, and analysis gaps [35]. Studies by Yiğit [36] and Jahanvand *et al.*, [15] showed that the integration was successful, and the final results reported were also effective. Therefore, the fuzzy evaluation method can be accepted as a transformation in the assessment strategy in today's education system.

### Fuzzy Set, Fuzzy Logic Theory and Fuzzy-Based Evaluation Method

Fuzzy set theory was introduced due to the need to produce a mathematical representation in the analysis of a phenomenon [2]. According to Tseng [37], fuzzy set theory can determine and handle judgments in situations that are not clear or precise in terms of mathematical analysis. In 1965, Lotfi A. Zadeh has introduced this fuzzy logic from the extension of classic Boolean logic that is also able to implement the concept of partial truth to make decisions and validate ambiguous and unclear issues related to humans as decision-makers in doubt [10, 38]. When statistical data deals with the ambiguity, uncertainty and vagueness of individual arbitration and valuation in the process of decision-making, accurate mathematical analysis methods are needed [39]. Fuzzy evaluation methods are required when the decision-making process of some of the available parameters and their ranking is based on several attributes that have different importance and roles [38]. Data analysis in an evaluation adheres to the theory of fuzzy sets and fuzzy logic. Their implications and adaptations are very important when used in analysis to find the best approximate solution in complex phenomena that are difficult to interpret by traditional mathematical methods. Fuzzy set, fuzzy logic, and fuzzy evaluation have been applied to virtually all branches of science, engineering, and socio-economic sciences [35, 40].

In short, fuzzy-based evaluation methods are based on a mathematical fuzzy system that includes steps such as:

- i. Fuzzification: determine all input values into fuzzy membership functions
- ii. Execute all applicable fuzzy evaluation to compute the fuzzy output functions
- iii. Defuzzification, converting output functions to get crisp output values

### Triangular Fuzzy Number and Fuzzy Conjoint (FC) Model

In general, the description of triangular fuzzy number,  $A$  is as follows.

Definition 1. [41] A triangular fuzzy number  $A$  can be defined by a triplet  $(a_1, a_2, a_3)$ . The membership function  $\mu_A(x)$  is

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_2 - a_3}{x - a_3}, & a_2 \leq x \leq a_3 \\ 0, & a_3 < x \end{cases}$$

where  $0 \leq a_1 \leq a_2 \leq a_3 \leq 1$ , the value of  $a_1$  dan  $a_3$  respectively for the lower and upper values of  $A$ , and  $a_2$  is the middle value.

Definition 2. [42] For triangular fuzzy numbers  $A = (a_1, a_2, a_3)$  and  $B = (b_1, b_2, b_3)$ ,  $height(A) = height(B) = 1, A, B \in [0,1]$ , where  $*$  =  $\{+, -, \times, \div\}$  be arithmetic operations on the triangular fuzzy numbers are defined by  $A * B = \{a_i * b_j, a_i \in A, b_j \in B\}$ . In particular, for any two triangular fuzzy numbers  $A = (a_1, a_2, a_3)$  and  $B = (b_1, b_2, b_3)$ , then

i. Addition (+):  $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$  (1)

ii. Subtraction (-):  $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$  (2)

iii. Multiplication ( $\times$ ):  $k \times A = (ka_1, ka_2, ka_3), k \in R, k \geq 0$  (3)  
 $A \times B = (a_1b_1, a_2b_2, a_3b_3)$  (4)  
 with  $a_i \geq 0$  and  $b_j > 0$  for  $i, j = 1, 2, 3$

iv. Division ( $\div$ ):  $A^{-1} = (a_1, a_2, a_3)^{-1} \cong (\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}), a_1 > 0, a_2 > 0, a_3 > 0$   
 $A \div B \cong (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}), a_1 \geq 0, b_1 \geq 0$  (5)

Definition 3. [43] The similarity degree between  $A$  and  $B$  can be calculated using the formula:

$$Sim(A, B) = \frac{1}{1+d(A, B)} \tag{6}$$

where  $d(A, B) = |P(A) - P(B)|$  is the distance measure of  $A$  and  $B$ , with  $P(A) = \frac{a_1+4a_2+a_3}{6}$  and  $P(B) = \frac{b_1+4b_2+b_3}{6}$ , are the centroid value respectively.

### Fuzzy Delphi (FD) Method

The fuzzy Delphi (FD) method is a mathematical fuzzy analysis procedure that combines the classical Delphi method and fuzzy set theory [44]. This analysis procedure is administered to collect and classify the knowledge and opinions of experts based on qualifications, using questionnaires to obtain feedback and judgment from the experts [17]. The validity and reliability of any attributes or criteria related to any phenomenon can be ensured through expert consensus in the administration of the FD method [44]. Based on this consistency, the FD method is widely applied, such as in the fields of social science [19, 44], humanities [45], business [16], management [17, 20], physical science [15], information science and engineering [46, 47].

Kaufman and Gupta pioneered the FD method to solve problems and weaknesses in the classic Delphi method [48]. Subsequently, modifications and developments were carried out by several researchers to increase efficiency in the procedure of this FD method, including those carried out by Ishikawa *et al.* [49], who used the max-min FD method and the new Delphi method through fuzzy integration. This improved version is introduced to achieve better analytical conclusions and decision-making. The system in this FD method is based on uncertainty and linguistic variables. A triangular fuzzy number is adopted and applied to determine the appropriateness of an indicator. The process of integrating and organizing experts' evaluations is to build priority relationships in ambiguity and find the best solutions in those priority relationships, which are grouped based on the tendencies of the participating experts. FD method is a simpler and more systematic analysis procedure in solving evaluation issues involving multi-criteria or various attributes. Furthermore, the reliability and validity of the fuzzy linguistic scale are higher than the traditional scale [48].

### Methodology

The proposed integrated procedure for identifying attributes and analysing parameters first applies the literature review (LR) to select the most important attributes and then uses the FC model to identify the priority and position among attributes. The importance weights and final ranking of parameters are calculated next by the FD method. Figure 1 shows the implementation procedure with three phases:

- i. Phase 1: Select and determine the attributes and parameters from the literature review
- ii. Phase 2: Identify the priority and position among the attributes by using the FC model
- iii. Phase 3: Evaluate and rank the parameters by using the FD method

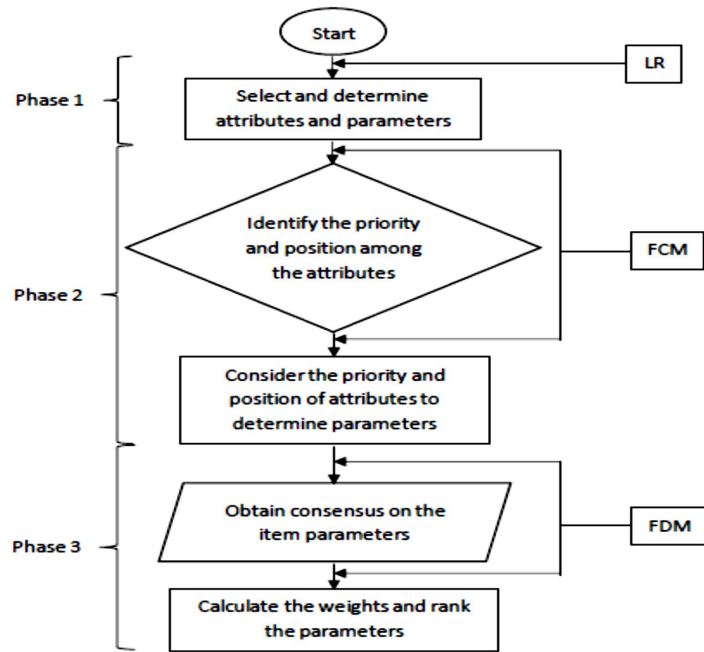


Figure. 1. The proposed integrated procedure

### Hybrid FC-FD Operator

The first Phase is FC administration.

Step 1: Determine the attribute set, denoted as  $A = \{A_i\}$  ( $i = 1,2,3 \dots n$ ) to represent the input data.

Step 2: Define appropriate predefined linguistic values for assessment using Triangular Fuzzy Numbers (TFN), represented as  $V_j = (b_1^j, b_2^j, b_3^j)$  where  $j = 1,2,3 \dots k$

Table 1. Membership function of linguistic values in triangular fuzzy number format

Linguistic values	Triangular fuzzy number
Very strongly disagree	(0.0, 0.0, 0.1)
Strongly disagree	(0.0, 0.1, 0.3)
Disagree	(0.1, 0.3, 0.5)
Neutral	(0.3, 0.5, 0.7)
Agree	(0.5, 0.7, 0.9)
Strongly agree	(0.7, 0.9, 1.0)
Very strongly agree	(0.9, 1.0, 1.0)

Step 3: Collect the number of responses,  $r_{ij}$  corresponding to linguistic values,  $V_j$ , where  $j = 1,2,3 \dots k$  for attributes  $A_i$ .

Step 4: Calculate the weight of attribute  $A_i$  with linguistic value  $V_j$  using formula:

$$w_{ij} = \frac{r_{ij}}{\sum_{j=1}^k r_{ij}} \tag{7}$$

Step 5: Determine the overall membership function of attribute  $A_i$  denoted as  $\tilde{A}_i = (a_1^i, a_2^i, a_3^i)$ , as:

$$\tilde{A}_i = \sum_{j=1}^k w_{ij} V_j, \text{ for } i = 1,2,3 \dots n, \text{ and } j = 1,2,3 \dots k \tag{8}$$

Step 6: Compute the degree of similarity between the aggregated linguistic ratings for the  $i$ -th attributes  $\tilde{A}_i = (a_1^i, a_2^i, a_3^i)$ , where  $i = 1,2,3 \dots n$ , and the linguistic ratings,  $V_j = (b_1^j, b_2^j, b_3^j)$ , where  $j = 1,2,3, \dots, k$  using the similarity measure:

$$S_{ij}(\tilde{A}_i, V_j) = \frac{1}{1+d(P(\tilde{A}_i)-P(V_j))}, i = 1,2,3 \dots, n, j = 1,2,3, \dots, k \tag{9}$$

$$\text{where } P(\tilde{A}_i) = \frac{a_1^i + 4a_2^i + a_3^i}{6} \text{ and } P(V_j) = \frac{b_1^j + 4b_2^j + b_3^j}{6}.$$

In the next phase of the approach, the corresponding weights of the parameter items are determined by FD based on the judgments of a selected group of experts.

**Step 7:** The selection of experts is based on expertise in the fields of mathematics, mathematics education and neuroscience or cognitive science. The expert will act to determine the importance of the evaluation criteria on the parameters to be measured using linguistic variables (Table 1).

**Step 8:** Determining the linguistic scale. This process involves the process of converting all linguistic variables into triangular fuzzy numbers ( $m_1, m_2, m_3$ );  $m_1$  represents the minimum value,  $m_2$  represents the moderate value and,  $m_3$  represents the maximum value.

**Step 9:** Conversion using the formula:

$$f_{ij} = \frac{1}{k} (f_{ij}^1 \pm f_{ij}^2 \pm f_{ij}^k) \quad ; i = 1,2,3, \dots, k. \tag{10}$$

$f_{ij}$  is the mean of triangular fuzzy number and  $k$  is the total number of items.

**Step 10:** For each expert, the vertex method is used to calculate the distance (threshold value,  $d$ ) between the mean of  $f_{ij}$ . The distance of two fuzzy numbers  $m = (m_1, m_2, m_3)$  and  $n = (n_1, n_2, n_3)$  is calculated using the formula:

$$d(\tilde{m}\tilde{n}) = \sqrt{\frac{1}{k} [(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]} \tag{11}$$

If the threshold value,  $d$  is less than or equal to 0.2, then it is considered that expert agreement has been reached. The overall agreement (group consensus) should exceed 75% agreement for each item. Otherwise, the second round should be implemented.

**Step 11:** Aggregate fuzzy ratings with:

$$\tilde{P} = \begin{bmatrix} \tilde{P}_1 \\ \tilde{P}_2 \\ \vdots \\ \tilde{P}_m \end{bmatrix} \text{ where } \tilde{P} = f_{i1} \times w_1 + f_{i2} \times w_2 + \dots + f_{in} \times w_n, i = 1,2, \dots, m \tag{12}$$

$w_i$  is a corresponding weight for  $i$  items.

**Step 12:** Next is the defuzzification step to determine the ranking attribute of each parameter using a formula,  $P_{max} = \frac{1}{3} (m_1 + m_2 + m_3)$  (13)

where  $m_1, m_2$  and  $m_3$  refer to the lower bound, peak, and upper bound of the aggregated triangular fuzzy number, respectively.

## Application Case

A survey using a fuzzy questionnaire was carried out to investigate teachers' perceptions and justifications to determine the need for the development of the students' mathematics problem-solving ability (SMPSA) measurement model. Before that, a content analysis was carried out first to see the problems and gaps in SMPSA through the literature review and detailing the issues that lead to the need for model development. In this regard, based on research gaps, several past studies suggest the development of alternative measurement models to overcome problems in SMPSA. Therefore, a need assessment study should be conducted to see if this happens in the researcher's local environment. This study focuses specifically on the perception of teachers' knowledge and the level of SMPSA which consists of thirteen attributes that use symbols with ( $A_{1-13}$ ), teachers' perspectives on SMPSA which consists of seven attributes represented as ( $A_{14-20}$ ) as well as needs for the development of alternative assessment models denoted as ( $A_{21-26}$ ) which consist of six attributes. The questionnaire was randomly distributed to secondary school Mathematics Teachers in Pasir Gudang, Johor, Malaysia where the total number of respondents was 37 teachers. The respondents involved are male and female teachers who teach mathematics subjects and can make choices and decisions based on their respective experiences.

### Selection of Attributes and Parameters

Through a systematic review of the related literature in the domain of mathematics learning, students' mathematics problem-solving, brain mechanisms of mathematics learning, neurocognitive, metacognitive and so on, attributes and parameters were identified. Labeled with a combination of  $A_iP_j$  which means sequence or number respectively ( $i = 1, 2, \dots, n; j = 1, 2, \dots, k$ ) according to attribute ( $A_i$ ) and item parameter ( $P_j$ ), as shown in Table 2 below.



**Table 2.** A summary of attributes and parameters

Attributes	Parameters	References
Emotion ( $A_1$ )	Achievement emotion <ul style="list-style-type: none"> <li>feeling will succeed (<math>A1P1</math>), (<math>A1P3</math>)</li> <li>feeling will learn new things (<math>A1P4</math>), (<math>A1P5</math>)</li> <li>positive feedback (proud) about ability in problem-solving (<math>A1P6</math>), (<math>A1P20</math>), (<math>A1P21</math>)</li> <li>care about other people's (peer) performance (<math>A1P13</math>), (<math>A1P12</math>), (<math>A1P14</math>), (<math>A1P15</math>)</li> </ul>	[50-59]
	Anxiety <ul style="list-style-type: none"> <li>anxiety control, know about relaxation techniques, stay calm (<math>A1P2</math>), (<math>A1P16</math>), (<math>A1P17</math>)</li> <li>feeling of nervous, worried, afraid, confused, threatened (<math>A1P7</math>), (<math>A1P8</math>), (<math>A1P9</math>), (<math>A1P10</math>), (<math>A1P11</math>), (<math>A1P18</math>), (<math>A1P19</math>)</li> </ul>	
Motivation ( $A_2$ )	Value <ul style="list-style-type: none"> <li>importance of subjects, help with daily practice and human being (<math>A2P1</math>), (<math>A2P5</math>), (<math>A2P14</math>), (<math>A2P15</math>)</li> <li>valuable for self-development (<math>A2P2</math>), (<math>A2P4</math>), (<math>A2P7</math>), (<math>A2P8</math>), (<math>A2P13</math>)</li> </ul>	[52-65]
	Attitude <ul style="list-style-type: none"> <li>expressing the own ideas, skills, performance (<math>A2P3</math>), (<math>A2P9</math>), (<math>A2P10</math>), (<math>A2P11</math>), (<math>A2P16</math>), (<math>A2P18</math>)</li> <li>inquiry, to do more, want to try (<math>A2P6</math>), (<math>A2P12</math>), (<math>A2P17</math>)</li> </ul>	
	Interest, Enjoyment <ul style="list-style-type: none"> <li>like, fun (<math>A2P19</math>), (<math>A2P20</math>)</li> </ul>	
Attention ( $A_3$ )	Readiness, Curiosity <ul style="list-style-type: none"> <li>slow down when encountering important information (<math>A3P6</math>)</li> <li>cannot concentrate/focus while solving problems (<math>A3P14</math>)</li> </ul>	[7], [22] [54-57], [66-70]
	Efficacy, Mindfulness <ul style="list-style-type: none"> <li>able to solve mathematics problems without too much difficulty, maintaining proper attention, shifting attention (<math>A3P5</math>), (<math>A3P7</math>), (<math>A3P11</math>), (<math>A3P12</math>)</li> <li>feeling reduces concentration (<math>A3P13</math>)</li> </ul>	
	Self-concept, Belief <ul style="list-style-type: none"> <li>trust the ability to solve new and difficult problems, should solve them successfully, self-confidence, not scare (<math>A3P1</math>), (<math>A3P2</math>), (<math>A3P3</math>), (<math>A3P4</math>), (<math>A3P8</math>), (<math>A3P9</math>), (<math>A3P10</math>), (<math>A3P15</math>), (<math>A3P16</math>), (<math>A3P17</math>)</li> </ul>	
Executive function ( $A_4$ )	Translating, Representation <ul style="list-style-type: none"> <li>translating, understanding information, repeatedly reading (<math>A4P3</math>), (<math>A4P6</math>), (<math>A4P17</math>)</li> <li>marking, take note (<math>A4P10</math>), (<math>A4P18</math>), (<math>A4P19</math>)</li> <li>representing variables, organizing information (<math>A4P11</math>), (<math>A4P21</math>), (<math>A4P29</math>)</li> </ul>	[52], [56-59], [71-79]
	Visualising <ul style="list-style-type: none"> <li>drawing, illustrating the problem (<math>A4P2</math>), (<math>A4P20</math>)</li> <li>following a map (<math>A4P5</math>)</li> </ul>	
	Abstracting, Hypothesizing <ul style="list-style-type: none"> <li>predicting the result, outcome (<math>A4P1</math>), (<math>A4P9</math>)</li> <li>categorize the type of questions (<math>A4P30</math>)</li> </ul>	
	Modelling, Shifting <ul style="list-style-type: none"> <li>breaking down into smaller steps (<math>A4P4</math>)</li> <li>formulating (<math>A4P12</math>), (<math>A4P13</math>), (<math>A4P27</math>), (<math>A4P31</math>), (<math>A4P32</math>)</li> <li>associating the given problem to the real world (<math>A4P26</math>)</li> </ul>	
	Computation, Reasoning <ul style="list-style-type: none"> <li>planning, handling, completing tasks (<math>A4P7</math>), (<math>A4P8</math>), (<math>A4P14</math>), (<math>A4P15</math>), (<math>A4P16</math>)</li> <li>identifying quantities, the unknown, the series of operations (<math>A4P22</math>), (<math>A4P23</math>), (<math>A4P24</math>)</li> <li>looking for patterns in the problem (<math>A4P25</math>)</li> <li>finding creative and effective solutions (<math>A4P28</math>)</li> </ul>	

Attributes	Parameters	References
Metacognition (A <sub>5</sub> )	Conditional, Procedural <ul style="list-style-type: none"> <li>think up creative and effective alternatives (A5P1)</li> </ul> Planning <ul style="list-style-type: none"> <li>think of several ways and alternatives (A5P4), (A5P5)</li> <li>planning short-and long-term goals, target (A5P8), (A5P18)</li> </ul> Insight <ul style="list-style-type: none"> <li>operating, solve step-by-step (A5P3), (A5P13)</li> <li>use real examples to verify the math theory conclusion (A5P17)</li> </ul> Monitoring <ul style="list-style-type: none"> <li>surveying and considering the relevant information, the meaning of the words, options (A5P2), (A5P6), (A5P7), (A5P10)</li> <li>make sure that you have the goal in mind (A5P11)</li> <li>act very patient (A5P15)</li> </ul> Evaluating <ul style="list-style-type: none"> <li>checking the accuracy, comparing and confirmation using another way, (A5P9), (A5P12), (A5P14), (A5P16)</li> <li>clarifying a mistake or a miscalculation (A5P19), (A5P20)</li> </ul>	[52], [58-59], [68-70]
	Working memory (A <sub>6</sub> )	Short-term memory (STM) <ul style="list-style-type: none"> <li>remember the details of the instructions, key math formulas, steps (A6P2), (A6P4), (A6P14), (A6P22)</li> <li>difficulty remembering after reading, re-read, re-doing (A6P6), (A6P15)</li> </ul> Long-term memory (LTM) <ul style="list-style-type: none"> <li>develop memory techniques (A6P1)</li> <li>recall knowledge, past lessons (A6P3), (A6P20), (A6P21)</li> <li>forget to do things that can be done in a sequence (A6P13)</li> </ul> Processing speed (PS) <ul style="list-style-type: none"> <li>getting problems with remembering sequences of numbers, difficulty understanding, forgetting what is going to solve (A6P5), (A6P8), (A6P12)</li> <li>difficult to change the strategy (A6P10)</li> <li>do not return to planned tasks if get interrupted (A6P18)</li> </ul> Verbal and visuo-spatial. <ul style="list-style-type: none"> <li>trouble remembering directions or instructions, difficulty recognizing diagram (A6P11) (A6P16), (A6P17), (A6P23)</li> <li>must re-read, looking many times (A6P7), (A6P9), (A6P19)</li> </ul>

### Acquiring Teachers' Opinions

In this second phase, a total of 37 teachers with more than ten years of experience were selected to identify the priority of six selected attributes to determine the position and influence on students' mathematics learning ability based on the FC model analysis procedure.

**Table 3.** The weight  $w_{ij}$  and overall membership function for attribute  $A_i$  related to linguistic values,  $V_j$

Attribute s	V <sub>1</sub>	V <sub>2</sub>	V <sub>3</sub>	V <sub>4</sub>	V <sub>5</sub>	V <sub>6</sub>	V <sub>7</sub>	Overall membership function
A <sub>1</sub>	0.027 0	0	0	0.135 1	0.297 3	0.1622	0.378 4	(0.612, 0.784, 0.912)
A <sub>2</sub>	0	0	0.027 0	0.162 2	0.189 2	0.2703	0.351 4	(0.628, 0.792, 0.9)
A <sub>3</sub>	0	0	0	0.162 2	0.243 2	0.1622	0.432 4	(0.668, 0.824, 0.924)
A <sub>4</sub>	0	0	0.027 0	0.081 1	0.108 1	0.3784	0.405 4	(0.7, 0.856, 0.936)
A <sub>5</sub>	0	0	0.054 1	0.108 1	0.189 2	0.2432	0.405 4	(0.66, 0.816, 0.908)
A <sub>6</sub>	0	0	0.054 1	0.081 1	0.216 2	0.3514	0.297 3	(0.612, 0.784, 0.9)

To get the position, analysis through similarity degree using equation (9) is carried out.



**Table 4.** Similarity degree  $S(A_i, V_j)$  of attributes

$A_i$	$V_1$	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_7$	$S_{max}$	$V(S_{max})$	Rank
$A_1$	0.5682	0.6024	0.6772	0.7833	0.9288	0.9036	0.8287	0.9288	$V_5$	4
$A_2$	0.5663	0.6002	0.6745	0.7796	0.9236	0.9085	0.8329	0.9236	$V_5$	6
$A_3$	0.5562	0.5889	0.6602	0.7606	0.8971	0.9357	0.8557	0.9357	$V_6$	2
$A_4$	0.5474	0.5792	0.6479	0.7444	0.8746	0.9615	0.8772	0.9615	$V_6$	1
$A_5$	0.5591	0.5922	0.6643	0.7661	0.9047	0.9276	0.8489	0.9276	$V_6$	5
$A_6$	0.5688	0.6031	0.6781	0.7845	0.9305	0.9021	0.8274	0.9305	$V_5$	3

From the results of this calculation, three items at position  $V_6$  are for items  $A_3$ ,  $A_4$  and  $A_5$ . While the remaining three items namely  $A_1$ ,  $A_2$  and  $A_6$  are at the  $V_5$  position. The highest similarity degree value is for item  $A_4$  which is 0.9615, while the lowest similarity degree value is 0.9236 for item  $A_2$ .

### Obtaining Expert Judgements

From the second phase, weighting is given to the more important attributes to determine the number of parameters required. After the parameters are determined, the third phase is the screening process, judgments and getting consensus from 10 experts administered through the FD method to assess the suitability of the parameter items contained in the six attributes.

#### Emotion

**Table 5.** Findings of expert consensus on emotion

Parameters' Item	Condition of triangular fuzzy numbers		Condition of defuzzification process	Position	Experts' consensus
	Threshold value, d	Percentage of experts group consensus, %	Fuzzy Score		
$A_1P_1$	0.233	80	0.827	10	Accepted
$A_1P_2$	0.373	40	0.713	20	Rejected
$A_1P_3$	0.234	90	0.827	9	Accepted
$A_1P_4$	0.358	30	0.753	17	Rejected
$A_1P_5$	0.073	100	0.927	2	Accepted
$A_1P_6$	0.064	100	0.937	1	Accepted
$A_1P_7$	0.248	80	0.810	11	Accepted
$A_1P_8$	0.231	90	0.853	7	Accepted
$A_1P_9$	0.132	100	0.893	5	Accepted
$A_1P_{10}$	0.321	80	0.800	12	Accepted
$A_1P_{11}$	0.428	20	0.717	19	Rejected
$A_1P_{12}$	0.264	90	0.773	14	Accepted
$A_1P_{13}$	0.357	80	0.770	15	Accepted
$A_1P_{14}$	0.259	80	0.787	13	Accepted
$A_1P_{15}$	0.385	30	0.760	16	Rejected
$A_1P_{16}$	0.132	100	0.893	6	Accepted
$A_1P_{17}$	0.322	30	0.710	21	Rejected
$A_1P_{18}$	0.216	90	0.853	8	Accepted
$A_1P_{19}$	0.345	30	0.743	18	Rejected
$A_1P_{20}$	0.132	100	0.893	4	Accepted
$A_1P_{21}$	0.132	100	0.903	3	Accepted

Motivation

**Table 6.** Findings of expert consensus on motivation

Parameters' Item	Condition of triangular fuzzy numbers		Condition of defuzzification process	Position	Experts' consensus
	Threshold value, d	Percentage of experts group consensus, %	Fuzzy Score		
<i>A<sub>2</sub>P<sub>1</sub></i>	0.309	80	0.837	11	Accepted
<i>A<sub>2</sub>P<sub>2</sub></i>	0.300	80	0.807	15	Accepted
<i>A<sub>2</sub>P<sub>3</sub></i>	0.275	80	0.780	18	Accepted
<i>A<sub>2</sub>P<sub>4</sub></i>	0.137	90	0.880	7	Accepted
<i>A<sub>2</sub>P<sub>5</sub></i>	0.172	90	0.873	8	Accepted
<i>A<sub>2</sub>P<sub>6</sub></i>	0.076	100	0.917	1	Accepted
<i>A<sub>2</sub>P<sub>7</sub></i>	0.103	90	0.910	2	Accepted
<i>A<sub>2</sub>P<sub>8</sub></i>	0.137	90	0.880	6	Accepted
<i>A<sub>2</sub>P<sub>9</sub></i>	0.312	80	0.760	19	Accepted
<i>A<sub>2</sub>P<sub>10</sub></i>	0.141	90	0.890	3	Accepted
<i>A<sub>2</sub>P<sub>11</sub></i>	0.166	90	0.863	9	Accepted
<i>A<sub>2</sub>P<sub>12</sub></i>	0.192	90	0.810	14	Accepted
<i>A<sub>2</sub>P<sub>13</sub></i>	0.137	90	0.880	5	Accepted
<i>A<sub>2</sub>P<sub>14</sub></i>	0.193	90	0.847	10	Accepted
<i>A<sub>2</sub>P<sub>15</sub></i>	0.308	60	0.700	20	Rejected
<i>A<sub>2</sub>P<sub>16</sub></i>	0.314	80	0.790	16	Accepted
<i>A<sub>2</sub>P<sub>17</sub></i>	0.234	90	0.827	12	Accepted
<i>A<sub>2</sub>P<sub>18</sub></i>	0.275	80	0.780	17	Accepted
<i>A<sub>2</sub>P<sub>19</sub></i>	0.250	90	0.810	13	Accepted
<i>A<sub>2</sub>P<sub>20</sub></i>	0.094	100	0.890	4	Accepted

Attention

**Table 7.** Findings of expert consensus on attention

Parameters' Item	Condition of triangular fuzzy numbers		Condition of defuzzification process	Position	Experts' consensus
	Threshold value, d	Percentage of experts group consensus, %	Fuzzy Score		
<i>A<sub>3</sub>P<sub>1</sub></i>	0.345	30	0.743	15	Rejected
<i>A<sub>3</sub>P<sub>2</sub></i>	0.240	90	0.800	10	Accepted
<i>A<sub>3</sub>P<sub>3</sub></i>	0.446	20	0.647	17	Rejected
<i>A<sub>3</sub>P<sub>4</sub></i>	0.203	90	0.820	8	Accepted
<i>A<sub>3</sub>P<sub>5</sub></i>	0.312	80	0.760	13	Accepted
<i>A<sub>3</sub>P<sub>6</sub></i>	0.386	50	0.677	16	Rejected
<i>A<sub>3</sub>P<sub>7</sub></i>	0.137	90	0.880	1	Accepted
<i>A<sub>3</sub>P<sub>8</sub></i>	0.159	90	0.853	5	Accepted
<i>A<sub>3</sub>P<sub>9</sub></i>	0.166	90	0.863	4	Accepted
<i>A<sub>3</sub>P<sub>10</sub></i>	0.225	80	0.817	9	Accepted
<i>A<sub>3</sub>P<sub>11</sub></i>	0.118	100	0.873	2	Accepted
<i>A<sub>3</sub>P<sub>12</sub></i>	0.185	90	0.837	7	Accepted
<i>A<sub>3</sub>P<sub>13</sub></i>	0.155	100	0.867	3	Accepted
<i>A<sub>3</sub>P<sub>14</sub></i>	0.262	90	0.847	6	Accepted
<i>A<sub>3</sub>P<sub>15</sub></i>	0.285	90	0.793	11	Accepted
<i>A<sub>3</sub>P<sub>16</sub></i>	0.354	20	0.753	14	Rejected
<i>A<sub>3</sub>P<sub>17</sub></i>	0.307	80	0.780	12	Accepted

Executive Function

**Table 8.** Findings of expert consensus on executive function

Parameters' Item	Condition of triangular fuzzy numbers		Condition of defuzzification process	Position	Experts' consensus
	Threshold value, d	Percentage of experts group consensus, %	Fuzzy Score		
<i>A<sub>4</sub>P<sub>1</sub></i>	0.305	40	0.747	31	Rejected
<i>A<sub>4</sub>P<sub>2</sub></i>	0.200	90	0.857	13	Accepted
<i>A<sub>4</sub>P<sub>3</sub></i>	0.259	90	0.820	19	Accepted
<i>A<sub>4</sub>P<sub>4</sub></i>	0.162	100	0.877	5	Accepted
<i>A<sub>4</sub>P<sub>5</sub></i>	0.372	40	0.707	32	Rejected
<i>A<sub>4</sub>P<sub>6</sub></i>	0.234	90	0.827	18	Accepted
<i>A<sub>4</sub>P<sub>7</sub></i>	0.225	80	0.817	22	Accepted
<i>A<sub>4</sub>P<sub>8</sub></i>	0.321	80	0.800	28	Accepted
<i>A<sub>4</sub>P<sub>9</sub></i>	0.234	90	0.827	17	Accepted
<i>A<sub>4</sub>P<sub>10</sub></i>	0.304	80	0.817	21	Accepted
<i>A<sub>4</sub>P<sub>11</sub></i>	0.145	100	0.857	14	Accepted
<i>A<sub>4</sub>P<sub>12</sub></i>	0.128	100	0.883	3	Accepted
<i>A<sub>4</sub>P<sub>13</sub></i>	0.310	80	0.763	30	Accepted
<i>A<sub>4</sub>P<sub>14</sub></i>	0.319	70	0.770	29	Rejected
<i>A<sub>4</sub>P<sub>15</sub></i>	0.155	100	0.867	11	Accepted
<i>A<sub>4</sub>P<sub>16</sub></i>	0.275	80	0.807	27	Accepted
<i>A<sub>4</sub>P<sub>17</sub></i>	0.258	80	0.820	20	Accepted
<i>A<sub>4</sub>P<sub>18</sub></i>	0.128	100	0.883	3	Accepted
<i>A<sub>4</sub>P<sub>19</sub></i>	0.103	90	0.910	1	Accepted
<i>A<sub>4</sub>P<sub>20</sub></i>	0.132	100	0.893	2	Accepted
<i>A<sub>4</sub>P<sub>21</sub></i>	0.193	90	0.847	15	Accepted
<i>A<sub>4</sub>P<sub>22</sub></i>	0.274	90	0.810	24	Accepted
<i>A<sub>4</sub>P<sub>23</sub></i>	0.211	90	0.843	16	Accepted
<i>A<sub>4</sub>P<sub>24</sub></i>	0.155	100	0.867	9	Accepted
<i>A<sub>4</sub>P<sub>25</sub></i>	0.162	100	0.877	5	Accepted
<i>A<sub>4</sub>P<sub>26</sub></i>	0.155	100	0.867	8	Accepted
<i>A<sub>4</sub>P<sub>27</sub></i>	0.274	90	0.810	23	Accepted
<i>A<sub>4</sub>P<sub>28</sub></i>	0.248	80	0.810	25	Accepted
<i>A<sub>4</sub>P<sub>29</sub></i>	0.166	90	0.863	12	Accepted
<i>A<sub>4</sub>P<sub>30</sub></i>	0.162	100	0.877	5	Accepted
<i>A<sub>4</sub>P<sub>31</sub></i>	0.248	80	0.810	26	Accepted
<i>A<sub>4</sub>P<sub>32</sub></i>	0.155	100	0.867	10	Accepted

Metacognition

**Table 9.** Findings of expert consensus on metacognition

Parameters' Item	Condition of triangular fuzzy numbers		Condition of defuzzification process	Position	Experts' consensus
	Threshold value, d	Percentage of experts group consensus, %	Fuzzy Score		
<i>A<sub>5</sub>P<sub>1</sub></i>	0.172	90	0.873	3	Accepted
<i>A<sub>5</sub>P<sub>2</sub></i>	0.172	90	0.873	2	Accepted
<i>A<sub>5</sub>P<sub>3</sub></i>	0.274	90	0.810	15	Accepted
<i>A<sub>5</sub>P<sub>4</sub></i>	0.155	100	0.867	4	Accepted
<i>A<sub>5</sub>P<sub>5</sub></i>	0.155	100	0.867	5	Accepted
<i>A<sub>5</sub>P<sub>6</sub></i>	0.320	80	0.773	17	Accepted
<i>A<sub>5</sub>P<sub>7</sub></i>	0.281	90	0.820	12	Accepted
<i>A<sub>5</sub>P<sub>8</sub></i>	0.310	80	0.763	18	Accepted
<i>A<sub>5</sub>P<sub>9</sub></i>	0.234	90	0.827	9	Accepted
<i>A<sub>5</sub>P<sub>10</sub></i>	0.305	40	0.747	19	Rejected
<i>A<sub>5</sub>P<sub>11</sub></i>	0.259	90	0.837	7	Accepted
<i>A<sub>5</sub>P<sub>12</sub></i>	0.321	80	0.800	16	Accepted
<i>A<sub>5</sub>P<sub>13</sub></i>	0.281	90	0.820	10	Accepted
<i>A<sub>5</sub>P<sub>14</sub></i>	0.281	90	0.820	11	Accepted
<i>A<sub>5</sub>P<sub>15</sub></i>	0.286	20	0.727	20	Rejected
<i>A<sub>5</sub>P<sub>16</sub></i>	0.281	90	0.820	13	Accepted
<i>A<sub>5</sub>P<sub>17</sub></i>	0.193	90	0.847	6	Accepted
<i>A<sub>5</sub>P<sub>18</sub></i>	0.281	90	0.820	14	Accepted
<i>A<sub>5</sub>P<sub>19</sub></i>	0.162	100	0.877	1	Accepted
<i>A<sub>5</sub>P<sub>20</sub></i>	0.286	90	0.830	8	Accepted

Working Memory

**Table 10.** Findings of expert consensus on working memory

Parameters' Item	Condition of triangular fuzzy numbers		Condition of defuzzification process	Position	Experts' consensus
	Threshold value, d	Percentage of experts group consensus, %	Fuzzy Score		
<i>A<sub>6</sub>P<sub>1</sub></i>	0.267	80	0.797	16	Accepted
<i>A<sub>6</sub>P<sub>2</sub></i>	0.166	90	0.863	2	Accepted
<i>A<sub>6</sub>P<sub>3</sub></i>	0.162	100	0.877	1	Accepted
<i>A<sub>6</sub>P<sub>4</sub></i>	0.188	100	0.860	3	Accepted
<i>A<sub>6</sub>P<sub>5</sub></i>	0.281	90	0.820	10	Accepted
<i>A<sub>6</sub>P<sub>6</sub></i>	0.259	90	0.837	4	Accepted
<i>A<sub>6</sub>P<sub>7</sub></i>	0.281	80	0.817	11	Accepted
<i>A<sub>6</sub>P<sub>8</sub></i>	0.409	80	0.773	19	Accepted
<i>A<sub>6</sub>P<sub>9</sub></i>	0.367	80	0.763	20	Accepted
<i>A<sub>6</sub>P<sub>10</sub></i>	0.397	30	0.753	21	Rejected
<i>A<sub>6</sub>P<sub>11</sub></i>	0.281	90	0.820	9	Accepted
<i>A<sub>6</sub>P<sub>12</sub></i>	0.259	90	0.837	5	Accepted
<i>A<sub>6</sub>P<sub>13</sub></i>	0.254	90	0.827	7	Accepted
<i>A<sub>6</sub>P<sub>14</sub></i>	0.274	90	0.810	12	Accepted
<i>A<sub>6</sub>P<sub>15</sub></i>	0.254	90	0.827	6	Accepted
<i>A<sub>6</sub>P<sub>16</sub></i>	0.281	90	0.820	8	Accepted
<i>A<sub>6</sub>P<sub>17</sub></i>	0.266	90	0.800	15	Accepted

Parameters' Item	Condition of triangular fuzzy numbers		Condition of defuzzification process	Position	Experts' consensus
	Threshold value, d	Percentage of experts group consensus, %	Fuzzy Score		
$A_6P_{18}$	0.397	30	0.753	22	Rejected
$A_6P_{19}$	0.437	10	0.727	23	Rejected
$A_6P_{20}$	0.329	80	0.783	18	Accepted
$A_6P_{21}$	0.274	90	0.810	13	Accepted
$A_6P_{22}$	0.314	80	0.790	17	Accepted
$A_6P_{23}$	0.274	90	0.810	14	Accepted

## Results and Discussion

In this study, the integration of FC and FD methods has been used as a mathematical analysis procedure, firstly, performing a literature review to obtain attributes that affect students' mathematics learning ability and can be used as indicators to measure that ability. The results were obtained with six attributes that were shortlisted, namely emotion ( $A_1$ ), motivation ( $A_2$ ), attention ( $A_3$ ), executive function ( $A_4$ ), working memory ( $A_5$ ), and metacognition ( $A_6$ ). The results of the second phase, which is from the similarity degree analysis Table 4, show that attributes in ascending ranking are  $A_2$ - $A_5$ - $A_1$ - $A_6$ - $A_3$ - $A_4$ , and according to position and priority are executive function, attention, working memory, emotion, metacognition, and motivation. These results show that executive function plays a major role in students' performance during mathematics learning, and this attribute becomes the main condition in determining students' ability. So, based on this justification, the parameters to measure executive function will be detailed, and the number must be reasonable and appropriate. Next, five more attributes are not separated and are also placed as elements that support the ability to learn mathematics. They should not be ignored because these attributes have a very complex relationship, especially in assessing students' intellectual abilities more accurately [7, 30, 32]. This result supports the theory of Schoenfeld [21] and the view of Mayer and Wittrock [23], which states that learning is a mixture of systems and cognitive processes with external attributes such as emotions and motivation that support student ability.

In the third phase, evaluation and decision-making made by experts determine the suitability of parameters based on the FD method analysis. The analysis results of Table 5 to Table 10 show the percentage of expert agreement and the value of threshold, d obtained. Based on those results, some parameter items were rejected. For emotion ( $A_1$ ) attribute, parameter items  $A_1P_2$ ,  $A_1P_4$ ,  $A_1P_{11}$ ,  $A_1P_{15}$ ,  $A_1P_{17}$  and  $A_1P_{19}$  have been rejected. Only one item was rejected from attribute motivation ( $A_2$ ),  $A_2P_{15}$ . Next for attention ( $A_3$ ), four items were rejected:  $A_3P_1$ ,  $A_3P_3$ ,  $A_3P_6$ , and  $A_3P_{16}$ . Three parameter items from the attribute executive function ( $A_4$ ) are rejected:  $A_4P_1$ ,  $A_4P_5$  and  $A_4P_{14}$ . Whereas two and three items from metacognition ( $A_5$ ) and working memory ( $A_6$ ), are items  $A_5P_{10}$ ,  $A_5P_{15}$ ,  $A_6P_{10}$ ,  $A_6P_{18}$ , and  $A_6P_{19}$ . Based on the results obtained, most of the rejected parameter items are because the items are negative, and there is an incorrect use of words in the item statement. This is accepted as a reasonable decision because every parameter contained in the attributes to measure positive things such as the level of students' mathematics learning ability, must be positive. Then, the results tend to be normal and positive, such as the order of ability levels, for example, excellent, good, medium, and poor [2].

In addition, the results through the FD method can also determine the ranking of parameter items of each attribute. This allows the items to be arranged in a model that is ready to be produced according to priority more systematically. In conclusion, the final results of the students' mathematics learning ability assessment model include 114 parameter items based on six main attributes with executive function having 29 items, attention 13 items, working memory 20 items, emotion 15 items, metacognition 18 items, and motivation 19 items, respectively.

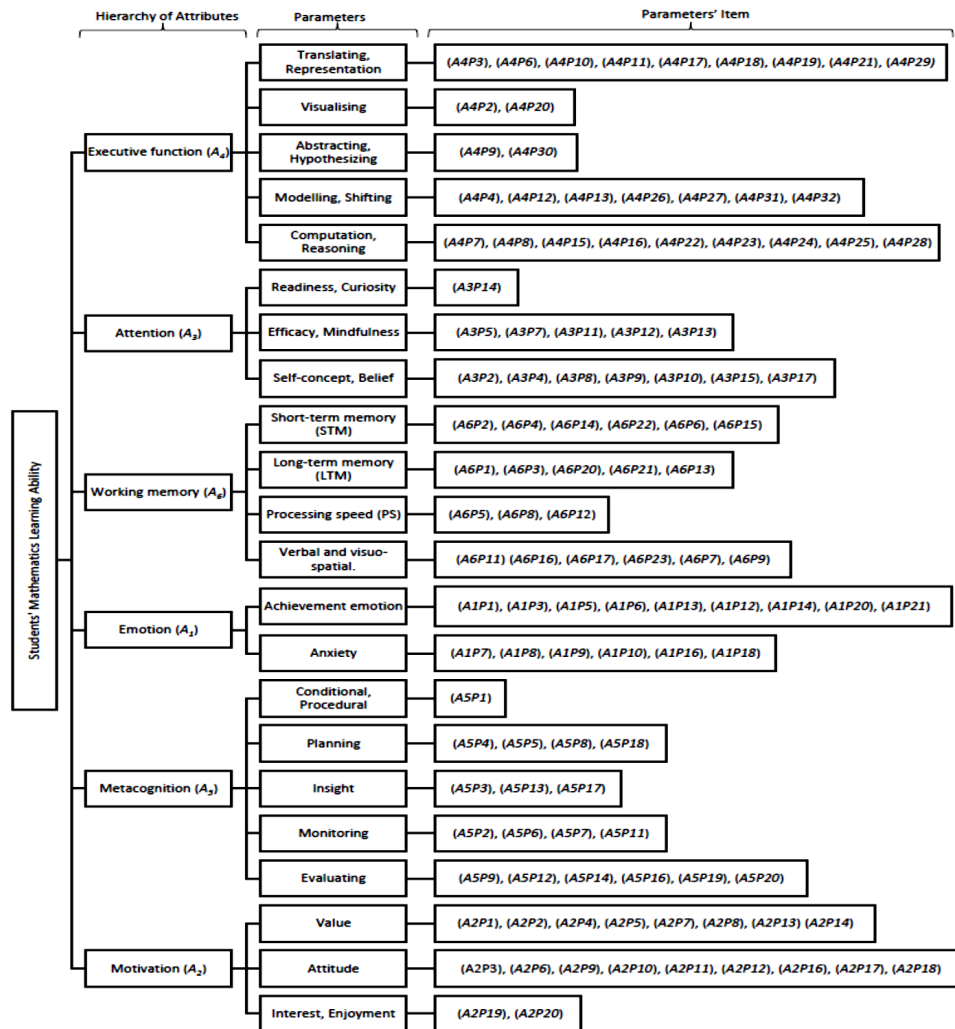


Figure 2. The final model of students' mathematics learning ability assessment

## Conclusions

This study has successfully demonstrated the mathematics analysis procedure from the integration of the fuzzy evaluation method, triangular fuzzy conjoint, and fuzzy Delphi for the process of identification and analysis of attributes and parameters in developing an assessment model. An assessment model to determine the level of students' mathematics learning ability has been developed in a more structured and systematic way.

However, there are some constraints in applying this procedure. Among them is the initial determination of attributes and parameters, which only depend on the findings during the literature review. So, it depends on the search method determined by the researcher and will be directly related to the designated database source. A possible method recommended to future researchers is to use thematic or content analysis as an additional condition. In addition, the fuzzy evaluation procedure requires an effective number of experts. Although this study was conducted with several experts in the appropriate category, it should also be emphasized to obtain more significant results. It is not a limitation that can lead to a major influence on the development of the model.

In addition, the evaluation and applicability of this evaluation model have not been fully described and require further study. A possible approach that can be highlighted as a model evaluation procedure, which is also in the fuzzy evaluation method group, is ANFIS or Artificial Neural Network (ANN). Indirectly, this study can be used as a starting point for the development of fuzzy mathematics analysis



and fuzzy evaluation, and it can also be improved to type-2 fuzzy procedures such as fuzzy interval neutrosophic set.

## Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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