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Some fundamental properties for regular element of electroencephalography signals semigroup during epileptic seizure

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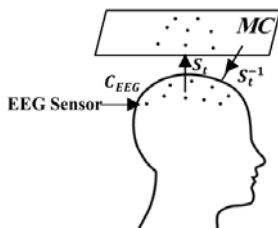*Corresponding Author: tahir@ibnusina.utm.my

GRAPHICAL ABSTRACT

ABSTRACT

Electroencephalography (EEG) is a record of electrical activity along the scalp. It is measures voltage fluctuations resulting from ionic current flows within the neurons of the brain. EEG is most often used to diagnose epilepsy, which causes understandable abnormalities in EEG readings. The mathematical analysis of EEG signals assists medical specialists by providing a clarification of the brain activity being observed, so increasing the information about the structure and function of the human brain. EEG signals during epileptic seizure can be viewed as a semigroup of square matrices under matrix multiplication, and every element in that semigroup is shown to be regular. In this paper, we will present some main properties of regular element of EEG signals during epileptic seizure.

Keywords: EEG, semigroup, regular element

1. INTRODUCTION

Epilepsy is a group of long term neurological disorders categorized by seizures. These seizures are episodes that can differ from brief and nearly undetectable to long periods of strong trembling. In epilepsy, seizures tend to recur, and have no direct underlying cause while seizures that occur due to an exact cause are not deemed to represent epilepsy [1]. It is categorized into two major groups, partial and generalized. Partial seizures are those in which the clinical or electroencephalographic evidence recommends that the attacks have a localized onset in the brain [2]. This class of seizure affects only a part of the cerebral hemisphere at seizure onset and produces symptoms in corresponding parts of the body or disturbances in some related mental functions. Perversely, generalized seizures are said to occur if the evidence proposes that the attacks were well spread [3].

An instrument that measures electrical potentials (brain waves) on the scalp and produces a record of the electrical activity of the brain called Electroencephalography (EEG). It is a key tool in the diagnosis and management of epilepsy and further seizure disorders, as well as in the diagnosis of brain damage related to trauma and diseases such as strokes, tumors, encephalitis, drug, and alcohol intoxication. Furthermore, EEG is also useful in control brain wave activity and in the brain death determination [4].

The mathematical analysis of EEG signals may assist the medical professionals by providing an illustration of the brain activity being observed, therefore increasing the understanding of the brain function of human. Up to the moment, the Fast Fourier Transform (FFT) is the most used quantitative technique for EEG signals analysis. However, it has some disadvantages that limit its applicability. Hence, further methods for extracting "hidden" information from the EEG signals are required.

2. FLATTENING THE EEG SIGNALS

A new technique to map high dimensional signal, namely EEG into low dimensional space had been developed by Zakaria and Ahmad in 2007 [5]. The procedure of this technique contained three important sections. The first one deals with flattening the EEG data which essentially entails transformation of three-dimensional spaces into two-dimensional spaces. This procedure contains the position of sensors on the patient's head with EEG signal. The second section comprises processing EEG signals via Fuzzy c-Means clustering. The last section contains finding the optimal number of clusters using cluster validity analysis.

The EEG coordinate system (Figure 1(a)) is defined by Zakaria and Ahmad in 2007 as follows:

$$C_{EEG} = \left\{ \left((x, y, z), e_p \right) : x, y, z, e_p \in \mathbb{R} \text{ and } \right.$$

$$\left. x^2 + y^2 + z^2 = r^2 \right\}$$ where r is the radius of a patient head [5]. In addition, a function is defined from C_{EEG} to MC plan as the following: $S_t : C_{EEG} \rightarrow MC$ (Figure 1 (b)) such that $S_t\left((x, y, z), e_p \right) = \left(\frac{rx + iry}{r + z}, e_p \right) = \left(\frac{rx}{r+z}, \frac{rx}{r+z} \right)_{e_p(x,y,z)}$.

Both, C_{EEG} and MC were designed and verified as 2-manifolds [6]. In this situation, it must be well-known that S_t is an injective mapping of a conformal structure.

Therefore, S_t can preserve information in a particular angle and orientation of the surface through the recorded EEG signals. In addition, they implemented this method followed by clustering on real time EEG data obtained from patients who suffer from seizure.
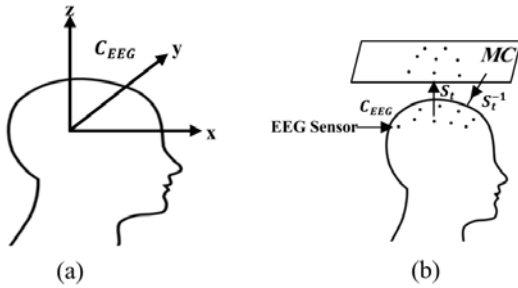

Figure 1. (a) EEG coordinate system (b) EEG projection

The signals (Figure 2) were digitized at 256 samples per second using Nicolet One EEG software. The average potential difference was calculated from the 256 samples of raw data at every second. Similarly to the location of the electrodes, the EEG signal was preserved through this new method. Every single second of the particular average potential difference was stored in a file which contains the position of the electrode on MC plane (Figure 3).
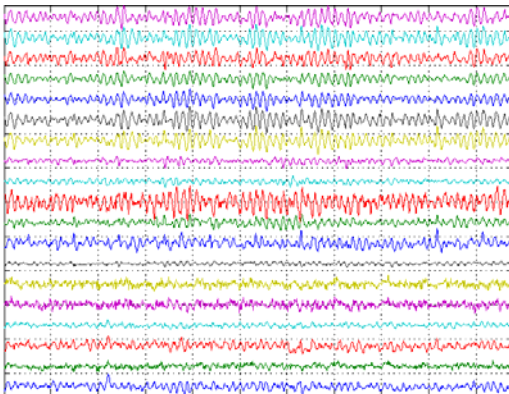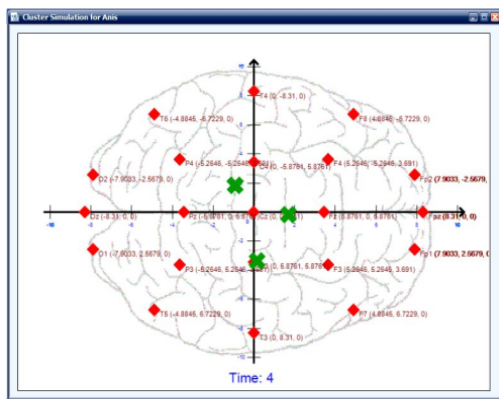

Figure 2. Sample of EEG signal

Figure 3. Flat EEG signal

3. SEMIGROUP OF EEG SIGNALS DURING EPILEPTIC SEIZURE

Binjadhnan and Ahmad (2010) [7] shown EEG signals during epileptic seizure can be composed into a set of $n \times n$ square matrices

$$MC_n(\mathbb{R}) = \left\{ \left[\beta_{ij}(z)_t \right]_{n \times n} : i,j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}$$

where, $\beta_{ij}(z)_t$ is a potential difference reading of EEG signals from a particular ij sensor at time t. In addition, they transformed the set $MC_n(\mathbb{R})$ to the set of upper triangular matrices denoted by $MC_n''(\mathbb{R})$ using *QR-real Schur triangularization* (Figure 4) as following:

$$MC_n''(\mathbb{R}) = \left\{ \left[\beta_{ij}(z)_t \right]_{n \times n} : \beta_{ij}(z)_t = 0, \forall 1 \le j < i \le n, \ i,j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}.$$
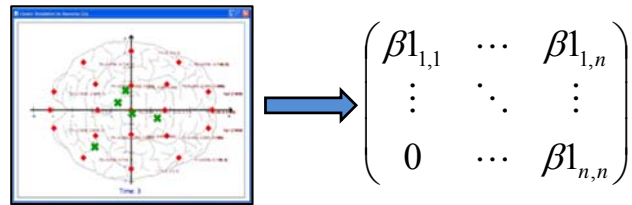

Figure 4. MC plan transformed into an upper triangular matrix

Furthermore, the set $MC_n''(\mathbb{R})$ satisfies all the axioms of a semigroup under matrix multiplication. Particularly, $MC_n''(\mathbb{R})$ are closed and associative under matrix multiplication.

4. MATERIALS AND METHODS

In this section, some existing definitions and results that will be used throughout this paper are presented.

Definition1 [8]: A subset $\alpha \subseteq \{1,2,\dots\dots,n\}$ with the property that for any $i \le j \le k \in \{1,2,\dots\dots,n\}$ with $i,k \in \alpha$ with $j \in \alpha$ is called $n-interval$. Let A be $n \times n$ matrix over a field F and α an $n-interval$ the restriction $A|_\alpha$ is the matrix $\left(A_{ij} \right)_{i,j \in \alpha}$ with rows and columns indexed by α. Two matrices A and B agree on α if $A_{ij} = B_{ij}$ for all $i,j \in \alpha$. We say that A and B are scalar multiples on α if there exists a non-zero field element $\lambda \in F$ such that $A_{ij} = \lambda B_{ij}$ for all $i,j \in \alpha$, such that A and λB agree on α.

Definition2 [9]: Let $S(n,F)$ be a semigroup of all $n \times n$ upper triangular matrices with entries drawn from field F, with usual operation (matrix multiplication). Let $A \in S(n,F)$, the diagonal shape of A is the set $shape(A) = \{i \in Z | 1 \le i \le n \ , \ a_{ii} \ne 0\}$. Note that two matrices have the same diagonal shape if they have zeros in exactly the same positions on the main diagonal.

Theorem1 [10]: If A, B are upper triangular matrices. Then $shape(AB) = shape(A) \cap shape(B)$.

Let A_t, B_t two matrices in $MC_n^{"}(\mathbb{R})$ and we define a relation Ω on a semigroups $MC_n^{"}(\mathbb{R})$ by $A_t \Omega B_t$ if and only if $A_t = \lambda B_t$ for some non-zero field element λ. A relation Ω on the $MC_n^{"}(\mathbb{R})$ is called left compatible if $\forall A_t, B_t, C_t \in MC_n^{"}(\mathbb{R})$, $A_t \Omega B_t \Rightarrow C_t A_t \Omega C_t B_t$ and right compatible if $\forall A_t, B_t, C_t \in MC_n^{"}(\mathbb{R})$, $A_t \Omega B_t \Rightarrow A_t C_t \Omega B_t C_t$ and it is called compatible if $\forall A_t, B_t, A_t', B_t' \in MC_n^{"}(\mathbb{R})$, $A_t \Omega A_t'$ and $B_t \Omega B_t' \Rightarrow A_t B_t \Omega A_t' B_t'$. A left (right) compatible equivalence relation is called a left (right) congruence. A compatible equivalence relation is called congruence.

Definition3 [11]: A row operation on an upper triangular matrix is said to be invertible if we can add a multiple of one row to a row above or scaling a row by non-zero field element.
Remark 1: Column operations are defined analogously.

Let $A_t, B_t \in MC_n^{"}(\mathbb{R})$. It is easy to see that $B_t A_t$ can be obtained from A_t by a certain sequence of row operation determined by the matrix A_t. Conversely, every row operation can be represented as left-multiplication by a certain triangular matrix. There is an analogous relationship between right-multiplication and column operations.

A direct consequence of these explanations is the following characterization of Green's relations $\mathcal{L}, \mathfrak{R}$ and $\mathcal{J}$ on the semigroups $MC_n^{"}(\mathbb{R})$.

In 1951 James Alexander Green introduced five equivalence relations that characterize the elements of a semigroup in terms of the principal ideals. These relations are useful for understanding the nature of divisibility in semigroup [The prime decomposition theorem Krohn-Rhodes 1965] . Moreover, Green relations are particularly significant in the study of regular semigroup.

Definition4 [13]: An element s of a semigroup S is called a regular element of S if there is an element t of S such that $s = sts$ and S is said to be a regular semigroup if every element of S is regular. Note that the regular elements of semigroup of upper triangular matrices are characterized as those matrices whose rank is equal to the number of their non-zero diagonal entries.

If A_t is an element of a semigroup $MC_n^{"}(\mathbb{R})$, the smallest left ideal contain A_t is $MC_n^{"}(\mathbb{R})A_t \cup \{A_t\}$, which may conveniently write as $MC_n^{"\backslash}(\mathbb{R})A_t$, and called it the principal left ideal generated by A_t. An equivalence relation $\mathcal{L}$ on $MC_n^{"}(\mathbb{R})$ is defined by the rule that $A_t \mathcal{L} B_t$ if and only if A_t and B_t generate the same principal left ideal, in other words, $MC_n^{"\backslash}(\mathbb{R})A_t = MC_n^{"\backslash}(\mathbb{R})B_t$. Similarly, define $\mathfrak{R}$ by the rule that $A_t \mathfrak{R} B_t$ if and only if A_t and B_t generate the same principle right ideal, in other words, $A_t MC_n^{"\backslash}(\mathbb{R}) = B_t MC_n^{"\backslash}(\mathbb{R})$. Furthermore, define $\mathcal{J}$ by the rule that

$A_t \mathcal{J} B_t$ if and only $MC_n^{"\backslash}(\mathbb{R})A_t MC_n^{"\backslash}(\mathbb{R}) = MC_n^{"\backslash}(\mathbb{R})B_t MC_n^{"\backslash}(\mathbb{R})$.

Definition5 [14]: The elementary EEG signals is a square matrix of EEG signals reading at time t in terms of one of the following types:
 (i) Diagonal matrix (special case sub-identity matrix).
 (ii) Unipotent matrix.
(iii) Permutation.

Proposition1 [15]: Let $S(n, F)$ be a semigroup of all $n \times n$ upper triangular matrices with entries drawn from field F. Let $A_1, A_2 \in S(n, F)$ then:
 (i) A_1, A_2 are $\mathcal{L} - related$ precisely if each can be acquired from the other by row operation.
 (ii) A_1, A_2 are $\mathfrak{R} - related$ precisely if each can be acquired from the other by column operation.
(iii) A_1, A_2 are $\mathcal{J} - related$ precisely if each can be acquired from the other by row and column operation.

Theorem2 [16]: Assume that A_t is an upper triangular matrix of EEG signals during epileptic seizure $\left(A_t \in MC_n^{"}(\mathbb{R})\right)$. Then the following are equivalent:
 (i) A_t is regular;
 (ii) Every row (column) in A_t is a linear combination of rows (columns) in A_t with non-zero diagonal entries;
(iii) A_t is $\mathcal{J} - related$ to sub-identity.

5. CHARACTERIZATION OF REGULAR ELEMENTS OF SEMIGROUP OF EEG SIGNALS DURING EPILEPTIC SEIZURE

The following is a corollary of Theorem 1 which characterize the $\mathcal{J} - related$ of regular elements of $MC_n^{"}(\mathbb{R})$ in terms of diagonal shape of EEG signals matrix.

Corollary 1: Let A_t, B_t are upper triangular matrices of EEG signals during epileptic seizure $\left(A_t, B_t \in MC_n^{"}(\mathbb{R})\right)$ with A_t, B_t regulars. Both A_t, B_t are $\mathcal{J} - related$ if and only if they have the same diagonal shape.

Proof.

Let $A_t, B_t \in MC_n^{"}(\mathbb{R})$ and A_t, B_t are $-related$.
Since the diagonal shape is preserved by invertible row and column operation, and if A_t, B_t have zeros in exactly the same positions on the main diagonal, then $\mathcal{J} - related$ for both EEG signals matrices (A_t and B_t) needed to have the same diagonal shape (by Proposition 1). Conversely, let A_t and B_t are regular elements and have the same diagonal shape, then A_t and B_t are $\mathcal{J} - related$ to the unique sub-identity EEG signals matrices with that diagonal shape (by theorem 2). Thus, they are $\mathcal{J} - related$ to each other. □

Theorem 3: Assume that A_t, B_t are upper triangular matrices of EEG signals during epileptic seizure $\left(A_t, B_t \in MC_n^{''}(\mathbb{R})\right)$ with A_t regular. Then the following are equivalent:

(i) $shape(A_t) \subseteq shape(B_t)$;
(ii) $A_t B_t \, \Re \, A_t$;
(iii) $A_t B_t \, \mathcal{J} \, A_t$

Proof.

(i)$\Rightarrow$(ii)

Let $shape(A_t) \subseteq shape(B_t)$ where $A_t, B_t \in MC_n^{''}(\mathbb{R})$. By applying column operation (Definition 3) for A_t we have $A_t B_t$ can be acquired from A_t. Therefore, every column of A_t is a linear combination of columns with nonzero diagonal entries (by theorem 2). But since $shape(A_t) \subseteq shape(B_t)$, then every column of A_t will be a combination of columns of $A_t B_t$ with nonzero diagonal entries. Thus, $A_t B_t C_t = A_t$ for some C_t, which mean $A_t B_t \, \Re \, A_t$ (definition of $\Re - relation$).

(ii)$\Rightarrow$(iii)

If A_t and B_t are regular elements and $\mathcal{J} - related$, then $shape(A_t) = shape(B_t)$ which implies to $A_t B_t \, \mathcal{J} \, A_t$ (by Corollary 1).

(iii)$\Rightarrow$(i)

Let $A_t B_t \, \mathcal{J} \, A_t$, then
$shape(A_t) = shape(A_t B_t)$ (by Proposition 1)
$= shape(A_t) \cap shape(B_t)$ (by theorem 1)
Hence, $shape(A_t) \subseteq shape(B_t)$. □

6. CONCLUSION

The paper presented the use of Green's relations to characterize the regularity of semigroup elements of EEG signals during epileptic seizure.

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