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RESEARCH ARTICLE

Interval Neutrosophic Cubic Bézier Curve Approximation Model for Complex Data

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Abstract Complex data is defined as data that has the qualities of huge data, a lack of data information, and uncertainty. This paper discussed constructing the interval neutrosophic cubic Bézier curve (INCBC) approximation model for complex data. To construct the interval neutrosophic data point (INDP) based on the definition of interval neutrosophic set (INS), interval neutrosophic relation (INR) and interval neutrosophic point (INP). Next is the introduction of an interval neutrosophic control point (INCP) that blends with the theory of interval neutrosophic set and the Bernstein basis function. Later, the interval neutrosophic cubic Bézier curve (INCBC) model is visualizing with a four-by-four control points relation approximates the curves for truth, false, and indeterminacy membership. At the end of this paper will demonstrate the algorithm for the creation of the interval neutrosophic cubic Bézier curve (INCBC). The scientific value of this work is the acceptance of complex uncertainty data. As a result, due to the fact it combines fuzzy geometric modelling, this approach has the potential to make a significant contribution to complex uncertainty modelling by using this spline which is Bézier curve model.

Keywords: Interval Neutrosophic Set, Control Point, Bézier Curve, Approximation Method, Complex Data.

Introduction

Data is an important element in processing information obtained either in the form of linguistics or numbers. In this paper, complex data will be used to model it by using interval neutrosophic theory and the Bernstein basis function. Ayasdi [1] states that complex data can be referred to as big data sets or big data. He also stated that a relatively small data set is also a complex data set and difficult to deal with. Data that is difficult to interpolate is also categorized as complex by Ayasdi [1]. According to the ETL-tools [2], complex data refers to big data with less uncertain information. As a result of the references obtained, the complex data employed in this study may be classified as huge data, difficult to work with it, and less uncertain information.

The fuzzy set (FS) model is a theoretical framework for dealing with imprecision and ambiguity in data. Lotfi Zadeh developed it in the 1960s to address the inherent ambiguity and vagueness of natural language and human intellect [3]. As a result, FS only examines true and false membership data and disregards inconsistent data. Krassimir Atanassov [4] developed the intuitionistic fuzzy set (IFS) theory, a generalization of FS that includes true, false, and uncertain information, in 1986. It's great for dealing with ambiguity. Due to FS theory only considering entire membership data, the IFS concept is an alternative method for establishing FS when the amount of information recorded is insufficient to classify and process. Nevertheless, approaching an advanced problem with intuitive and fuzzy components is difficult, and it is rarely handled in the context of spline modelling. [5-13] contains research on fuzzy and intuitionistic fuzzy set theory, as well as spline modelling.

Florentin Smarandache [14] developed the neutrosophic technique as a mathematical application of the concept of neutrality that works with uncertain data. Membership degrees, non-membership, and indeterminacy define the neutrosophic set (NS) concept. NS in this context, refers to the resolution and representation of problems spanning multiple domains. Since in NS theory, true, false, and indeterminate

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membership degrees are independent, an element can have any value at the same time. Wang et al. [15] introduced the set-theoretic operators on an instance of a NS called interval neutrosophic set (INS). They also emphasized that an INS is an instance of NS which can be used in real scientific and engineering applications. The main motivation of this study is INS allows for the modelling of more complex types of uncertainty and indeterminacy, such as when a statement can be true and false at the same time. Besides that, INS be the theory of this study instead of NS and other higher-order fuzzy sets (HOFS) due to the existence of footprints of uncertainty (FOU) can consider the uncertainty of the complex data. As mentioned above, complex data refers to large data with less and uncertain information. Therefore, the concept of INS can consider the uncertainty in complex data as an indeterminacy membership degree and take into account the FOU of the indeterminacy degree to get a precise result.

Tas and Topal [16,17] generated the Bézier curve and surface in general without focusing on the specifics of blending the Bernstein function with NS theory. Meanwhile, Rosli and Zulkifly [18] go into detail about the use of B-spline curve interpolation, and they also show a neutrosophic bicubic B-spline surface interpolation model for uncertainty data [19]. In 2019, Zulkifly and teams [30] introduced an approximation of the intuitionistic fuzzy Bézier curve model. After that, they expanded the study to discuss an intuitionistic fuzzy Bézier curve approximation model for uncertainty data [27]. Recently, Rosli and Zulkifly emphasized Bézier approximation models and introduced a 3-dimensional quartic Bézier curve approximation model by using neutrosophic approach [28], and a neutrosophic bicubic Bezier surface approximation model for uncertainty data [29]. However, there aren't studies that emphasize and discuss interval neutrosophic geometric modeling yet, especially for Bézier approximation models. Therefore, this study will demonstrate interval neutrosophic control points approximating the interval neutrosophic Bézier curve using a numerical example to visualize the interval neutrosophic Bézier curve for the cubic version.

The development of a geometric model for the Bézier curve that can deal with complex data is the focus of this study; specifically, the interval neutrosophic cubic Bézier curve (INCBC) model by using the approximation approach will be the model's primary focus. Before creating the INCBC, it is necessary to determine the interval neutrosophic control point (INCP) first that replaces complex data and will control the behaviour of INCBC by approximating the curve using the INS theories and the properties it possesses. To construct INCBC models, which are subsequently visualized using an approximation approach, these INCP are used in combination with the Bernstein basis function. The following describes how the format of this document should be used. The first section of this paper presented some background information about the subject. In Section 2, some definitions such as interval neutrosophic points (INP) and interval neutrosophic data points (INDP) are introduced to define the INCP, especially for the cubic case. In the third section, the method and mathematical representation that may INCP be used to approximate the INCBC using INCP and the properties of INCBC are discussed. In the fourth section, there is both a numerical example and a graphic representation of INCBC for truth, false, indeterminacy, and the combination of the degrees. At the end of this section, the algorithm for creating the INCBC will be demonstrated. This paper will be finished with the fifth section as the conclusion part.

Preliminaries

The goal of this section is to introduce the INDP and INCP that refer as the complex data, and the INDP will treat as INCP to represent the INCBC. As a result, before introducing it, the definition of INS, interval neutrosophic relation (INR) and interval neutrosophic point (INP) must be introduced first for the use of INDP. The fundamental concept of INS and INR was introduced by Wang et al. [15] in 2005 as follows. While the idea for the definition of INP was inspired by Tas and Topal [16,17] for the neutrosophic points. Besides that, the idea of INDP comes from the study by Zakaria and teams [20-22] that discussed the type-2 fuzzy data points in the representative of geometric modelling.

Definition 1. [15]
Let X be the main conversation that elements in X denoted as x . An **interval neutrosophic set (INS)** A is expressed by truth membership function T_A, indeterminacy membership function I_A, and false membership function F_A. Where $x \in X, T_A(x), I_A(x), F_A(x) \subseteq [0,1]$. Figure 1 depicts an interval neutrosophic set (INS) in A.

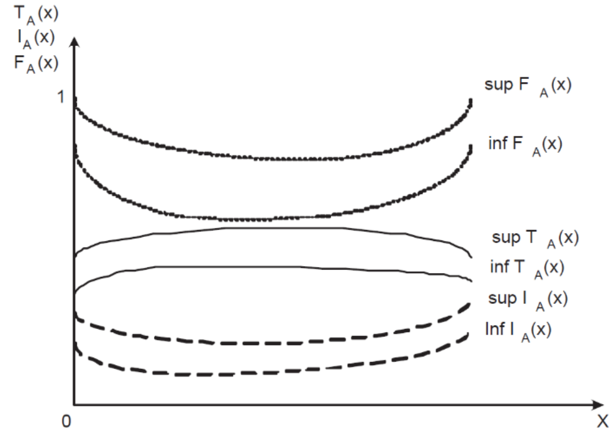

Figure 1. An interval neutrosophic set (INS) in A [15]

- When X is continuous, an INS A can be expressed as

$$A=\int_X \langle T(x), I(x), F(x)\rangle / x, x \in X \tag{1}$$

- When X is discrete, an INS A can be expressed as

$$A=\sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i)\rangle / x_i, x_i \in X \tag{2}$$

Definition 2. [15]
Suppose X and Y be non-empty crisp set. $R(X,Y)$ denoted as **interval neutrosophic relation (INR)** in a subset of product space $X \times Y$ and containing the truth membership function $T_R(x,y)$, indeterminacy membership function $I_R(x,y)$ and false membership function $F_R(x,y)$ where $x \in X$ and $y \in Y$, and $T_R(x,y), I_R(x,y), F_R(x,y) \subseteq [0,1]$.

Definition 3. [Interval neutrosophic point (INP)]
Suppose A in the space of $x \in X$ is an interval neutrosophic point (INP) and $x = \{x_i\}$ is a set of INPs where there exists $T_A(x) = [\sup(T_A), \inf(T_A)] : X \to [0,1]$ defining as the supremum and infimum of truth membership, $I_A(x) = [\sup(I_A), \inf(I_A)] : X \to [0,1]$ defining as the supremum and infimum of indeterminacy membership and $F_A(x) = [\sup(F_A), \inf(F_A)] : X \to [0,1]$ defining as the supremum and infimum of false membership where

$$T_A(x)=\begin{cases} 0 & \text{if } x_i \notin X \\ a \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \tag{3}$$

$$I_A(x)=\begin{cases} 0 & \text{if } x_i \notin X \\ b \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \tag{4}$$

$$F_A(x)=\begin{cases} 0 & \text{if } x_i \notin X \\ c \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \tag{5}$$

Based on definition above, consider three key considerations when releasing a new product: pricing, packaging, and product name. These metrics are widely used to determine the product's development. This section will use the evaluation of the product's development as an example to demonstrate the basic relation and set-theoretic operation such as complement and intersection between two INSs.

Example 1

Let $X=[x_1,x_2,x_3]$. x_1 is the pricing, x_2 is the packaging and x_3 is the product name where $[x_1,x_2,x_3]\in[0,1]$ .Some experts' questionnaires were collected to evaluate the new product, and their options included degrees of excellent, indeterminacy, and bad. Assume A and B is an INS of X defined as follows:

$$A=\frac{\langle[0.3,0.5],[0.4,0.6],[0.4,0.6]\rangle}{x_1}+\frac{\langle[0.6,0.8],[0.1,0.3],[0.3,0.4]\rangle}{x_2}+\frac{\langle[0.7,0.9],[0.3,0.4],[0.3,0.4]\rangle}{x_3}$$

$$B=\frac{\langle[0.6,0.8],[0.2,0.4],[0.2,0.4]\rangle}{x_1}+\frac{\langle[0.3,0.4],[0.3,0.5],[0.6,0.9]\rangle}{x_2}+\frac{\langle[0.5,0.7],[0.1,0.2],[0.4,0.5]\rangle}{x_3}$$

Example 2

Let A and B is an INS based on Example 1. The complement of A and B denoted as $\bar{A}$ and $\bar{B}$. Then,

$$\bar{A}=\frac{\langle[0.4,0.6],[0.4,0.6],[0.3,0.5]\rangle}{x_1}+\frac{\langle[0.3,0.4],[0.7,0.9],[0.6,0.8]\rangle}{x_2}+\frac{\langle[0.3,0.4],[0.6,0.7],[0.7,0.9]\rangle}{x_3}$$

$$\bar{B}=\frac{\langle[0.2,0.4],[0.6,0.8],[0.6,0.8]\rangle}{x_1}+\frac{\langle[0.6,0.9],[0.5,0.7],[0.3,0.4]\rangle}{x_2}+\frac{\langle[0.4,0.5],[0.8,0.9],[0.5,0.7]\rangle}{x_3}$$

Example 3

Let A and B is an INS based on Example 1. Then, $A\cap B=$

$$\frac{\langle[0.2,0.4],[0.6,0.8],[0.6,0.8]\rangle}{x_1}+\frac{\langle[0.3,0.4],[0.7,0.9],[0.6,0.8]\rangle}{x_2}+\frac{\langle[0.3,0.4],[0.8,0.9],[0.7,0.9]\rangle}{x_3}$$

Definition 4. [Interval neutrosophic data point (INDP)]

Suppose $A=\{x|x \text{ interval neutrosophic point}\}$ and $D=\{D_i|D_i \text{ data point}\}$ is a set of interval neutrosophic data points with $D_i\in D\subset X$, where X is a universal set and $T_A(D_i)=[\sup(T_A),\inf(T_A)]:D\to[0,1]$ for truth membership function which defined as $T_A(D_i)=1$, $I_A(D_i)=[\sup(I_A),\inf(I_A)]:D\to[0,1]$ for indeterminacy membership function which defined as $I_A(D_i)=1$, $F_A(D_i)=[\sup(F_A),\inf(F_A)]:D\to[0,1]$ for falsity membership function which defined as $F_A(D_i)=1$ and formulated by $D=\{(D_i,T_A(D_i),I_A(D_i)F_A(D_i)|D_i\in\square)\}$. Thus,

$$T_A(D_i)=\begin{cases}0 & \text{if } D_i\notin X\\ a\in(0,1) & \text{if } D_i\hat{\in} X\\ 1 & \text{if } D_i\in X\end{cases} \tag{6}$$

$$I_A(D_i)=\begin{cases}0 & \text{if } D_i\notin X\\ b\in(0,1) & \text{if } D_i\hat{\in} X\\ 1 & \text{if } D_i\in X\end{cases} \tag{7}$$

$$F_A(D_i)=\begin{cases}0 & \text{if } D_i\notin X\\ c\in(0,1) & \text{if } D_i\hat{\in} X\\ 1 & \text{if } D_i\in X\end{cases} \tag{8}$$

- For equation (6), with $T_A(D_i)=\langle T_A(D_i^L),T_A(D_i),T_A(D_i^R)\rangle$ where $T_A(D_i^L)$ and $T_A(D_i^R)$ are left and right footprint of truth membership values with $T_A(D_i^L)=\langle T_A(D_i^{LL}),T_A(D_i^L),T_A(D_i^{LR})\rangle$ where $T_A(D_i^{LL})$, $T_A(D_i^L)$, and $T_A(D_i^{LR})$ are left-left, left, left-right truth membership grade values, while $T_A(D_i^R)=\langle T_A(D_i^{RL}),T_A(D_i^R),T_A(D_i^{RR})\rangle$ where $T_A(D_i^{RL})$, $T_A(D_i^R)$, and $T_A(D_i^{RR})$ are right-left, right, right-right truth membership grade values.

- For equation (7), with $I_A(D_i)=\left\langle I_A(D_i^L),I_A(D_i),I_A(D_i^R)\right\rangle$ where $I_A(D_i^L)$ and $I_A(D_i^R)$ are left and right footprint of indeterminacy membership values with $I_A(D_i^L)=\left\langle I_A(D_i^{LL}),I_A(D_i^L),I_A(D_i^{LR})\right\rangle$ where $I_A(D_i^{LL})$, $I_A(D_i^L)$, and $I_A(D_i^{LR})$ are left-left, left, left-right indeterminacy membership grade values, while $I_A(D_i^R)=\left\langle I_A(D_i^{RL}),I_A(D_i^R),I_A(D_i^{RR})\right\rangle$ where $I_A(D_i^{RL})$, $I_A(D_i^R)$, and $I_A(D_i^{RR})$ are right-left, right, right-right indeterminacy membership grade values.
- For equation (8), with $F_A(D_i)=\left\langle F_A(D_i^L),F_A(D_i),F_A(D_i^R)\right\rangle$ where $F_A(D_i^L)$ and $F_A(D_i^R)$ are left and right footprint of falsity membership values with $F_A(D_i^L)=\left\langle F_A(D_i^{LL}),F_A(D_i^L),F_A(D_i^{LR})\right\rangle$ where $F_A(D_i^{LL})$, $F_A(D_i^L)$, and $F_A(D_i^{LR})$ are left-left, left, left-right falsity membership grade values, while $F_A(D_i^R)=\left\langle F_A(D_i^{RL}),F_A(D_i^R),F_A(D_i^{RR})\right\rangle$ where $F_A(D_i^{RL})$, $F_A(D_i^R)$, and $F_A(D_i^{RR})$ are right-left, right, right-right falsity membership grade values.

For all i, $D_i=\left\langle D_i^L,D_i,D_i^R\right\rangle$ with $D_i^L=\left\langle D_i^{LL},D_i^L,D_i^{LR}\right\rangle$ where D_i^{LL}, D_i^L and D_i^{LR} are left-left, left and left-right of INDP and $D_i^R=\left\langle D_i^{RL},D_i^R,D_i^{RR}\right\rangle$ where D_i^{RL}, D_i^R, and D_i^{RR} are right-left, right, and right-right of INDP respectively. This is shown in Figure 2 for the truth, indeterminacy and falsity memberships. From the figure also demonstrate that the upper bound is $\left[D^{LL},D,D^{RR}\right]$ while the lower bound is $\left[D^{LR},D,D^{RL}\right]$.

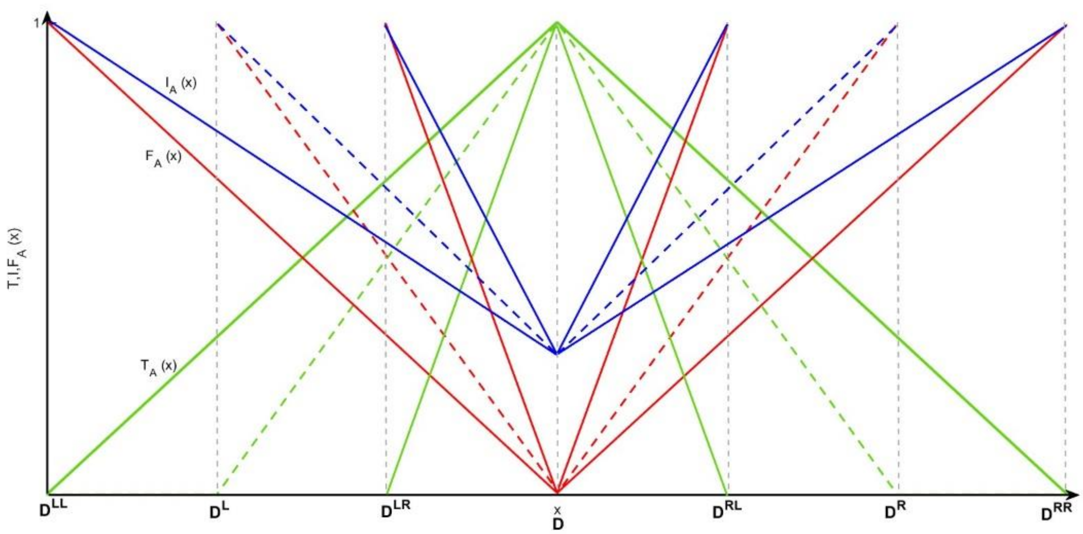

Figure 2. Interval Neutrosophic Data Points for truth, indeterminacy, and membership degrees

Interval Neutrosophic Control Point (INCP)

A control point (CP) is a single point or set of points in computer graphics and mathematical modelling that influences the shape or behaviour of a curve, surface, or other geometric object. Techniques that use CPs include in non-uniform rational B-splines modelling (NURBS), B-splines, and Bézier curves or surface. The geometric object's qualities and deformation are defined by the location and properties of the CPs. Aside from data manipulation, the form, curvature, and other characteristics of the curve or surface can be altered. In this work, the CPs are a collection of points used to define the contours of an interval neutrosophic Bézier curve. It is also important in geometric modelling for deriving and producing smooth curves. The concept of INCP was inspired by Wahab and Colleagues [7-9] for FS and IFS as follows:

Definition 5 [Interval neutrosophic control point (INCP)]
Let A be an INDP, then INCP is viewed as a group of points that denotes a locations and coordinates and is used to describe the curve and is indicated by

$$\begin{aligned} P_i^T &= \left\{ p_0^T, p_1^T, \ldots, p_n^T \right\} \\ P_i^I &= \left\{ p_0^I, p_1^I, \ldots, p_n^I \right\} \\ P_i^F &= \left\{ p_0^F, p_1^F, \ldots, p_n^F \right\} \end{aligned} \tag{9}$$

where P_i^T, P_i^I and P_i^F are interval neutrosophic control points for truth, false and indeterminacy membership function and i is one less than n. This study concentrates on **cubic case** when $n = 3$ to create the INCBC. Thus, the INCP as follows:

$$\begin{aligned} P_i^T &= \left\{ p_0^T, p_1^T, p_2^T, p_3^T \right\} \\ P_i^I &= \left\{ p_0^I, p_1^I, p_2^I, p_3^I \right\} \\ P_i^F &= \left\{ p_0^F, p_1^F, p_2^F, p_3^F \right\} \end{aligned} \tag{10}$$

Approximation of Interval Neutrosophic Cubic Bézier Curve (INCBC)

Bézier curves which are parameterized curves guided by a control polygon, are frequently used in geometric simulation [23, 24]. The degree of the polynomial corresponds to the number of data points used to construct the curve [25]. The illustration below shows a Bézier curve created by integrating the Bernstein polynomial or basis function with INCP. The INCBC is constructed using INCP and **Definition 1**, which is then combined in a geometric model with the Bézier blending function. The Bézier curve for approximation method is derived from Piegl and Tiller [26] and is then combined with INCP as follows:

Definition 6 [Approximation of interval neutrosophic cubic Bézier curve (INCBC)]
Suppose $P_i^T = \left\{ p_0^T, p_1^T, p_2^T, p_3^T \right\}, P_i^I = \left\{ p_0^I, p_1^I, p_2^I, p_3^I \right\}$, and $P_i^F = \left\{ p_0^F, p_1^F, p_2^F, p_3^F \right\}$ where $i = 0,1,2,3$ is INCP. An INCBC is defined as $BC(t)$ with the curve position vector depending on the value of t, then blending with J_i by Bézier curves and represented as follows:

$$\begin{aligned} BC(t)^T &= \sum_{i=0}^{3} P_i^T J_{3,i}(t) \\ BC(t)^I &= \sum_{i=0}^{3} P_i^I J_{3,i}(t) \\ BC(t)^F &= \sum_{i=0}^{3} P_i^F J_{3,i}(t) \end{aligned} \tag{11}$$

where $0 \le t \le 1$ and the blending function is a Bézier or Bernstein basis, J_i :

$$J_{(3,i)}(t) = \binom{3}{i} t^i (1-t)^{3-i} \qquad (0)^0 \equiv 1 \tag{12}$$

with

$$\binom{3}{i} = \frac{3!}{i!(3-1)!} \qquad (0)^0 \equiv 1 \tag{13}$$

The INCBC equation can also be expressed in matrix multiplication using the approach of Zaidi and Zulkifly [27]. INCBC can be represented as a matrix by extending the analytic formulation of the curve into its Bernstein polynomial coefficients and then expressing these coefficients using the polynomial power basis [27]:

$$BC(t) = [J][P] \tag{14}$$

where

$$[J] = \left[J_{3,0}, J_{3,1}, J_{3,2}, J_{3,3} \right] \tag{15}$$

$$[P]^T = \left[P_0, P_1, P_2, P_3 \right] \tag{16}$$

Visualization of Interval Eutrophic Cubic Bézier Curve (INCBC)

This section visualizes the INCBC approximation model for truth, indeterminacy, and falsity membership function. At the end of this section also will demonstrate the combination of all memberships in one axis and an algorithm to create the INCBC based on previous sections that already discussed. Let consider **Table 1** is a complex data that represent as INCP.

Table 1. INCP with its membership degrees

INCP, P_i	Truth Degree, P_i^T	Indeterminacy Degree, P_i^I	False Degree, P_i^F
$P_0=(2,2)$	0.7	0.2	0.4
$P_1=(7,8)$	0.6	0.3	0.4
$P_2=(11,13)$	0.8	0.2	0.3
$P_3=(25,23)$	0.3	0.6	0.4

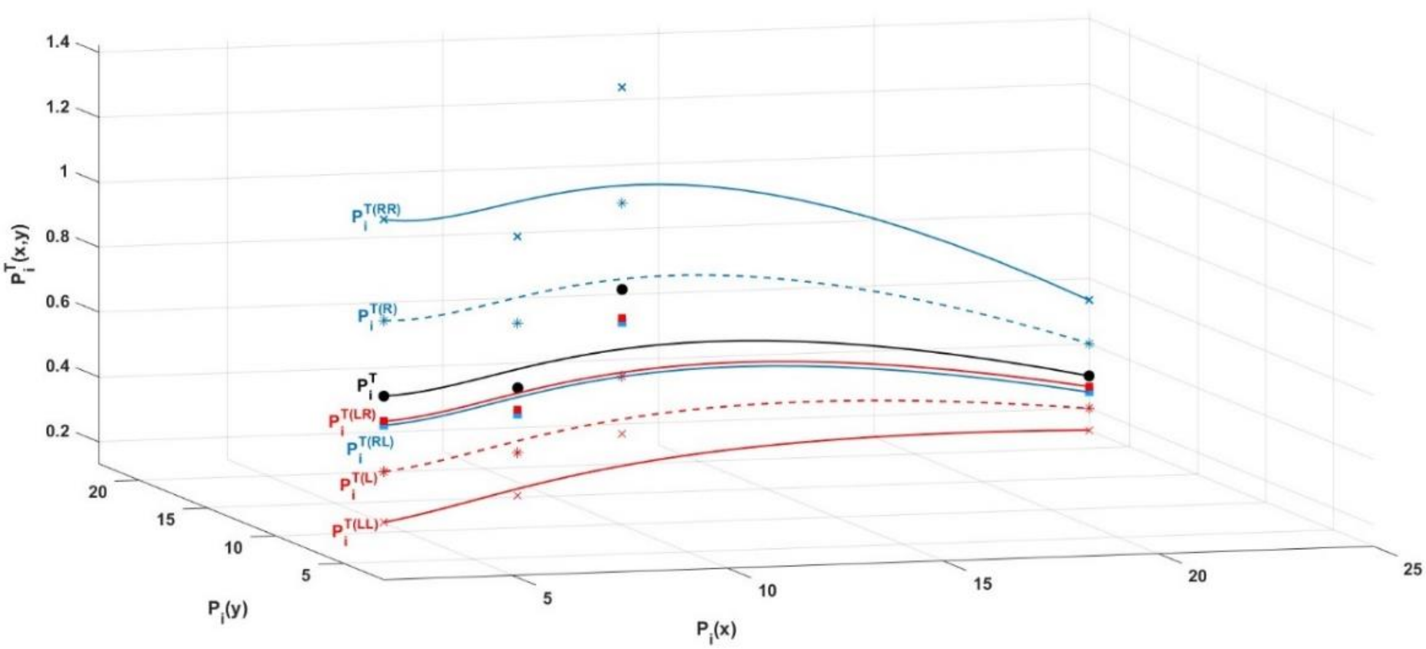

Figure 3. INCBC for truth membership with its respective INCP

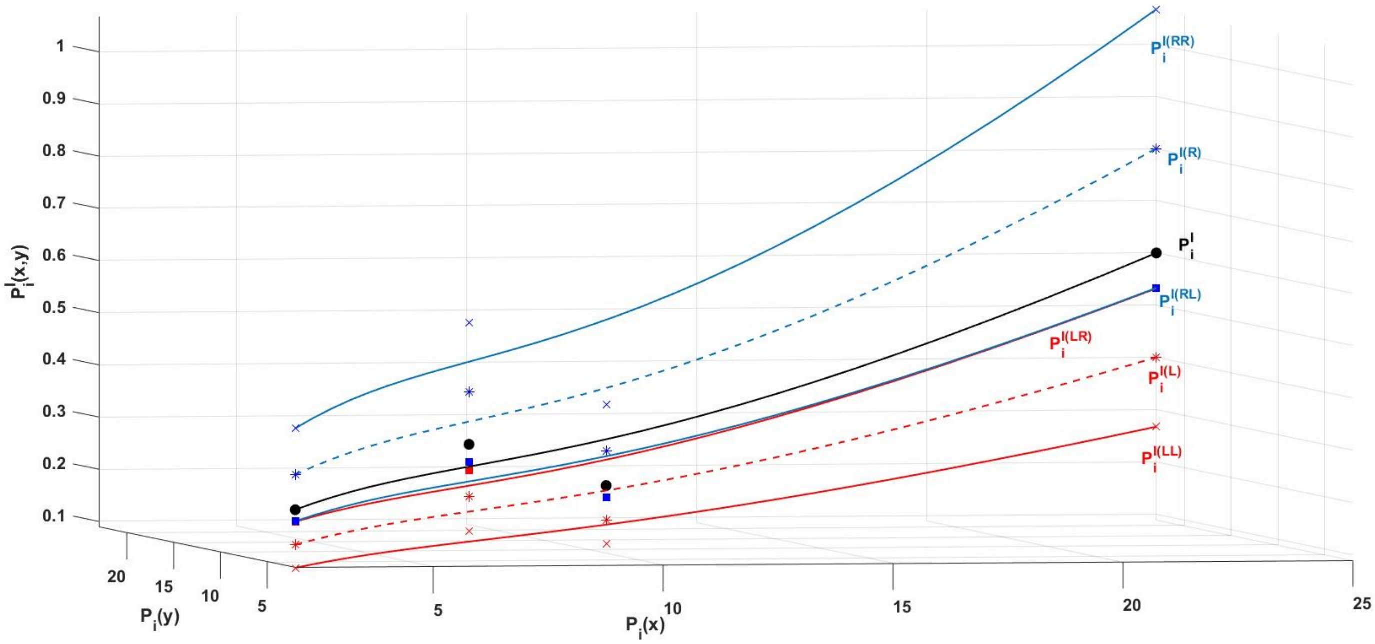

Figure 4. INCBC for indeterminacy membership with its respective INCP

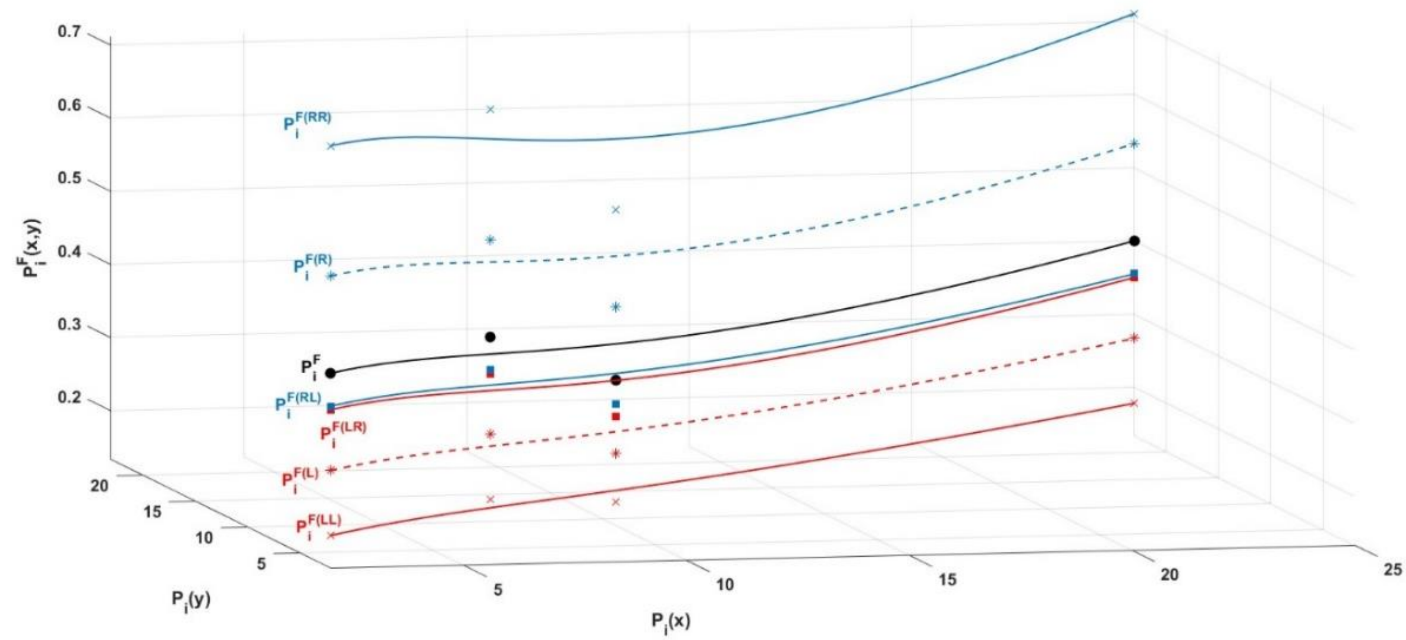

Figure 5. INCBC for falsity Membership with its INCP

As mentioned in Figure 2, the INDP in this case was refer to INCP P_i as in Table 1. Based on **Figure 3** to **Figure 5**, the P_i^T, P_i^I and P_i^F denoted as INCP and INCBC for truth, indeterminacy, and falsity degrees respectively and they were demonstrate in black curve for the INCBC and black dot for the INCP. Thus, $\left[P_i^{T(LL)}, P_i^T, P_i^{T(RR)}\right]$ refer as upper bound while $\left[P_i^{T(RL)}, P_i^T, P_i^{T(LR)}\right]$ refer as lower bound for truth membership, $\left[P_i^{I(LL)}, P_i^I, P_i^{I(RR)}\right]$ refer as upper bound while $\left[P_i^{I(RL)}, P_i^I, P_i^{I(LR)}\right]$ refer as lower bound for indeterminacy membership, and $\left[P_i^{F(LL)}, P_i^F, P_i^{F(RR)}\right]$ refer as upper bound while $\left[P_i^{F(RL)}, P_i^F, P_i^{F(LR)}\right]$ refer as lower bound for false membership. The right footprint values $\left[P_i^{T(RR)}, P_i^{T(R)}, P_i^{T(RL)}\right]$ was shown in blue while the left footprint values $\left[P_i^{T(LR)}, P_i^{T(L)}, P_i^{T(LL)}\right]$ for truth membership in **Figure 3**, the right footprint values $\left[P_i^{I(RR)}, P_i^{I(R)}, P_i^{I(RL)}\right]$ was demonstrated in blue while the left footprint values $\left[P_i^{I(LR)}, P_i^{I(L)}, P_i^{I(LL)}\right]$ for indeterminacy membership in **Figure 4** and The right footprint values, $\left[P_i^{F(RR)}, P_i^{F(R)}, P_i^{F(RL)}\right]$ was denoted in blue while the left footprint values $\left[P_i^{F(LR)}, P_i^{F(L)}, P_i^{F(LL)}\right]$ for false membership in **Figure 5.** Since that the Bernstein basis function was used in this study, the following is some INCBC properties:

i. The interval neutrosophic control polygon's initial and last points coincide with the first and last points on the INCBC.
ii. In general, the interval neutrosophic control polygon's shape is followed by the INCBC.
iii. The convex hull contains INCBC for every parametric parameter.
iv. The number of intervals neutrosophic control polygon points is one fewer than the degree of the polynomial defining the curve segment.
v. Under an affine transformation, the INCBC remains invariant.
vi. The sum of the Bernstein basis function equals one and it is non-negative.
vii. The generated INCBC demonstrates the property of variation-diminishing.
viii. The first and last spans of intervals neutrosophic control polygons and the tangent vector at the ends of INCBC have the same direction.

Figure 6 shows the INCBC consists of truth, indeterminacy and falsity membership including for each of their left and right footprint. The green curves demonstrate for truth membership, the blue curves show for indeterminacy curves and the red curves denote for falsity membership. At the end of this section, **Figure 7** demonstrates an algorithm to create the INCBC.

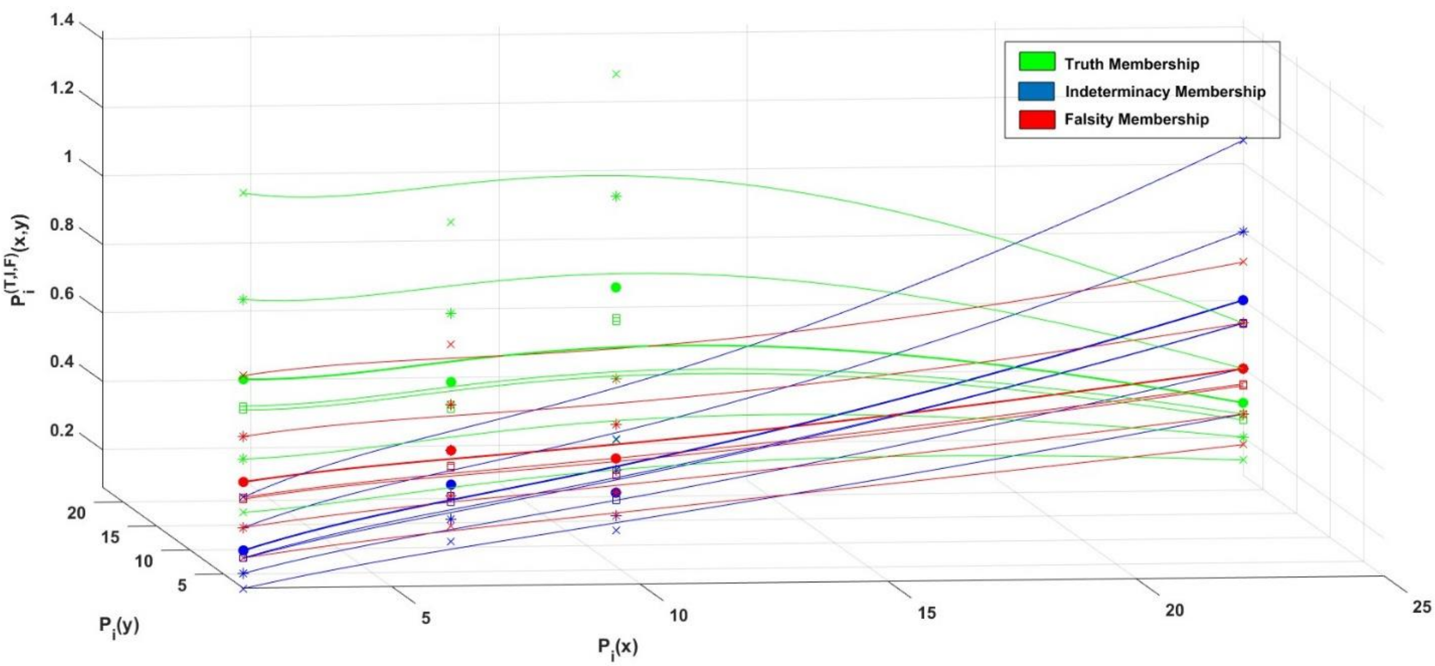

Figure 6. INCBC with its respective INCP (Truth, indeterminacy and false)

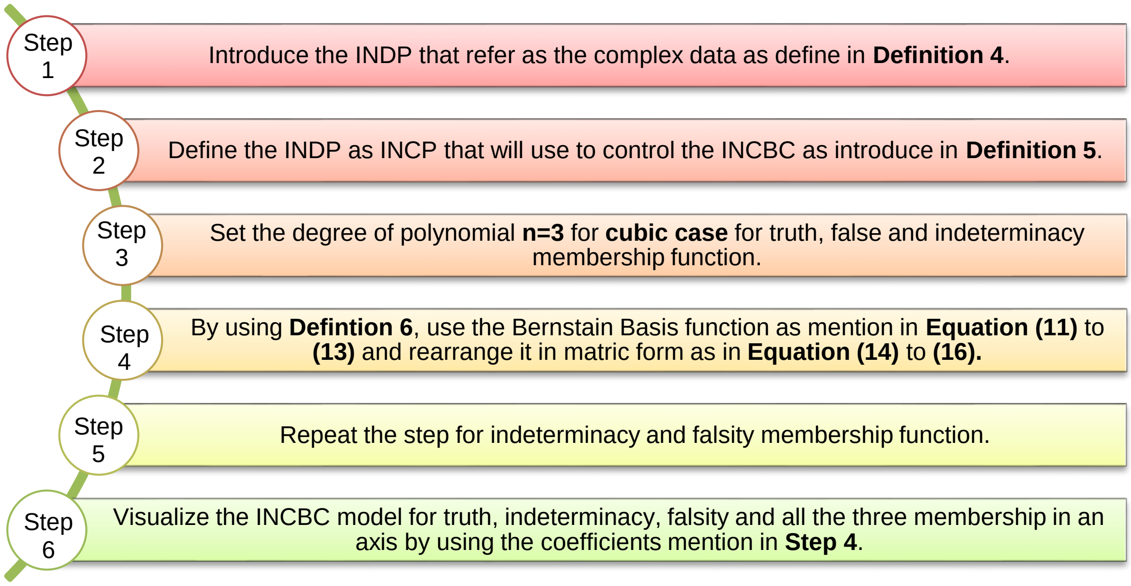

Figure 7. An Algorithm for INCBC

Conclusions

This work proposed the concepts of INP and INDP to establish the INCP that controls the behaviour of INCBC. The model demonstrates that the visualisation of complex uncertain data can be achieved through the use of the theory of INS. This approach has the potential to make valuable contributions to areas characterised by high levels of uncertainty data, such as bathymetry data. In addition to these applications, predictive models are also utilised in other medical disciplines, including cancer level prediction, picture blurring detection, and disaster warning systems. The scope of this study can be expanded to include more complex problem, specifically the type-2 neutrosophic set. Additionally, it is worth considering the other geometric models, such as B-spline and non-uniform rational B-spline (NURBS), in future investigations to enhance the visualisation capabilities of this study. In addition, this work has the potential for expansion through the use of surface modelling techniques or the utilisation of interpolation methods.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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