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RESEARCH ARTICLE

Fuzzy Intuitionistic Alpha-cut Interpolation Rational Bézier Curve Modeling for Shoreline Island Data

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Abstract The problem of uncertain data cannot be solved by conventional methods, which results in inaccurate data analysis and prediction. During the data collecting phase, ambiguous data are often collected, but they cannot be used immediately to generate geometric models. In this case, the new approaches to intuitionistic fuzzy sets will be used to determine the alpha cut value for uncertainty data sets. To solve the uncertainty data and build the mathematical model, this study applied fuzzy set theory, intuitionistic fuzzy sets, and rational Bézier curve geometric modelling. There are three main methods in this study. The triangular fuzzy number is used to define the uncertainty data in the first place. The alpha value can then be found using a centre of mass alpha-cut. The intuitionistic alpha-cut can then be applied to both membership and non-membership data. This procedure, also called fuzzification, is defined as fuzzy intuitionistic into alpha-cut values. The data set will then undergo the defuzzification procedure to get single value data. For the purpose of analysis and conclusion-making, the modeling data for each process will be visualised using an interpolation rational Bézier curve. The findings demonstrate that using the intuitionistic fuzzy set for the alpha-cut value was more effective than the previous method without considering both membership and non-membership values.

Keywords: Uncertainty data, fuzzy set theory, intuitionistic fuzzy set, rational Bézier curve, shoreline island data.

Introduction

Two important fields of study that use the fuzzy geometric modeling paradigm are fuzzy set theory and geometric modeling [22]. Both is one of the crucial strategies in the process of analyzing data or making decisions for the problem in defining and subsequently modeling uncertainty data. This technique also applied in other fields where the data visualisation is not uncertain or ambiguous. When modeling a collection of data sets, issues arise when the data become ambiguous which make it unable to model them [20]. Uncertainty data occurred due to some factors of unavoidable circumstances, including the environment, human mistake, tool limits, and so on. The data must be defined using a new method that takes measurement uncertainties into account in order for the uncertainty data to be useful for analysis and model creation [13].

Fuzzy set theory was first proposed by Zadeh in 1965 since it has been frequently used in dealing with uncertainty in various decision-making processes [19]. The fuzzy number idea was utilized in dealing with uncertain data and part of the fuzzy theory notion [21, 23]. To simulate the real data points, the geometric model that will be proposed is an interpolation rational Bézier curve applying the interpolation method. Rational Bézier curves can include conic sections in Bézier form [5]. Meanwhile, interpolation is a method for creating data points within a range of discrete data of known data points with a curve that passes through each control point [14].

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Fuzzy interpolation rational Bézier curve model has been used to verify offline handwriting signatures in circumstances when the signatures become fuzzy cases [15]. Apart from that, cubic cases will be discussed because it has been applied in numerous research and has proven to be a useful tool in curve design [16]. The alpha value of the fuzzification procedure will be determined using the fuzzy intuitionistic alpha-cut. Previously, the intuitionistic fuzzy B-Spline curve model was visualised using the interpolation approach [25]. In addition, the control point of data can be used with the fundamental geometric modelling operations, such splines to create a variety of models in the form of intuitive fuzzy spline curves or surfaces that are easily to figure out [24]. These models can be created using the intuitionistic concept and relation to represent the relevant data. Atanassov in 1985 was the first who suggested an intuitionistic fuzzy set (IFS) [1, 3]. In comparison to intuitionistic fuzzy sets, which have two geometric interpretations, ordinary fuzzy sets only have one [2]. The intuitionistic fuzzy set considers more uncertainty than fuzzy theory, such as the degree of membership and non-membership [6].

The coastal zone's resources and environmental protection rely greatly on the extraction and analysis of the shoreline, which acts as the boundary between the land and the water [7]. The study will employ data from shoreline islands as it provides uncertainty information brought on by processes such as erosion, accretion, sedimentation, wind and wave patterns, and sea level rise. The shoreline data results vary due to the different resolutions of satellites rise [18]. Additionally, the effects of image resolution and scale of interpretation have rarely been taken for consideration when examining the results of such investigations [9].

The modelling of interpolation rational Bézier curve using fuzzy set theory will be covered in this study, which is structured as follows. Geometric modelling is mostly used to analyse a set of data represented by a curve. While finding the alpha-cut value for both membership and non-membership degrees was the purpose of fuzzy intuitionistic. In the methodology section, fuzzy set theory and triangular fuzzy number concepts are covered in the representation of data points. Fuzzification techniques covered in this section include the centre of mass alpha-cut and intuitionistic alpha-cut processes. It also discusses defuzzification using weighted average and interpolation modelling functions for rational Bézier curves. The next part shows the application of the fuzzy interpolation rational Bézier curve model. The proposed method is used to model a set of uncertain data. Last but not least, by comparing the crisp data curves with the defuzzification data curve, this part demonstrates the effectiveness of the proposed fuzzy rational Bézier interpolation model.

Materials and Methods

Shoreline island data will be used in this study as uncertainty data. From crisp uncertainty shoreline island data points, it can be defined into fuzzy data points by applying fuzzy set theory and fuzzy number concepts. To model the curve geometrically, interpolation rational Bézier geometric modeling will be used. Then, centre of mass alpha-cut and an intuitionistic alpha-cut will be applied throughout the fuzzification process to determine the alpha value. Through the centre of mass alpha-cut method in the fuzzification process, the issue is the setting of the alpha value itself, which is how far the confidence of the alpha value from the centre of mass method is also questioned. That’s why the second method is applied, which is intuitionistic alpha-cut to find out the real alpha value. To obtain a single data value, the defuzzification procedure will be used. The purpose of this method is to distinguish between crisp data points and defuzzified data points, particularly to identify errors.

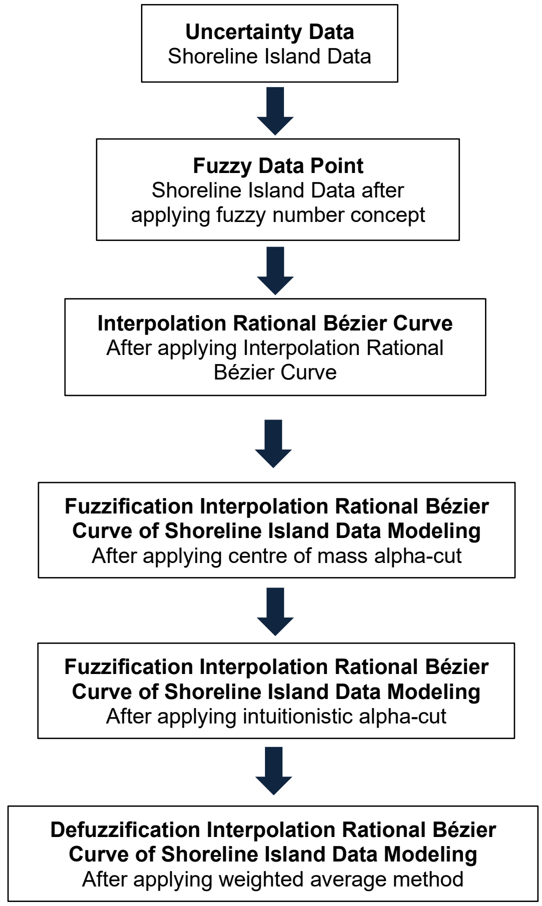

Figure 1. Process of modeling uncertainty shoreline island data by using fuzzy intuitionistic interpolation rational Bézier curve

Fuzzy Set Theory

Let x be an element in set X. If x has full membership of A, then the membership function is one. For x that has half-membership, the membership function is in open interval (0,1). If x is not in A then the membership function is 0. Generally, the membership function of fuzzy set $\overleftrightarrow{A}$ can be summarized as:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \text{ (full membership)} \\ 0 < c < 1 & \text{if } x \tilde{\in} A \text{ (non-full membership)} \\ 0 & \text{if } x \notin A \text{ (non-membership)} \end{cases} \quad (1)$$

With c denoted as membership value. Then, fuzzy set $\overleftrightarrow{A}$ can be expressed as $\overleftrightarrow{A} = \{(x, \mu_A(x))\}$ which is $\overleftrightarrow{A}$ in X is a collection of order pairs denoted by x in X with grade of membership $\mu_A(x)$ in $[0,1]$.

If the following conditions are satisfied, a fuzzy set $\overleftrightarrow{A}$ is referred to as a fuzzy number.

i. There exist $x \in R$, as a result $\mu_A(x) = 1$

ii. For any $\alpha \in (0,1]$ the set $\{x: \mu_A(x) \geq \alpha\}$ is a closed interval characterized by $\langle \overleftarrow{A}_\alpha^-, A, \overrightarrow{A}_\alpha^+ \rangle$

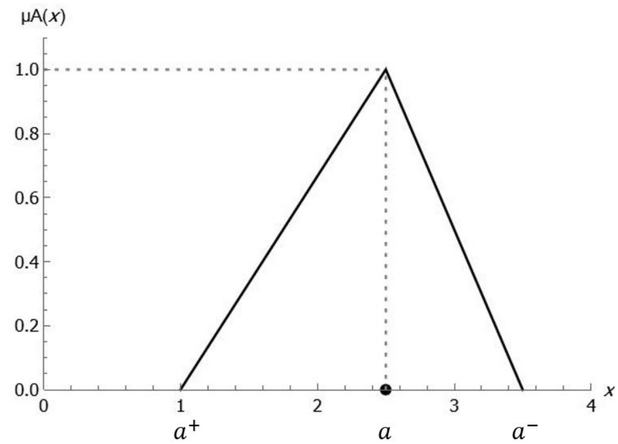

Figure 2. Triangular fuzzy number

The mean of the membership function is a fuzzy number, which is actually a fuzzy set defined on a set of real numbers. The fuzzy number concept that is applied to defined fuzzy data points is the triangular fuzzy number. Figure 2 above shows the triangular fuzzy number that based on the three-value judgement including the left fuzzy number a^-, the crisp number a and the right fuzzy number a^+. All the uncertainty data will be defined by using a triangular fuzzy number from fuzzy set theory to get the three-value judgement [10].

Centre of Mass Alpha-Cut

To determine the alpha value, two methods will be applied. Since the triangular fuzzy number is used, the centroid of a triangle polygon will be determined. The centroid will always be triangular, while the intersection of the three medians on each side of the triangle determines the centroid, which is also known as the centre of mass [22]. Equation 2 is the calculation step to obtain the centre of mass alpha-cut value.

$$C=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right) \tag{2}$$

Intuitionistic Alpha-cut

Intuitionistic fuzzy sets are an extension of fuzzy sets which the theory containing new operations, relations and operators that essentially extend the operators defined over fuzzy sets [4]. After getting the alpha-cut value from the centre of mass, the second alpha-cut value will be determined by using the intuitionistic alpha-cut. This process is to consider both membership and non-membership functions. In Eq. (3), let X be as the given set. Then an intuitionistic fuzzy set A^* in X is provided as below:

$$A^*=\left\{\left(x,\mu_{A^*}(x),v_{A^*}(x)\right)\mid x\in X\right\} \tag{3}$$

Where the degree of membership and non-membership is respectively represented by$\mu_{A^*}, v_{A^*}: X \to [0,1]$, of the element x in A and $0 \le \mu_{A^*}(x) + v_{A^*}(x) \le 1$ [8]. For each $x \in X, n_{A^*}(x) = 1 - \mu_{A^*}(x) - v_{A^*}(x)$ is the degree of hesitation. Then, let $\hat{A}^*$ be a triangular intuitionistic fuzzy number. Then the measure of membership value $\mu(\hat{A}^*)$ is given as in Eq. (4),

$$\mu\left(\hat{A}^*\right)=\frac{1}{4}\left(a_1+2a_2+a_3\right) \tag{4}$$

Similarly, the measure of non-membership value $v(\hat{A}^*)$ is given ad in Eq. (5),

$$v\left(\hat{A}^*\right)=-\frac{1}{4}\left(a_1^{'}+2a_2^{'}+a_3^{'}\right) \tag{5}$$

Next is the degree of hesitancy and the intuitionistic alpha value is given by Eq. (6) and Eq. (7) respectively,

$$\pi_{\hat{A}^*}\left(a_{ij}\right) = 1 - \mu\left(\hat{A}^*\right) - v\left(\hat{A}^*\right) \tag{6}$$

$$\alpha_{\hat{A}^*} = \mu\left(\hat{A}^*\right) + \pi_{A^*}\left(a_{ij}\right)\mu\left(\hat{A}^*\right) \tag{7}$$

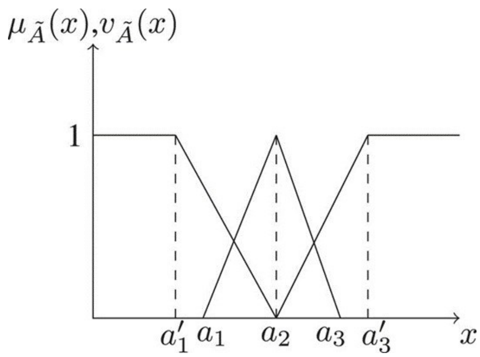

Figure 3. Triangular intuitionistic alpha-cut

Figure 3 illustrates the method of how to find the second alpha value by using the triangular intuitionistic alpha-cut [8].

Fuzzification Process

After find out the second alpha value through intuitionistic alpha-cut, next is the fuzzification process towards fuzzy data point. As in Eq. (8) and Eq. (9) below, let $\overleftrightarrow{D}$ be a set of fuzzy control points and $\alpha_{\hat{A}^*}$ is an intuitionistic alpha value for all fuzzy data point. Then, with $\overleftrightarrow{D}^L_{\alpha_{\hat{A}^*}}$ and $\overleftrightarrow{D}^R_{\alpha_{\hat{A}^*}}$ is a left and right fuzzy data point respectively after α-cut process with $\overleftrightarrow{D}_{\alpha_{\hat{A}^*}}$ is the intuitionistic α-cut operation. Then, D_i is a crisp data point value for all $\alpha_{\hat{A}^*} \in (0,1]$ and $i = 0,1,\dots,n$ which is given as the following equation [21],

$$\overleftrightarrow{D}_{\alpha_{\hat{A}^*}} = \left\langle \overleftrightarrow{D}^L_{\alpha_{\hat{A}^*}}, D_i, \overleftrightarrow{D}^R_{\alpha_{\hat{A}^*}} \right\rangle \tag{8}$$

$$\overleftrightarrow{D}_{\alpha_{\hat{A}^*}} = \left\langle \left[\left(D_{(i,j)} - \overleftrightarrow{D}^L_{\alpha_{\hat{A}^*}}\right)\alpha_{\hat{A}^*} + \overleftrightarrow{D}^L_{\alpha_{\hat{A}^*}}\right], D_{(i,j)}, \left[-\left(\overleftrightarrow{D}^L_{\alpha_{\hat{A}^*}} - D_{(i,j)}\right)\alpha_{\hat{A}^*} + \overleftrightarrow{D}^R_{\alpha_{\hat{A}^*}}\right] \right\rangle \tag{9}$$

Defuzzification Process

The method that will be applied to get the defuzzification data point is the weighted average method. In Eq. (10), let $\overleftrightarrow{D}_{\alpha_{\hat{A}^*}}$ is as fuzzy data point after intuitionistic α-cut process. Then, $\tilde{D}_{\alpha_{\hat{A}^*}}$ is the defuzzification for $\overleftrightarrow{D}_{\alpha_{\hat{A}^*}}$ that can be shown as follow [11],

$$\tilde{D}_{\alpha_{\hat{A}^*}} = \frac{(\alpha_{\hat{A}^*} \times D^L_{\alpha_{\hat{A}^*}}) + (\alpha_{\hat{A}^*} \times D_{\alpha_{\hat{A}^*}}) + (\alpha_{\hat{A}^*} \times D^R_{\alpha_{\hat{A}^*}})}{(\alpha_{\hat{A}^*} + \alpha_{\hat{A}^*} + \alpha_{\hat{A}^*})} \tag{10}$$

Fuzzy Interpolation for Rational Bézier Curve Modeling

This study will apply interpolation curves, which means that all data points are interpolated by a single curve, which must go through all data points. Hence, the fuzzy interpolation rational Bézier curve was addressed in this section. Fuzzy interpolation rational Bézier curve are built to maintain the continuity and smoothness of the curve [14]. In general, fuzzy rational Bézier curve is defined as in Eq. (11) and the basic functions as in Eq. (12),

$$\vec{R}(t)=\frac{\sum_{i=0}^{n}\vec{w}_i B_i^n(t)\vec{P}_i}{\sum_{i=0}^{n}\vec{w}_i B_i^n(t)} \tag{11}$$

$$\frac{\vec{W}_i B_i^n(t)}{\sum_{i=0}^{n}\vec{W}_i B_i^n(t)} \tag{12}$$

Next is fuzzy interpolation rational Bézier curve if $\alpha = \beta = 3$ for cubic cases. Let $\vec{P}_i, \vec{D}_i \in R, i = 0,1,\dots,n$, be a set of fuzzy data points with fuzzy derivative value at t as shown in Eq. (13) and by using the fuzzy blending functions provide in Eq. (14),

$$\vec{R}(t)=\frac{\vec{F}_0(t)\vec{P}_i+\vec{F}_1(t)\vec{v}_i\vec{Q}_i+\vec{F}_2(t)\vec{w}_i S_i+\vec{F}_3(t)\vec{P}_{i+1}}{\vec{F}_0(t)+\vec{F}_1(t)\vec{v}_i+\vec{F}_2(t)\vec{w}_i+\vec{F}_3(t)} \tag{13}$$

$$\begin{aligned}\vec{F}_0(t)&=(1-t)^2(1+(2-\alpha)t)\\ \vec{F}_1(t)&=\alpha(1-t)^2 t\\ \vec{F}_2(t)&=\beta t^2(1-t)\\ \vec{F}_3(t)&=t^2(1+(2+\beta)(1-t))\end{aligned} \tag{14}$$

In Eq. (15) shows that shows that $\vec{v}_i$ and $\vec{w}_i$ are fuzzy weights values. Which $\vec{D}_i$ and $\vec{D}_{i+1}$ are the tangent vectors at $\vec{P}_i$ and $\vec{P}_{i+1}$. Then, Eq. (16) and Eq. (17) are the definitions of the tangent vectors for open curves [12],

$$\begin{aligned}\vec{Q}_i&=\frac{\vec{D}_i}{3\vec{v}_i}+\vec{P}_i\\ \vec{S}_i&=\vec{P}_{i+1}-\frac{\vec{D}_{i+1}}{3\vec{w}_i}\end{aligned} \tag{15}$$

$$\begin{aligned}\vec{D}_0&=2(\vec{P}_1-\vec{P}_0)-\frac{\vec{P}_2-\vec{P}_0}{2}\\ \vec{D}_i&=\vec{a}_i(\vec{P}_i-\vec{P}_{i-1})+(1-\vec{a}_i)(\vec{P}_{i-1}-\vec{P}_i)\\ \vec{D}_n&=2(\vec{P}_n-\vec{P}_{n-1})-\frac{\vec{P}_n-\vec{P}_{n-2}}{2}\end{aligned} \tag{16}$$

$$\vec{a}_i=\frac{\left\|\vec{P}_{i-1}-\vec{P}_i\right\|}{\left\|\vec{P}_{i-1}-\vec{P}_i\right\|+\left\|\vec{P}_i-\vec{P}_{i-1}\right\|}, \qquad i=0,1,\dots,n \tag{17}$$

The modelling of shoreline island uncertainty data using fuzzy interpolation rational Bézier curve can be showed as in Figure in the results.

Results and Discussion

A fuzzy intuitionistic interpolation rational Bézier curve model of shoreline island data will be illustrated in this section. There are fifteen shoreline islands that have been plotted to model the fuzzy intuitionistic interpolation rational Bézier curve. By using concepts of fuzzy numbers, the crisp interpolation rational Bézier curve model can be seen in Figure 3 (a). After achieving the fuzzy interpolation rational Bézier curve model, Figures 4 and 5 apply the fuzzification process by using a triangular alpha-cut value and then an intuitionistic alpha-cut value. Both methods are used to determine which method has a better interval after plotting the curve. The results, which have a smaller interval, will be used for the defuzzification process, which is shown in Figure 6.

Crisp Interpolation Rational Bézier Curve Model of Shoreline Island

By using 15 control points from the uncertainty data of shoreline islands, the model of the rational Bézier curve will be shown. The data points were plotted manually using the Mathematica application.

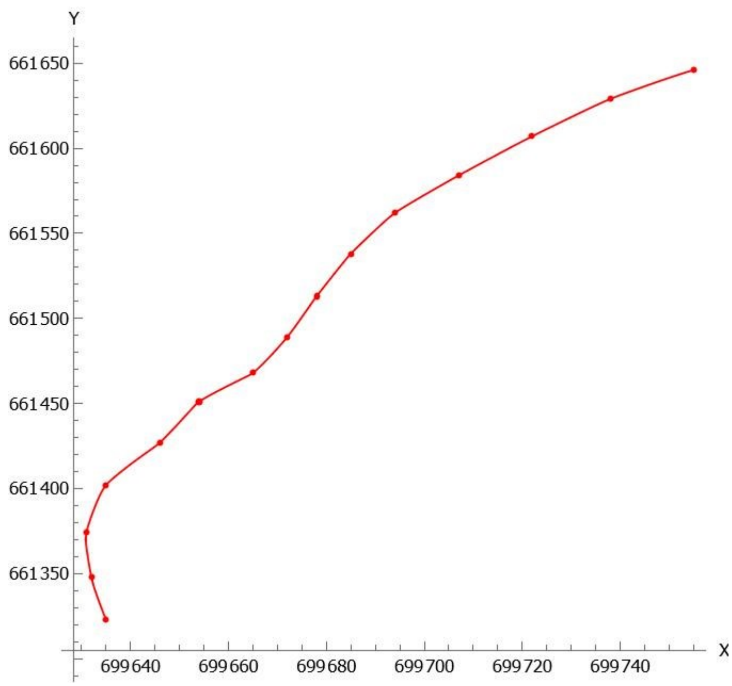

Figure 4. Crisp interpolation rational Bézier curve model of shoreline island data

The red curve in Figure 4 illustrates the model of crisp interpolation rational Bézier curve model using the uncertainty data of shoreline islands

Comparison between Fuzzification Interpolation Rational Bézier Curve Model of Shoreline Island by Using Triangular Alpha-Cut Value and Intuitionistic Alpha-Cut Value

From Figure 5 and Figure 6, each model shows the green, red, and blue curves, which represent the left, crisp, and right interpolation rational Bézier curve models respectively. By comparison of Figure 5 and Figure 6, the interval of fuzzification interpolation rational Bézier curve by using the triangular alpha-cut value shown in Figure 5 is wider than the interval of fuzzification interpolation rational Bézier curve by using the intuitionistic alpha-cut value illustrated in Figure 6. Based on past research, a method of identifying the location and analysing the precise position accuracy, the left fuzzy data point (green curve) used 4.38 and the right fuzzy data point (blue curve) used 5.38 since this value is more accurate [17].

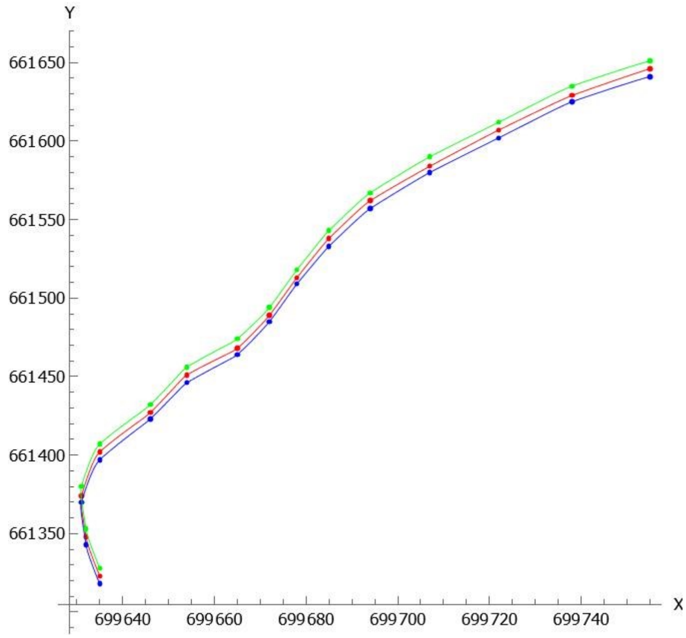

Figure 5. Fuzzification interpolation rational Bézier curve by using the triangular alpha-cut value

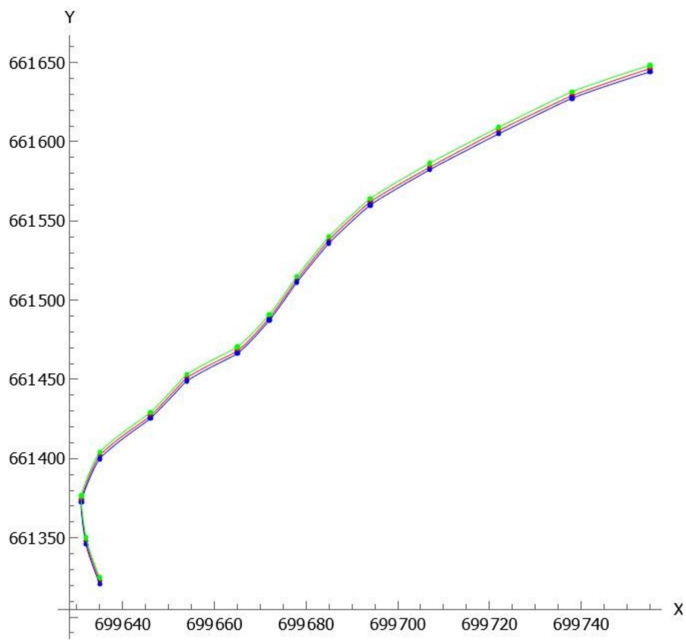

Figure 6 Fuzzification interpolation rational Bézier curve by using the intuitionistic alpha-cut value.

Figure 7 illustrate the comparison between crisp interpolation rational Bézier curve model and defuzzification by intuitionistic alpha-cut interpolation rational Bézier curve model. Eq. (10) is the equation used to determine the defuzzification of rational Bézier curve model. This comparison curve proves that there is little noticeable change. The defuzzification of rational Bézier model is illustrated by the purple curve, while the crisp curve is represented by the red curve. The curve that employs an intuitionistic alpha-cut value is the purple curve. In order to determine the alpha value, all three elements of an intuitionistic fuzzy set are taken into account. Despite the lack of a precise formula for calculating alpha value, this proves that the purple curve is more accurate than the red curve since better the alpha value, more elements are taken into consideration.

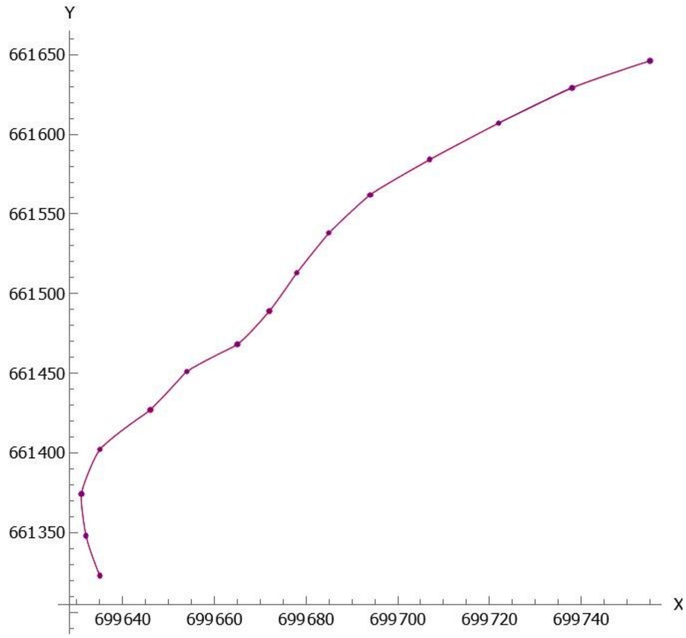

Figure 7. Comparison between crisp interpolation rational Bézier curve model and defuzzification of intuitionistic alpha-cut interpolation rational Bézier curve model

Average Percentage of Error

To investigate the effectiveness of output of uncertainty data, the errors between the crisp data points and defuzzification data points need to be identified as in Eq. (18).

$$\sum_{i=0,1,\ldots,n}^{n} D_{i_r} \text{ with } D_{i_r} = \frac{\tilde{D}_{i\alpha_{\hat{A}^*}} - D_i}{D_i}, i = 0,1,\ldots,n \text{ and } n = 15 \tag{18}$$

The average percentage of error is 2.2726×10^{-6}. This result proves that the methods for fuzzy intuitionistic alpha-cut are reliable and acceptable. The result of average percentage is in an acceptable range since the value of the error average for uncertainty data is less than 10%.

Conclusions

In conclusion, the fuzzy intuitionistic alpha-cut value towards interpolation rational Bézier curve modeling for shoreline island data was applied. The uncertainty data for shoreline islands was defined using the fuzzy number concept. This study contributes to handling the imprecise shoreline island uncertainty data since the fuzzy intuitionistic alpha-cut consists of membership degree, non-membership degree, and hesitancy degree. The data points of shoreline islands were identified by applying the interpolation rational Bézier that generated a curve that went across all the data points. This model benefits those who are doing prediction and data analysis. Furthermore, this approach can also advance the field of fuzzy modeling techniques, as are capable of the model it generates. For future research, this study can be extended by using the approximation curve or surface method. The Non-Uniform Rational B-Spline (NURBS) model can also be expanded using this model, as NURBS employs a more complex function with a more precise and flexible curve.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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