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RESEARCH ARTICLE

Stabilization of Fuzzy Prey-Predator Model Using Backstepping Method

Ibtisam Kamil Hanan*, Fatimah Al-Taie, Fadhel S. Fadhel, Akram Al-Sabbagh

Department of Mathematics and Computer Applications, College of Sciences, Al-Nahrain University, Jadriya, Baghdad, Iraq

Abstract Ecological system depends on prey predator interaction and therefore, as a result, diseases may spread among prey or predator or both of them. In this work, the fuzzy logic-based systems are used to elaborate a prey-predator model to study the effect of varying in the inflection rate. Formulation of prey predator model using fuzzy logic is more realistic depiction of the phenomena, since the initial population estimates may not be precisely known in the real-life situation, therefore the initial conditions may also be considered as fuzzy. The dynamical behaviour of the fuzzy exploited system is studied by using the backstepping method. Some references working on prey-predator model, in which they used classical control schemes with a very long and complicated steps. While the proposed method of this paper simplifies the work steps. Numerical simulation results are presented to validate the theoretical analysis.

Keywords: Backstepping method, Prey-Predator Model, Fuzzy numbers, Ecological Systems, Lyapunov function.

Introduction

Recently, researchers have shown an increased interest in bio-mathematical systems. Several works have been published in this field based on usual, classical and complicated controller design methods for stabilizing such systems, see e.g., [1,2]. The scientific study of ecological systems utilizing theoretical approaches, such as mathematical models, computational simulations, and advanced data analysis is known as theoretical ecology. Effective models increase comprehension of the natural world by illustrating how species population dynamics are frequently dependent on fundamental biological conditions and processes.

In many applications, uncertainties are usually appeared, which lead authors to defined fuzzy differential equations with a derivative based on Hukuhara derivative and its generalization (more details can be found in [3,4]). One other concept that drag some focus is the predator-prey models with uncertainty in the initial populations of the predator and the prey. This model is solved numerically as differential equations with fuzzy initial conditions and the stability of the solutions is discussed in [5].

As mentioned before, stability of bio-mathematical systems is proposed in many previous works via several different methods. For example, the author in [6] considered a model for an infectious disease that affect the prey population, in which a local stability of the system at equilibrium points. On the other hand, a system with two competing species that is affected by harvesting and the presence of a predator is considered in [7]. The work given in [1] and [8] for stabilizing prey-predator model is defined mathematically as the set of nonlinear differential equations. The strategy involves an assessment into the existence, uniqueness, and boundedness of the model's solution, which establishes the local and global stability conditions of all possible equilibrium points. While here, in our work, we will use the backstepping method, to guarantee the stability of the mentioned system in simplest and abbreviated steps without needing the previous investigation requires.

The backstepping technique is initially suggested in [9,10], where it gives a recursive approach for system stabilization. In most circumstances, a conventional feedback linearization strategy for nonlinear

***For correspondence:**
btisam.kamil@nahrainuniv.edu.iq

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systems results in the cancellation of valuable nonlinearities. Backstepping design is more flexible than feedback linearization since it does not need the resultant input-output dynamics to be linear. The backstepping controller design approach is a powerful tool for creating controllers for huge systems. The main idea behind backstepping is to decompose a design problem on a higher order system into a number of sub-problems on lower order systems, and then iteratively employ certain states as "virtual controls" to derive the intermediate control laws using the control Lyapunov function. The benefits of backstepping control include the assurance of global of regional stability, [11]. Recently we proposed the backstepping method for stabilizing nonlinear fractional order partial differential equation (FPDE). The semi-discretized fractional order backstepping approach introduced to find the boundary controller function which stabilizes nonlinear FPDE by transformation it into an equivalent stable closed loop [12-14], while in [15] we used backstepping method to stabilize fractional order Riccati matrix differential equations with and without delay term. In this paper the backstepping controller design is applied to guarantee the stability of the crisp and fuzzy prey- predator model and the result is presented as a numerical simulation.

Preliminaries and Basic Concepts

In this section, some basic definitions, that are used through this article, are presented starting with basic concepts of fuzzy set theory. Moreover, a brief knowledge of the mathematical model of an epidemiological pre-predator system is discussed here as well.

Definition 1, [16]. A triangular fuzzy number $\widetilde{N}$ can be defined as a fuzzy set with membership function $\mu_{\widetilde{N}}: R \to [0,1]$, where R is the set of all real numbers and $\mu_{\widetilde{N}}$ should satisfies the conditions:

1. $\mu_{\widetilde{N}}$ is the upper semi continuous.
2. $\mu_{\widetilde{N}}$ is fuzzy convex, that is, $\mu_{\widetilde{N}}(\beta u + (1-\beta)v) \geq min\{\mu_{\widetilde{N}}(u), \mu_{\widetilde{N}}(v)\}$, for all $u, v \in R, \beta \in [0,1]$.
3. $\mu_{\widetilde{N}}$ is normal, that is there exist a unique $u_0 \in R$, such that $\mu_{\widetilde{N}}(u_0) = 1$.
4. The support set of $\mu_{\widetilde{N}}$ is the set $\{u \in R,\ N(u) > 0\}$ and its closure is compact.

Triangular fuzzy number may be abbreviated as $\widetilde{N} = [\rho_1, \rho_2, \rho_3]$ with $\rho_1 < \rho_2 < \rho_3$ and it satisfies the following [17]:

$$\mu_{\widetilde{N}}(u) = \begin{cases} 0 & \text{if } u \leq \rho_1 \quad or \quad u \geq \rho_3, \\ 1 & \text{if } u = \rho_2, \\ \text{Straight line} & \text{if } u \in [\rho_1, \rho_2] or\ u \in [\rho_2, \rho_3]. \end{cases}$$

Now, it may be worth mentioned that the interval fuzzy function method can be used to build a model for fuzzy differential equations [18]. At this stage, consider the linear interval-valued fuzzy differential equations

$$\tilde{y}' = \tilde{f}(t, y), \qquad \tilde{y}(t_0) = \tilde{y}_0$$

Where $\tilde{y} = [\underline{y}_0, \overline{y}_0]$, $\tilde{y}' = [\underline{y}', \overline{y}']$, $\tilde{f}(t, \mathrm{y}) = [\underline{f}(t, \mathrm{y}), \overline{f}(t, \mathrm{y})]$ and $\tilde{y}_0 = [\underline{y}_0, \overline{y}_0]$. The fuzziness may occur either through triangular or trapezoidal fuzzy numbers.

Thus, based on [19], by using of Hukuhara differentiability of the last fuzzy ordinary differential equations, one can arrive at $\min[\underline{y}', \overline{y}'] = \underline{f}(t, \mathrm{y})$ and $\max[\underline{y}', \overline{y}'] = \overline{f}(t, \mathrm{y})$.

Now, the starting conditions, $\underline{y}(t_0) = \underline{y}_0$ and $\overline{y}(t_0) = \overline{y}_0$, can lead to two situations as follows:

Case 1: If $\underline{y}'(t) \leq \overline{y}'(t)$, then the hypothetical differential equation gives:

$\underline{y}'(t) = \underline{f}(t, y)$ and $\overline{y}'(t) = \overline{f}(t, y)$,

with initial conditions $\underline{y}(t_0) = \underline{y}_0$, $\overline{y}(t_0) = \overline{y}_0$.

Case 2: If $\underline{y}'(t) \geq \overline{y}'(t)$, then the differential equations gives:

$\underline{y}'(t) = \overline{f}(t, \mathrm{y})$ and $\overline{y}'(t) = \underline{f}(t, \mathrm{y})$,

with initial conditions $\underline{y}(t_0) = \underline{y}_0, \overline{y}(t_0) = \overline{y}_0$.

Definition 2 (Control Lyapunov Function), [20]. Consider the nonlinear controlled dynamical system:

$$\dot{x} = f(x(t), \xi(t)),\ x(t_0) = x_0,\ t \geq 0, \tag{1}$$

where $x(t) \in D \subseteq R^n, t \geq 0$ is the state vector, D is an open set with $0 \in D, \xi(t) \in \zeta \subseteq R^m$ is the control input and $f: D \times \zeta \longrightarrow R^n$ satisfies $f(0,0) = (0,0)$. A smooth positive definite and radially unbounded function $V: D \longrightarrow R$ is called a control Lyapunov function of system (1) if:

$$\inf_{\xi \in \zeta} \left\{\frac{\partial v}{\partial x} f(x, \xi)\right\} < 0,\ \ x \neq 0.$$

Definition 3 (Ecological System or Ecosystem), [1]. An ecosystem is a population of living species that interact as a system with the non-living components of their habitat.
More precisely, ecology is the study of the interactions of organisms to each other and with their physical environment.

Definition 4 (Epidemiological Model), [1]. The epidemiological model is a mathematical model that describes the contagious diseases spreading in the population.

The Crisp Mathematical Model, [1]:
An eco-epidemiological mathematical system that consists the prey-predator model involving SI disease in prey and SIS disease in predator population is given in [1], where the reality problem is converted and represented as a dimensionless mathematical system as follows (for more details, see [1]):

$$\left.\begin{aligned} \frac{dx}{dt} &= x\left(1 - x - y - u_1 y - (u_2+u_3) - \frac{u_4 z}{u_5+x}\right), \\ \frac{dy}{dt} &= y\big(u_1 x - u_6 z - u_7 w - (u_3+u_8+u_9)\big) + u_2 x, \\ \frac{dz}{dt} &= z\left(\frac{u_{10}x}{u_5+x} + u_{11}y - u_{12}w - u_{13}\right) + u_{14}w, \\ \frac{dw}{dt} &= w\big(u_{15}y + u_{12}z - (u_{13}+u_{14})\big), \end{aligned}\right\} \tag{2}$$

where:

$$u_1 = \frac{\gamma_1 k}{r}, u_2 = \frac{\alpha_1}{r}, u_3 = \frac{d_1}{r}, u_4 = \frac{a_1}{r}, u_5 = \frac{b}{k}, u_6 = \frac{a_2 k}{r}, u_7 = \frac{a_3 k}{r}, u_8 = \frac{\beta}{r}, u_9 = \frac{h}{r},$$

$$u_{10} = \frac{e_1 a_1}{r}, u_{11} = \frac{e_2 a_2 k}{r}, u_{12} = \frac{\gamma_2 k}{r}, u_{13} = \frac{d_2}{r}, u_{14} = \frac{\alpha_2}{r}, u_{15} = \frac{e_3 a_3 k}{r}$$

Details discussion of the variables appeared in the above expression are of biological meaning is given in [1]. In the later section, stability of such systems will be studied and guaranteed via applying backstepping method rather than the classical methods which has been used in [1]. Also, as mentioned before, our proposed method ensures the stability of the system in more simple steps and less difficulties.

Backstepping Method for Crisp Prey-Predator Model

In order to solve and stabilize the crisp system (2), apply the backstepping approach by introducing control functions $\xi_1, \xi_2, \xi_3, \xi_4$ which are defined below. Hence, the crisp nonlinear system (2) will be modified to:

$$\left.\begin{aligned} \frac{dx}{dt} &= x\left(1 - x - y - u_1 y - (u_2+u_3) - \frac{u_4 z}{u_5+x}\right) + \xi_1, \\ \frac{dy}{dt} &= y\big(u_1 x - u_6 z - u_7 w - (u_3+u_8+u_9)\big) + u_2 x + \xi_2, \\ \frac{dz}{dt} &= z\left(\frac{u_{10}x}{u_5+x} + u_{11}y - u_{12}w - u_{13}\right) + u_{14}w + \xi_3, \\ \frac{dw}{dt} &= w\big(u_{15}y + u_{12}z - (u_{13}+u_{14})\big) + \xi_4. \end{aligned}\right\} \tag{3}$$

Firstly, Assume the stabilizability of the first equation of system (3)

$$\frac{dx}{dt} = x\left(1 - x - y - u_1 y - (u_2+u_3) - \frac{u_4 z}{u_5+x}\right) + \xi_1, \tag{4}$$

where y is defined as virtual controller. The Lyapunov function defined by $v_1(x) = \frac{1}{2}x^2$ will be considered. Now, assume the controller function $y = \eta_1(x)$. If $\eta_1(x) = 0$ and $\xi_1 = -J_1 x + x^2 + \frac{u_4 z}{u_5+x}x$, then $\dot{v}_1 = -(J_1 - 1 + u_2 + u_3)x^2$, where $J_1 \geq 1 - u_2 - u_3$ which means that $\dot{v}_1 < 0$. The recursive feedback ξ_1 and $\eta_1(x)$ that make Eq.(4) asymptotically stable. Function $\eta_1(x)$ is an estimating function for the controller y. The error between y and $\eta_1(x)$ is $\zeta_2 = y - \eta_1(x)$.
As a second stage, consider the (x, ζ_2) -subsystem given by:

$$\left.\begin{aligned} \frac{dx}{dt} &= -(J_1 - 1 + u_2 + u_3)x - (1+u_1)x\zeta_2, \\ \frac{d\zeta_2}{dt} &= \zeta_2\big(u_1 x - u_6 z - u_7 w - (u_3+u_8+u_9)\big) + u_2 x + \xi_2, \end{aligned}\right\} \tag{5}$$

where z is defined as a virtual controller in system (5). Suppose that $z = \eta_2(x, \zeta_2)$, then system (5) become asymptotically stable. Consider Lyapunov function:
$v_2(x, \zeta_2) = v_1(x) + \frac{1}{2}\zeta_2^2$.
If $\eta_2(x, \zeta_2) = 0$ and $\xi_2 = -u_2 x - (J_2 + u_1 x - u_7 w)\zeta_2$, where $J_2 \geq -(u_3 + u_8 + u_9)$. Then:
$\dot{v}_2 = -(J_1 - 1 + u_2 + u_3)x^2 - (J_2 + u_3 + u_8 + u_9)\zeta_2^2$,
which means that $\dot{v}_2 < 0$.
Therefore, system (5) is asymptotically stable via the selection of ξ_2 and $\eta_2(x, \zeta_2)$. Define the error

between z and $\eta_2(x,\zeta_2)$ as $\zeta_3 = z - \eta_2(x,\zeta_2)$.
As a third stage, consider the (x,ζ_2,ζ_3)-subsystem given by:

$$\left.\begin{aligned} \frac{dx}{dt} &= -(J_1 - 1 + u_2 + u_3)x - (1+u_1)x\zeta_2, \\ \frac{d\zeta_2}{dt} &= -(J_2 + u_3 + u_8 + u_9)\zeta_2 - u_6\zeta_2\zeta_3, \\ \frac{d\zeta_3}{dt} &= \zeta_3\left(\frac{u_{10}x}{u_5+x} + u_{11}\zeta_2 - u_{12}w - u_{13}\right) + u_{14}w + \xi_3, \end{aligned}\right\} \quad (6)$$

where w is regarded as a virtual controller in system (6). Suppose that $w = \eta_3(x,\zeta_2,\zeta_3)$, then system (6) is asymptotically stable. The Lyapunov function is now defined by:

$v_3(x,\zeta_2,\zeta_3) = v_2(x,\zeta_2) + \frac{1}{2}\zeta_3^2$.

If $\eta_3(x,\zeta_2,\zeta_3) = 0$, and $\xi_3 = -(J_3 + \frac{u_{10}x}{u_5+x} + u_{11}\zeta_2)\zeta_3$, where, $J_3 \geq -u_{13}$. Then:

$\dot{v}_3 = -(J_1 - 1 + u_2 + u_3)x^2 - (J_2 + u_3 + u_8 + u_9)\zeta_2^2 - (J_3 + u_{13})\zeta_3^2$,

which means that $\dot{v}_3 < 0$. The recursive control function is selected ξ_3 and $\eta_3(x,\zeta_2,\zeta_3) = 0$ that make the crisp controllable system (6) asymptotically stable.
As a final stage, represent the error between ζ_4 and $\eta_3(x,\zeta_2,\zeta_3)$ as $\zeta_4 = w - \eta_3(x,\zeta_2,\zeta_3)$, and consider the $(x,\zeta_2,\zeta_3,\zeta_4)$-subsystem given by:

$$\left.\begin{aligned} \frac{dx}{dt} &= -(J_1 - 1 + u_2 + u_3)x - (1+u_1)x\zeta_2, \\ \frac{d\zeta_2}{dt} &= -(J_2 + u_3 + u_8 + u_9)\zeta_2 - u_6\zeta_2\zeta_3, \\ \frac{d\zeta_3}{dt} &= -(J_3 + u_{13})\zeta_3 - u_{12}\zeta_3\zeta_4 + u_{14}\zeta_4, \\ \frac{d\zeta_4}{dt} &= \zeta_4\big(u_{15}\zeta_2 + u_{12}\zeta_3 - (u_{13} + u_{14})\big) + \xi_4, \end{aligned}\right\} \quad (7)$$

so similarly, the Lyapunov function is defined by:

$v_4(x,\zeta_2,\zeta_3,\zeta_4) = v_3(x,\zeta_2,\zeta_3) + \frac{1}{2}\zeta_4^2$,

and when $\xi_4 = -(J_4 + u_{15}\zeta_2 - u_{12}\zeta_3)\zeta_4$, where $J_4 \geq -(u_{13} + u_{14})$, then:

$\dot{v}_4 = -(J_1 - 1 + u_2 + u_3)x^2 - (J_2 + u_3 + u_8 + u_9)\zeta_2^2 - (J_3 + u_{13})\zeta_3^2 - (J_4 + u_{13} + u_{15})\zeta_4^2$,

the above choice of recursive feedback ξ_4 makes the system (7) asymptotically stable since $\dot{v}_4$ is negative definite function.
Therefore, system (2) is asymptotically stable with the backstepping controls:

$$\left.\begin{aligned} \xi_1 &= -J_1x + x^2 + \frac{u_4 z}{u_5+x}x, && \text{if } J_1 \geq 1 - u_2 - u_3, \\ \xi_2 &= -u_2x - (J_2 + u_1x - u_7w)\zeta_2, && \text{if } J_2 \geq -(u_3 + u_8 + u_9), \\ \xi_3 &= -\left(J_3 + \frac{u_{10}x}{u_5+x} + u_{11}\zeta_2\right)\zeta_3, && \text{if } J_3 \geq -u_{13}, \\ \xi_4 &= -(J_4 + u_{15}\zeta_2 - u_{12}\zeta_3)\zeta_4, && \text{if } J_4 \geq -(u_{13} + u_{14}). \end{aligned}\right\} \quad (8)$$

Numerical simulation has been carried out to solve system (2) with backstepping controls ξ_1,ξ_2,ξ_3,ξ_4 given by Eqs.(8), with initial states are taken, to be $(0.2, 0.4, 0.3, 0.7)$. The values of the designed parameters (J_1,J_2,J_3,J_4) are chosen as $(2,1,1,1)$, and the dimensionless parameters of system (2) are chosen as they are selected in [1]:

$$\begin{aligned} &u_1 = 0.5, u_2 = 0.2, u_3 = 0.3, u_4 = 0.4, u_5 = 0.4, u_6 = 0.6, u_7 = 0.5, u_8 = 0.2, \\ &u_9 = 0.5, u_{10} = 0.2, u_{11} = 0.1, , u_{12} = 0.3, u_{13} = 0.01\ , u_{14} = 0.002, u_{15} = 0.01. \end{aligned} \quad (9)$$

The obtained stabilized solutions of the crisp system (2) are illustrated in Figure 1.

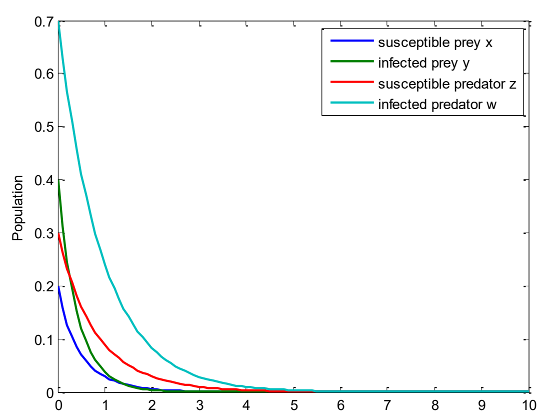


Figure 1. Time series of solutions of the crisp system (2) with backstepping controls given in Eqs. (8) with dimensionless parameters (9)

The simulation results show the performance of the controllers in regulations of the system state.

Backstepping Method for Fuzzy Prey-Predator Model

In reality, the eco-epidemiological system may contain some imprecise or vague parameters, which are the most case that simulate the real-life problem under consideration. Such cases may happen when some parameters have a range of occurrence. In this section, one parameter will be assumed ,for illustration purpose, to be fuzzy, which will affect the whole solution and the controllers of the system which will be fuzzy. In order to study this case, the effects of varying infection rate u_1 will be assumed to be, for simplicity of illustration only, a triangular fuzzy number. It also can be assumed to be some other type of fuzzy numbers such as trapezoidal fuzzy number, or Gaussian fuzzy number, etc., in which they satisfy $\mu_{\tilde{N}}(u_0) = 1$, [21]. The backstepping method is used to stabilize system (2) with some of its parameters are fuzzy numbers in addition to the initial conditions may be introduced also as fuzzy numbers.

$$\left.\begin{aligned}
\frac{d\tilde{x}}{dt} &= \tilde{x}\left(1-\tilde{x}-\tilde{y}-\tilde{u}_1\tilde{y}-(u_2+u_3)-\frac{u_4\tilde{z}}{u_5+\tilde{x}}\right),\\
\frac{d\tilde{y}}{dt} &= \tilde{y}\big(\tilde{u}_1\tilde{x}-u_6\tilde{z}-u_7\tilde{w}-(u_3+u_8+u_9)\big)+u_2\tilde{x},\\
\frac{d\tilde{z}}{dt} &= \tilde{z}\left(\frac{u_{10}\tilde{x}}{u_5+\tilde{x}}+u_{11}\tilde{y}-u_{12}\tilde{w}-u_{13}\right)+u_{14}\tilde{w},\\
\frac{d\tilde{w}}{dt} &= \tilde{w}\big(u_{15}\tilde{y}+u_{12}\tilde{z}-(u_{13}+u_{14})\big).
\end{aligned}\right\} \tag{10}$$

In order to stabilize this system, let $\tilde{x} = [\underline{x}, \overline{x}], \tilde{y} = [\underline{y}, \overline{y}], \tilde{z} = [\underline{z}, \overline{z}], \tilde{w} = [\underline{w}, \overline{w}]$ and $\tilde{u}_1 = [\underline{u}_1, \overline{u}_1]$ as the α-level sets corresponding to the fuzzy solution and parameters of system (10).

The first step in this approach is to decompose system (10) into two related subsystems. The first system is related to the lower solutions and the second one for the upper solutions of the original system. Then, in order to apply the backstepping method on system (10), let us introduce the controller functions $\xi_1, \xi_2, \xi_3, \xi_4$, and thus the following two situations follows:

Case 1: In this case the following subsystems are obtained:

$$\left.\begin{aligned}
\frac{d\underline{x}}{dt} &= \underline{x}\left(1-\overline{x}-\overline{y}-\overline{u}_1\overline{y}-(u_2+u_3)-\frac{u_4\overline{z}}{u_5+\overline{x}}\right)+\xi_1(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\underline{y}}{dt} &= \underline{y}\left(\underline{u}_1\underline{x}-u_6\overline{z}-u_7\overline{w}-(u_3+u_8+u_9)\right)+u_2\underline{x}+\xi_2(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\underline{z}}{dt} &= \underline{z}\left(\frac{u_{10}\underline{x}}{u_5+\underline{x}}+u_{11}\underline{y}-u_{12}\overline{w}-u_{13}\right)+u_{14}\underline{w}+\xi_3(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\underline{w}}{dt} &= \underline{w}\left(u_{15}\underline{y}+u_{12}\underline{z}-(u_{13}+u_{14})\right)+\xi_4(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),
\end{aligned}\right\} \tag{11}$$

and

$$\left.\begin{aligned}
\frac{d\overline{x}}{dt} &= \overline{x}\left(1-\underline{x}-\underline{y}-\underline{u}_1\underline{y}-(u_2+u_3)-\frac{u_4\underline{z}}{u_5+\underline{x}}\right)+\xi_1(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\overline{y}}{dt} &= \overline{y}\left(\overline{u}_1\overline{x}-u_6\underline{z}-u_7\underline{w}-(u_3+u_8+u_9)\right)+u_2\overline{x}+\xi_2(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\overline{z}}{dt} &= \overline{z}\left(\frac{u_{10}\overline{x}}{u_5+\overline{x}}+u_{11}\overline{y}-u_{12}\underline{w}-u_{13}\right)+u_{14}\overline{w}+\xi_3(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\overline{w}}{dt} &= \overline{w}\big(u_{15}\overline{y}+u_{12}\overline{z}-(u_{13}+u_{14})\big)+\xi_4(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}).
\end{aligned}\right\} \tag{12}$$

Now, consider the stability of the first equation of system (11)

$$\frac{d\underline{x}}{dt} = \underline{x}\left(1-\overline{x}-\overline{y}-\overline{u}_1\overline{y}-(u_2+u_3)-\frac{u_4\overline{z}}{u_5+\overline{x}}\right)+\xi_1(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}), \tag{13}$$

where $\underline{y}$ is defined as a virtual controller, and the Lyapunov function is defined by $v_1(\underline{x}) = \frac{1}{2}\underline{x}^2$, which has the time derivative:

$$\dot{v}_1 = \underline{x}\left(-\underline{y}-(J_1-1+u_2+u_3)\underline{x}\right). \tag{14}$$

Assume the controller function $\underline{y} = \eta_1(\underline{x})$. If $\eta_1(\underline{x}) = 0$ and $\xi_1 = \underline{x}\overline{x}+\underline{x}\overline{y}+\overline{u}_1\underline{x}\overline{y}+\frac{u_4\overline{z}}{u_5+\overline{x}}\,\underline{x}-\underline{y}-J_1\underline{x}$. Then $\dot{v}_1 = -(J_1-1+u_2+u_3)\underline{x}^2$, where $J_1 \geq 1-u_2-u_3$, which is negative definite function. It is notable that, the recursive feedback ξ_1 and $\eta_1(\underline{x})$ that make Eq.(13) asymptotically stable, where the function $\eta_1(\underline{x})$ is an estimating function when $\underline{y}$ is considered as a controller. The error between $\underline{y}$ and $\eta_1(\underline{x})$ is $\zeta_2 = \underline{y} - \eta_1(\underline{x})$.

Consider the $(\underline{x}, \zeta_2)$-subsystem:

$$\left.\begin{aligned}&\frac{d\underline{x}}{dt}=-(J_1-1+u_2+u_3)\underline{x}-\underline{\zeta_2},\\&\frac{d\underline{\zeta_2}}{dt}=\underline{\zeta_2}\left(\underline{u_1}\underline{x}-u_6\overline{z}-u_7\overline{w}-(u_3+u_8+u_9)\right)+u_2\underline{x}+\xi_2(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\end{aligned}\right\}\quad(15)$$

where $\underline{z}$ is considered as a virtual controller in system (15). Suppose that $\underline{z}=\eta_2(\underline{x},\underline{\zeta_2})$, then system (15) become asymptotically stable, by considering the Lyapunov function defined by $v_2(\underline{x},\underline{\zeta_2})=v_1(\underline{x})+\frac{1}{2}\underline{\zeta_2}^2$, in which $v_2(\underline{x},\underline{\zeta_2})$ time derivative is:

$$\dot{v}_2=-(J_1-1+u_2+u_3)\underline{x}^2+\underline{\zeta_2}\left(-(J_2+u_3+u_8+u_9)+\underline{z}\right).\quad(16)$$

If $\eta_2(\underline{x},\underline{\zeta_2})=0$ and $\xi_2=u_6\overline{z}\underline{\zeta_2}-\underline{u_1}\underline{x}\underline{\zeta_2}+u_7\overline{w}\underline{\zeta_2}-u_2\underline{x}-J_2\underline{\zeta_2}+\underline{z}$, $J_2\geq-(u_3+u_8+u_9)$, then $\dot{v}_2=-(J_1-1+u_2+u_3)\underline{x}^2-(J_2+u_3+u_8+u_9)\underline{\zeta_2}^2$, which is negative definite function. Then system (15) is asymptotically stable via choosing ξ_2 and $\eta_2(\underline{x},\underline{\zeta_2})$. Define the error between $\underline{z}$ and $\eta_2(\underline{x},\underline{\zeta_2})$ as $\underline{\zeta_3}=\underline{z}-\eta_2(\underline{x},\underline{\zeta_2})$ and consider the $(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$-subsystem:

$$\left.\begin{aligned}&\frac{d\underline{x}}{dt}=-(J_1-1+u_2+u_3)\underline{x}-\underline{\zeta_2},\\&\frac{d\underline{\zeta_2}}{dt}=-(J_2+u_3+u_8+u_9)\underline{\zeta_2}+\underline{\zeta_3},\\&\frac{d\underline{\zeta_3}}{dt}=\underline{\zeta_3}\left(\frac{u_{10}\underline{x}}{u_5+\underline{x}}+u_{11}\underline{\zeta_2}-u_{12}\overline{w}-u_{13}\right)+u_{14}\underline{w}+\xi_3(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\end{aligned}\right\}\quad(17)$$

where $\underline{w}$ as a virtual controller in system (17). Assume that if it is equal to $\eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$ and the Lyapunov function is defined by $v_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})=v_2(\underline{x},\underline{\zeta_2})+\frac{1}{2}\underline{\zeta_3}^2$, if $\eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})=0$ and $\xi_3=-J_3\underline{\zeta_3}-\frac{u_{10}\underline{x}}{u_5+\underline{x}}\underline{\zeta_3}-u_{11}\underline{\zeta_2}\underline{\zeta_3}+u_{12}\overline{w}\underline{\zeta_3}$, $J_3\geq-u_{13}$, then $\dot{v}_3=-(J_1-1+u_2+u_3)\underline{x}^2-(J_2+u_3+u_8+u_9)\underline{\zeta_2}^2-(J_3+u_{13})\underline{\zeta_3}^2$, which is negative definite function. The recursive control ξ_3 and $\eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$ make system (17) asymptotically stable. Now, define the error function $\underline{\zeta_4}$ as $\underline{\zeta_4}=\underline{w}-\eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$ and consider the $(\underline{x},\underline{\zeta_2},\underline{\zeta_3},\underline{\zeta_4})$-subsystem:

$$\left.\begin{aligned}&\frac{d\underline{x}}{dt}=-(J_1-1+u_2+u_3)\underline{x}-\underline{\zeta_2},\\&\frac{d\underline{\zeta_2}}{dt}=-(J_2+u_3+u_8+u_9)\underline{\zeta_2}+\underline{\zeta_3},\\&\frac{d\underline{\zeta_3}}{dt}=-(J_3+u_{13})\underline{\zeta_3}+u_{14}\underline{\zeta_4},\\&\frac{d\underline{\zeta_4}}{dt}=\underline{\zeta_4}\left(u_{15}\underline{\zeta_2}+u_{12}\underline{\zeta_3}-(u_{13}+u_{14})\right)+\xi_4(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\end{aligned}\right\}\quad(18)$$

with Lyapunov function defined by $v_4=v_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})+\frac{1}{2}\underline{\zeta_4}^2$. If $\xi_4=-J_4\underline{\zeta_4}-u_{15}\underline{\zeta_2}\underline{\zeta_4}-u_{12}\underline{\zeta_3}\underline{\zeta_4}$, $J_4\geq-(u_{13}+u_{14})$, then the time derivative of v_4 is:
$\dot{v}_4=-(J_1-1+u_2+u_3)\underline{x}^2-(J_2+u_3+u_8+u_9)\underline{\zeta_2}^2-(J_3+u_{13})\underline{\zeta_3}^2-(J_4+u_{13}+u_{14})\underline{\zeta_4}^2$.
Then the above recursive feedback ξ_4 which makes system (18) asymptotically stable, since $\dot{v}_4$ is a negative definite function.

Similarly, the control functions for the upper solutions $\overline{x},\overline{y},\overline{z}$ and $\overline{w}$ in system (12) are evaluated to be:

$$\left.\begin{aligned}&\xi_1=\underline{x}\overline{x}+\underline{y}\overline{x}+\underline{u_1}\underline{y}\overline{x}+\frac{u_4\underline{z}}{u_5+\underline{x}}\,\overline{x}-\overline{y}-J_1\overline{x},\\&\xi_2=u_6\underline{z}\overline{\zeta}_2-\overline{u}_1\overline{x}\overline{\zeta}_2+u_7\underline{w}\overline{\zeta}_2-u_2\overline{x}-J_2\overline{\zeta}_2+\overline{\zeta}_3,\\&\xi_3=-J_3\overline{\zeta}_3-\frac{u_{10}\overline{x}}{u_5+\overline{x}}\overline{\zeta}_3-u_{11}\overline{\zeta}_2\overline{\zeta}_3+u_{12}\underline{w}\overline{\zeta}_3,\\&\xi_4=-J_4\overline{\zeta}_4-u_{15}\overline{\zeta}_2\overline{\zeta}_4-u_{12}\overline{\zeta}_3\overline{\zeta}_4.\end{aligned}\right\}\quad(19)$$

Numerical simulation has been carried out to solve subsystems (11) and (12) with fuzzy initial conditions given as triangular fuzzy numbers and thus for $\alpha\in[0,1]$, the initial conditions can be represented using α-level sets, as:

$$\left.\begin{aligned}&\tilde{x}(0)\cong[\underline{x},\overline{x}]=[0.2\alpha,0.4-0.2\alpha],\\&\tilde{y}(0)\cong[\underline{y},\overline{y}]=[0.4\alpha,0.8-0.4\alpha],\\&\tilde{z}(0)\cong[\underline{z},\overline{z}]=[0.3\alpha,0.6-0.3\alpha],\\&\tilde{w}(0)\cong[\underline{w},\overline{w}]=[0.7\alpha,1.4-0.7\alpha],\end{aligned}\right\}\quad(20)$$

while the infection rate $\tilde{u}_1\cong[\underline{u}_1,\overline{u}_1]=[a+\alpha(b-a),c-\alpha(c-b)]$, where a,b,c are the parameters of the triangular fuzzy number defining $\tilde{u}_1$.

The values of the parameters $(J_1, J_2, J_3, J_4, \mathrm{a}, \mathrm{b}, \mathrm{c})$ are chosen as $(2, 1, 1, 1, 0, 0.7, 1.4)$, and the dimensionless parameters $u_2, u_3, \ldots, u_{15}$ as given in Eq.(9). Solving the subsystems (11) and (12) with α-levels equals to $0, 0.5$, and 1. The solutions are illustrated in Figures 2, 3 and 4, respectively.

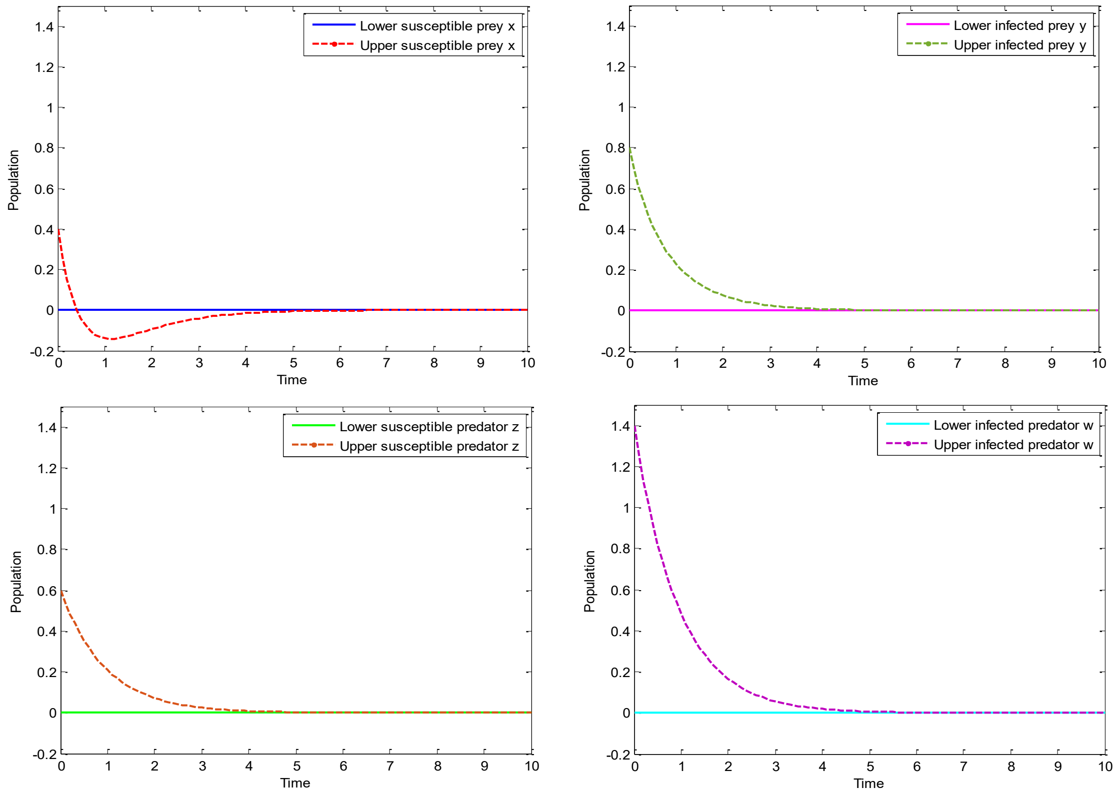
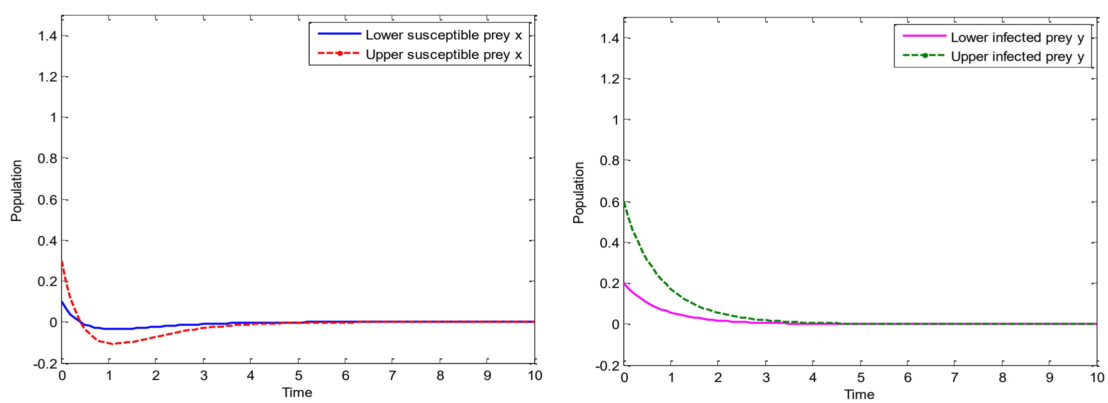


Figure 2. Time series of solution of systems (11) and (12) with $\alpha = 0$



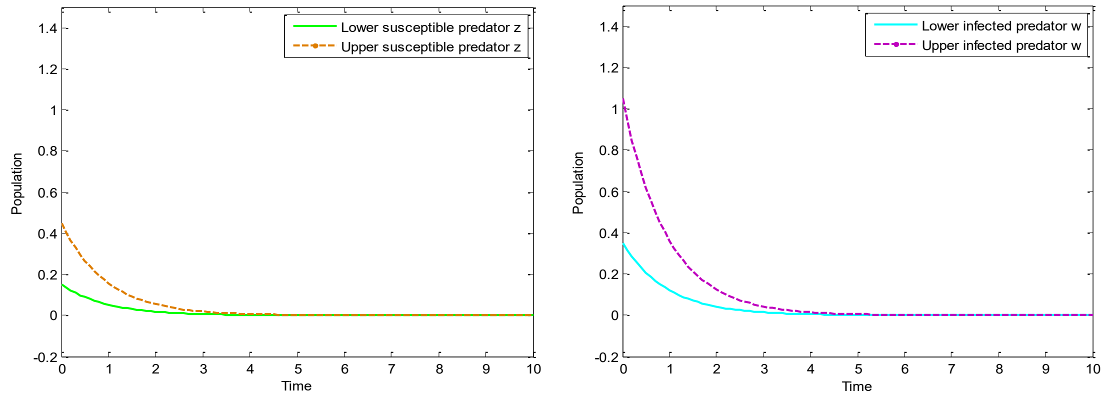


Figure 3. Time series of solution of systems (11) and (12) with α = 0.5

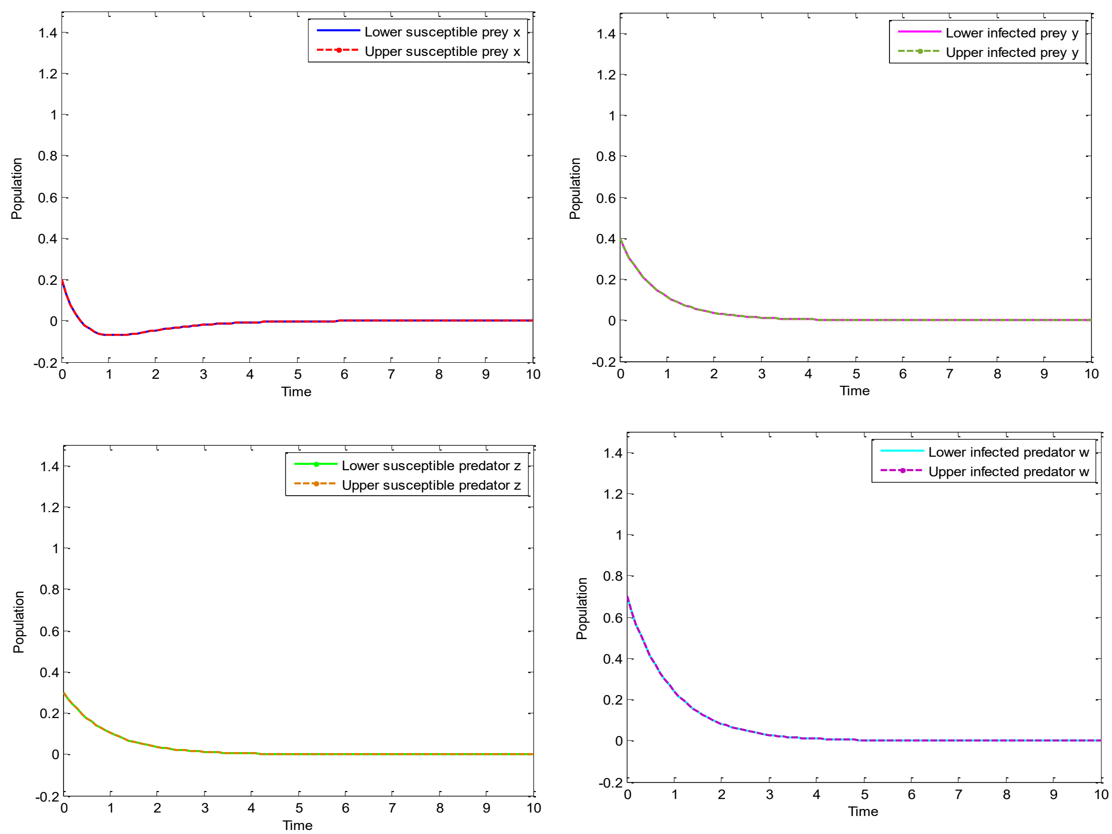


Figure 4. Time series of solution of systems (11) and (12) with α = 1

Case 2: In this case, the following subsystems are obtained:

$$\left.\begin{aligned}
\frac{d\underline{x}}{dt} &= \overline{x}\left(1-\underline{x}-\underline{y}-\underline{u}_1\underline{y}-(u_2+u_3)-\frac{u_4\underline{z}}{u_5+\underline{x}}\right)+\xi_1(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\underline{y}}{dt} &= \overline{y}\left(\overline{u}_1\overline{x}-u_6\underline{z}-u_7\underline{w}-(u_3+u_8+u_9)\right)+u_2\overline{x}+\xi_2(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\underline{z}}{dt} &= \overline{z}\left(\frac{u_{10}\overline{x}}{u_5+\overline{x}}+u_{11}\overline{y}-u_{12}\underline{w}-u_{13}\right)+u_{14}\overline{w}+\xi_3(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\underline{w}}{dt} &= \overline{w}\left(u_{15}\overline{y}+u_{12}\overline{z}-(u_{13}+u_{14})\right)+\xi_4(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),
\end{aligned}\right\} \quad (21)$$

and

$$\left.\begin{aligned}
\frac{d\overline{x}}{dt} &= \underline{x}\left(1-\overline{x}-\overline{y}-\overline{u}_1\overline{y}-(u_2+u_3)-\frac{u_4\overline{z}}{u_5+\overline{x}}\right)+\xi_1(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\overline{y}}{dt} &= \underline{y}\left(\underline{u}_1\underline{x}-u_6\overline{z}-u_7\overline{w}-(u_3+u_8+u_9)\right)+u_2\underline{x}+\xi_2(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\overline{z}}{dt} &= \underline{z}\left(\frac{u_{10}\underline{x}}{u_5+\underline{x}}+u_{11}\underline{y}-u_{12}\overline{w}-u_{13}\right)+u_{14}\underline{w}+\xi_3(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),\\
\frac{d\overline{w}}{dt} &= \underline{w}\left(u_{15}\underline{y}+u_{12}\underline{z}-(u_{13}+u_{14})\right)+\xi_4(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}).
\end{aligned}\right\} \quad (22)$$

Now, consider the stability of the first equation of system (21):

$$\frac{d\underline{x}}{dt} = \overline{x}\left(1-\underline{x}-\underline{y}-\underline{u}_1\underline{y}-(u_2+u_3)-\frac{u_4\underline{z}}{u_5+\underline{x}}\right)+\xi_1(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}), \quad (23)$$

where $\underline{y}$ is regarded as a virtual controller, and the Lyapunov function is defined by $v_1(\underline{x}) = \frac{1}{2}\underline{x}^2$. Assume the controller function $\underline{y} = \eta_1(\underline{x})$. If $\eta_1(\underline{x}) = 0$ and $\xi_1 = -\overline{x} + \underline{x}\overline{x} + (u_2+u_3)\overline{x} + \frac{u_4\underline{z}}{u_5+\underline{x}}\,\overline{x} - J_1\underline{x}$. Then $\dot{v}_1 = -J_1\underline{x}^2$, where $J_1 \geq 0$, which means that $\dot{v}_1 < 0$. Function $\eta_1(\underline{x})$ is an estimating function when $\underline{y}$ is considered as a controller. The error between $\underline{y}$ and $\eta_1(\underline{x})$ is $\underline{\zeta_2} = \underline{y} - \eta_1(\underline{x})$

Consider the $(\underline{x}, \underline{\zeta_2})$-subsystem:

$$\left.\begin{aligned}
\frac{d\underline{x}}{dt} &= -J_1\underline{x}-\overline{x}\underline{\zeta_2}-\underline{u}_1\overline{x}\underline{\zeta_2}\\
\frac{d\underline{\zeta_2}}{dt} &= \overline{y}\left(\overline{u}_1\overline{x}-u_6\underline{z}-u_7\underline{w}-(u_3+u_8+u_9)\right)+u_2\overline{x}+\xi_2\left(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}\right)
\end{aligned}\right\} \quad (24)$$

where $\underline{z}$ is defined as a virtual controller in system (24) which can be considered asymptotically stable system if $\underline{z}$ is equal to $\eta_2(\underline{x},\underline{\zeta_2})$. Now the Lyapunov function is defined by $v_2(\underline{x},\underline{\zeta_2}) = v_1(\underline{x}) + \frac{1}{2}\underline{\zeta_2}^2$.

If $\eta_2(\underline{x},\underline{\zeta_2}) = 0$ and $\xi_2 = -\overline{u}_1\overline{x}\,\overline{y} + u_7\underline{w}\overline{y} + (u_3+u_8+u_9)\overline{y} - u_2\bar{x} - J_2\underline{\zeta_2}$, $J_2 \geq 0$, then $\dot{v}_2 = -J_1\underline{x}^2 - J_2\underline{\zeta}_2^2$, which is negative definite function. The recursive control ξ_2 and $\eta_2(\underline{x},\underline{\zeta_2})$ make system (24) asymptotically stable. Define the error variable $\underline{\zeta_3} = \underline{z} - \eta_2(\underline{x},\underline{\zeta_2})$ and consider the $(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$-subsystem:

$$\left.\begin{aligned}
\frac{d\underline{x}}{dt} &= -J_1\underline{x}-\overline{x}\underline{\zeta_2}-\underline{u}_1\overline{x}\underline{\zeta_2},\\
\frac{d\underline{\zeta_2}}{dt} &= -J_2\underline{\zeta_2}-u_6\overline{y}\underline{\zeta_3},\\
\frac{d\underline{\zeta_3}}{dt} &= \overline{z}\left(\frac{u_{10}\overline{x}}{u_5+\overline{x}}+u_{11}\overline{y}-u_{12}\underline{w}-u_{13}\right)+u_{14}\overline{w}+\xi_3(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),
\end{aligned}\right\} \quad (25)$$

where $\underline{w}$ is considered as a virtual controller in system (25). Assume that if it is equal to $\eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$ and the Lyapunov function is defined by $v_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3}) = v_2(\underline{x},\underline{\zeta_2}) + \frac{1}{2}\underline{\zeta}_3^2$.

If $\eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3}) = 0$ and $\xi_3 = -J_3\underline{\zeta_3} - \frac{u_{10}\overline{x}}{u_5+\overline{x}}\overline{z} - u_{11}\overline{y}\,\overline{z} + u_{13}\overline{z} - u_{14}\overline{w}$, $J_3 \geq 0$, then $\dot{v}_3 = -J_1\underline{x}^2 - J_2\underline{\zeta}_2^2 - J_3\underline{\zeta}_3^2$, which is negative definite function. The recursive control ξ_3 and $\eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$ make system (25) asymptotically stable.

Now, define the error function $\underline{\zeta_4} = \underline{w} - \eta_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3})$ and consider the $(\underline{x},\underline{\zeta_2},\underline{\zeta_3},\underline{\zeta_4})$-subsystem:

$$\left.\begin{aligned}
\frac{d\underline{x}}{dt} &= -J_1\underline{x}-\overline{x}\underline{\zeta_2}-\underline{u}_1\overline{x}\underline{\zeta_2},\\
\frac{d\underline{\zeta_2}}{dt} &= -J_2\underline{\zeta_2}-u_6\overline{y}\underline{\zeta_3},\\
\frac{d\underline{\zeta_3}}{dt} &= -J_3\underline{\zeta_3}-u_{12}\overline{z}\underline{\zeta_4},\\
\frac{d\underline{\zeta_4}}{dt} &= \overline{w}\left(u_{15}\overline{y}+u_{12}\overline{z}-(u_{13}+u_{14})\right)+\xi_4(\underline{x},\overline{x},\underline{y},\overline{y},\underline{z},\overline{z},\underline{w},\overline{w}),
\end{aligned}\right\} \quad (26)$$

with Lyapunov function defined by $v_4 = v_3(\underline{x},\underline{\zeta_2},\underline{\zeta_3}) + \frac{1}{2}\underline{\zeta}_4^2$. If $\xi_4 = -J_4\underline{\zeta_4} - u_{15}\overline{y}\,\overline{w} - u_{12}\overline{z}\,\overline{w} + (u_{13}+u_{14})\overline{w}$, $J_4 \geq 0$, then the time derivative of v_4 is:

$\dot{v}_4 = -J_1\underline{x}^2 - J_2\underline{\zeta_2}^2 - J_3\underline{\zeta_3}^2 - J_4\underline{\zeta_4}^2$.

Then the above recursive feedback ξ_4 which makes system (26) asymptotically stable, since $\dot{v}_4$ is a negative definite function.

Similarly, the control functions for the upper solutions $\overline{x},\overline{y},\overline{z}$ and $\overline{w}$ in system (22) are evaluated to be:

$$\left.\begin{aligned}
\xi_1 &= -\underline{x} + \underline{x}\overline{x} + (u_2+u_3)\underline{x} + \frac{u_4\overline{z}}{u_5+\overline{x}}\,\underline{x} - J_1\overline{x},\\
\xi_2 &= -\underline{u}_1\underline{x}\,\underline{y} + u_7\underline{y}\overline{w} + (u_3+u_8+u_9)\underline{y} - u_2\underline{x} \;- J_2\overline{\zeta}_2,\\
\xi_3 &= -J_3\overline{\zeta}_3 - \frac{u_{10}\underline{x}}{u_5+\underline{x}}\underline{z} - u_{11}\underline{y}\,\underline{z} \;+ u_{13}\underline{z} - u_{14}\underline{w},\\
\xi_4 &= -J_4\overline{\zeta}_4 - u_{15}\underline{y}\,\underline{w} - u_{12}\underline{z}\,\underline{w} + (u_{13}+u_{14})\underline{w}.
\end{aligned}\right\} \tag{27}$$

Numerical simulation has been carried out to solve subsystems (21) and (22) with fuzzy initial conditions given in Eq. (20) for $\alpha \in [0,1]$. The values of the parameters $(J_1, J_2, J_3, J_4, \mathrm{a}, \mathrm{b}, \mathrm{c})$ are chosen as given in case 1, and the dimensionless parameters $u_2, u_3, \dots, u_{15}$ as given in Eq.(9). Solving the subsystems (21) and (22) with α-levels equals to $0, 0.5$ and 1.The solutions are illustrated in Figures 5, 6 and 7, respectively.

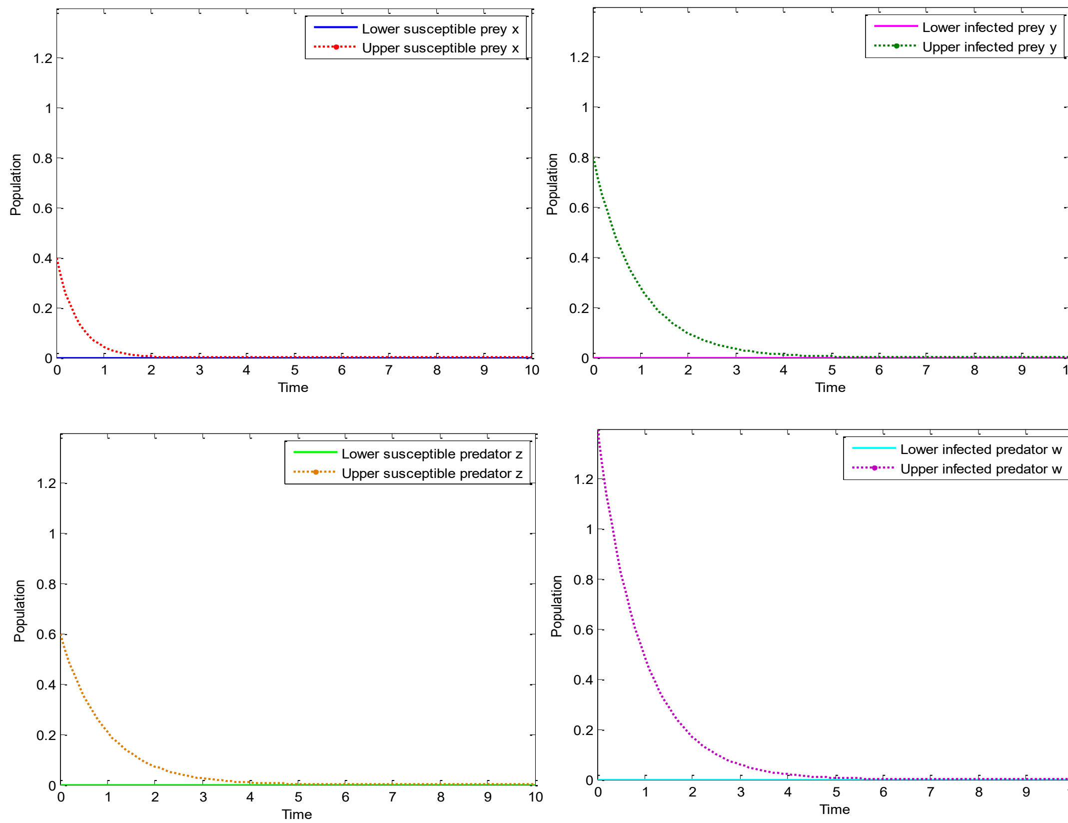


Figure 5. Time series of solution of systems (21) and (22) with α = 0

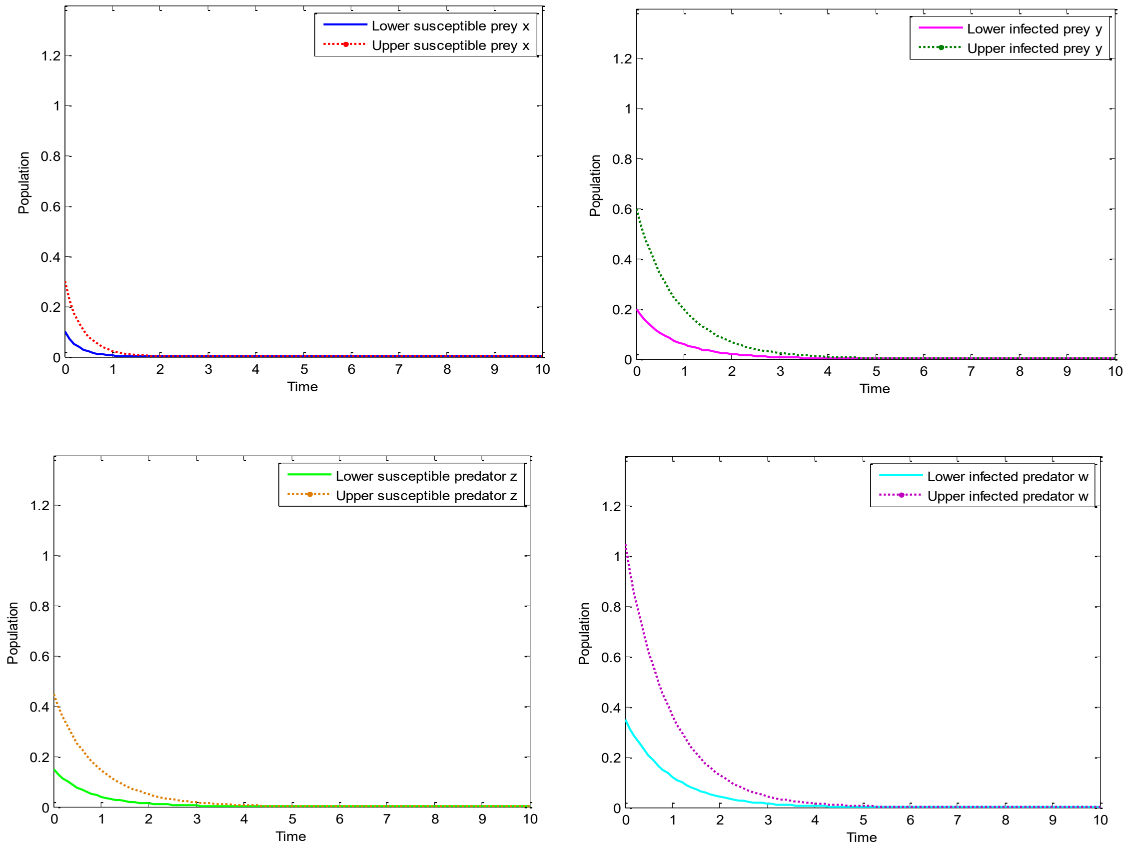
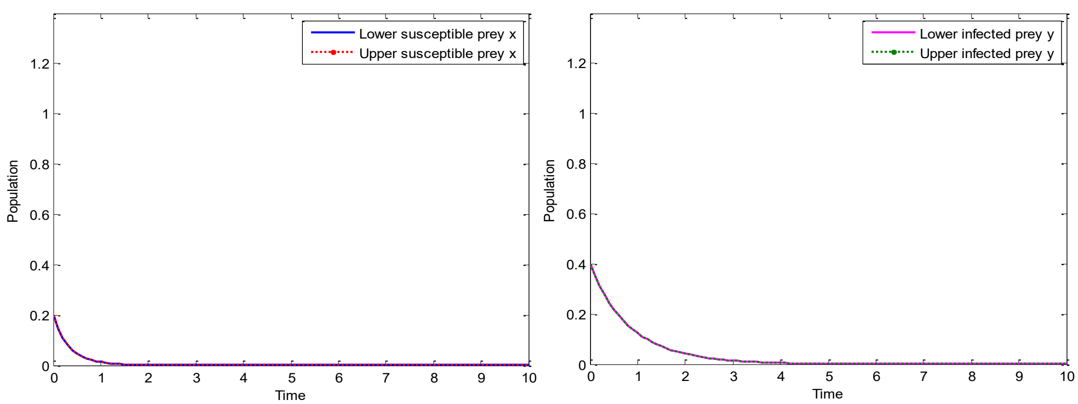


Figure 6. Time series of solution of systems (21) and (22) with $\alpha = 0.5$



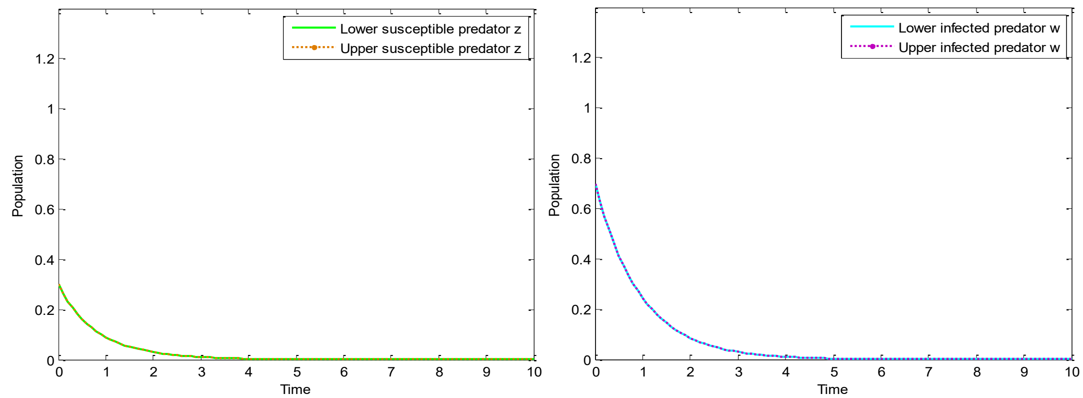


Figure 7. Time series of solution of systems (21) and (22) with $\alpha = 1$

From all above, the simulation results show the performance of the controller when using triangular fuzzy numbers. The solutions are asymptotically stable for different α-levels.
Also, other types of fuzzy numbers may be used, which depends on the nature of the fuzzy number appeared in system (10) and its related fuzzy initial conditions (20), such as using Gaussian fuzzy number that has the same behavior of triangular fuzzy number and has the membership function

$$\mu_{\tilde{N}}(u) = e^{-\frac{(u-a)}{b}} \tag{28}$$

where a and b are real numbers with $b \neq 0$. The difference between the triangular fuzzy number (given in Definition 1) and the Gaussian fuzzy number (28) is that, the first one has a linear membership function, while the second is an exponential function, i.e., nonlinear. But, both of them have the same behaviour, especially $\mu_{\tilde{N}}(a) = 1$.

Conclusions

In this work, an effective and more powerful approach than the methods used in previous works is presented here for stabilizing prey-predator model. Backstepping method has been utilized for that purpose. Lyapunov function is defined at each stage, and then the negativity of an overall Lyapunov function is ensured by proper selection of the control law. We propose a fuzzy prey- predator model simulating the reality. The initial conditions and the inflection rate are considered as triangular fuzzy numbers. We obtained biologically acceptable and asymptotically stable solutions when $\alpha < 1$, while if $\alpha = 1$, the solution equals to the exact or crisp solution, if we can find it. The results have been demonstrated via numerical simulation.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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