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A Multi-Criteria Generalised L-R Intuitionistic Fuzzy TOPSIS with CRITIC for River Water Pollution Classification

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Abstract A generalised L-R intuitionistic fuzzy numbers is an L-R intuitionistic fuzzy numbers that incorporates confidence level for both membership and non-membership functions. Therefore, this intuitionistic fuzzy number is suitable for classifying the river water pollution. This study aims to introduce the generalised L-R intuitionistic fuzzy numbers (GLRIFNs) which includes the membership and non-membership functions to classify the river water pollution using TOPSIS with CRITIC method. Due to the insufficient river data, this study has simulated the river data using the bootstrap method. This study had classified river water pollution for several rivers in Johor, Malaysia, namely Kim Kim River, Sayong River, Telor River, Pelepah River, and Bantang River from 2017 to 2021. The result shows that the Bantang River is the cleanest river, while the Kim Kim River is the most polluted river. The results proved that the GLRIFNs is quite a reliable method to classify river water pollution.

Keywords: Intuitionistic fuzzy numbers, L-R type, river pollution, CRITIC, TOPSIS.

1.0 Introduction

The most essential component of human life is water. Sufficient water supply is crucial to numerous socio-economic issues such as industrial production, agricultural activity, environmental protection, and local biodiversity [1]. According to the Department of Environment Malaysia (DOE Malaysia) [2], water resources in Malaysia come from rivers, lakes, and groundwater. The river has existed in almost every part of the country as a source of water supply for consumption. The river is one of the natural streams of flowing water. Rivers at their source are unpolluted, but as water flows downstream, the river receiving point and nonpoint pollutant sources negatively impact river water quality [3]. However, the discharge of numerous organic and inorganic contaminants into river systems has made water pollution a significant issue on a global scale. Therefore, classifying river water is required to classify river pollution accurately and effectively, allowing focused remediation actions and preserving water resources.

Water quality and water pollution are related concepts, but they refer to different aspects of water. Water quality describes the entire state of the water, considering all its chemical, physical, biological, and radiological characteristics. In contrast, water pollution mainly describes the degradation or contamination of water because of the presence of harmful substances. There are several methods evolved around the determination of river water pollution, such as the Water Quality Index (WQI) by the Department of Environment Malaysia, Fuzzy Comprehensive Evaluation (FCE), and Fuzzy Complex Index (FCI). In Malaysia, the WQI method was introduced almost 30 years ago by DOE Malaysia [2] to identify water quality level. The WQI method is a single value calculated for water quality level based on pools of some water quality parameters in a simple way [2]. For instance, complex information obtained from the water measurements is applied in the WQI method to obtain a single numeric expression [4]. However, this method cannot cater for the problem of uncertainty since this method relies on the single value of each parameter without considering the uncertainty in it. As for river water classification, according to DOE Malaysia [2], Malaysia divided the standard water quality into five classes which are Class I (very clean), Class II (clean), Class III (slightly polluted), Class IV (polluted), and Class V (very

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polluted). The water quality class used by DOE Malaysia will be used in this study to classify river water pollution.

The FCE method is one of the fuzzy approach methods to classify river water pollution. The FCE method applies fuzzy set theory in a fuzzy decision environment by combining qualitative appraisal and quantitative appraisal with multiple criteria [5, 6]. Jin-yao and Yuan-feng [7] applied the FCE method to Water Saving Irrigation System (WSIS) in Pingxiang, Guangxi, China; that contains six essential factors, including qualitative and quantitative indexes and claimed that the purpose of a comprehensive evaluation is to choose the most optimum plan used by decision-makers. Besides, Yang [8] used the FCE method to classify water quality and, at the same time, classify the water pollution level by observing Chaohu Lake's genetic toxicity in 2016 and evaluating the water quality of the lake. Yang [8] also claimed that the FCE method could implement simple analysis, decision-making, and evaluate those qualitative appraisals into the quantitative appraisal. Other than that, the FCI method can also classify the water quality and water pollution level. Zhu and Hu [9] introduced the FCI method to establish a comprehensive water quality index and investigate the trend in water quality in Donghu Lake, China, by classifying the river water. In summary, fuzzy approaches are already among the popular solutions for classifying river water based on the concept of fuzzy set theory. However, early classification of river water pollution is required to enable the river to be treated aptly with complete information, thus reducing the risk of river water pollution.

Multi-criteria decision-making (MCDM) is used to determine the best alternative by considering multiple criteria in the selection process [10]. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one of the MCDM methods that can solve the problem of classification of river water pollution by classifying the river pollution. In implementing the TOPSIS method in an intuitionistic fuzzy environment, Gautam *et al*. [11] introduced intuitionistic fuzzy TOPSIS using a ranking method based on the circumcentre of centroids for the selection of software engineers in software company. Besides, Memari *et al*. [12] used intuitionistic fuzzy TOPSIS for sustainable supplier selection, while Daneshvar Rouyendegh *et al*. [13] used intuitionistic fuzzy TOPSIS for site selection of wind power plants in Turkey. A few researchers incorporate Criteria Importance Through Intercriteria Correlation (CRITIC) with TOPSIS. CRITIC is used to determine the objective weights of criteria in decision-making issues. For example, Rostamzadeh *et al*. [14] used the fuzzy TOPSIS-CRITIC method to determine the most dominant sub-criteria for sustainable supply chain risk management. Next, Alipour-Bashary *et al*. [15] have used a hybrid method of fuzzy fault tree analysis and fuzzy CRITIC-TOPSIS for identifying, analysing, and evaluating the risks in building demolition operations. Additionally, Asante *et al*. [16] also used the CRITIC-fuzzy TOPSIS method to assess renewable energy barriers and prioritise renewable energy adoption in Ghana.

There are several methods to cater for the problem of classification of river water pollution; however, the current method does not consider the uncertainty and confidence level of the data. Therefore, the purpose of this study is to introduce the generalised L-R intuitionistic fuzzy numbers (GLRIFNs) with complete information, which includes the membership and non-membership functions to classify the river water pollution using TOPSIS with CRITIC method. The proposed procedure has the advantages of providing a comprehensive consideration of uncertainty of the data where the utilization of membership and the non-membership functions give a better representation of human evaluation process. Furthermore, the inclusion of confidence level values will give additional dimension of information in the evaluation process related to the judgment behaviour of the decision makers. This paper is organised in the following manner. The introduction and background of the study are discussed in Section 1. In Section 2, the definition of fuzzy set, intuitionistic fuzzy set, and membership function of L-R intuitionistic fuzzy numbers are given; meanwhile, Section 3 discussed the proposed method of GLRIFNs and its properties such as the membership and non-membership functions of GLRIFNs, alpha-cut and beta-cut, and Euclidean distance. In Section 4, the implementation of the proposed method with TOPSIS and CRITIC is discussed. In contrast, Section 5 discusses the case study of river water pollution using generalised trapezoidal L-R intuitionistic fuzzy TOPSIS (GTrLRIF TOPSIS) with CRITIC. Section 6 discusses the result and discussion, and finally, Section 7 concludes this study.

2.0 Preliminaries

In this section, definition of fuzzy sets, intuitionistic fuzzy sets, and membership function of L-R intuitionistic fuzzy numbers are given as follows:

Definition 1 ***[17]*** *Let X be the universe of discourse and A represent any fuzzy set on the universe X. A fuzzy set A on the universe X is a set defined by a membership function μ_A representing a mapping $\mu_A : X \to [0,1]$ and denoted as:*

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\}. \quad (2.1)$$

Definition 2 ***[18]*** *An intuitionistic fuzzy set considers two functions or uncertainties which are membership degree and non-membership degree. An intuitionistic fuzzy set A in a finite set X is denoted as:*

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\} \quad (2.2)$$

where $\mu_A(x), v_A(x) : X \to [0,1]$ are the membership function and non-membership function of an element x in a finite set X with the condition:

$$0 \le \mu_A(x) + v_A(x) \le 1. \quad (2.3)$$

Definition 3 ***[19]*** *An L-R intuitionistic fuzzy set A is an intuitionistic fuzzy set A that can be described as a trapezoidal L-R intuitionistic fuzzy numbers of the real line R for $A = (m_a, n_a; m_a', n_a'; l_a, r_a; l_a', r_a')_{LR}$. The membership functions of trapezoidal L-R intuitionistic fuzzy numbers are defined as follows:*

$$\mu_A(x) = \begin{cases} L\left(\dfrac{m_a - x}{l_a}\right) & ; \quad m_a - l_a \le x \le m_a \\ 1 & ; \quad m_a \le x \le n_a \\ R\left(\dfrac{x - n_a}{r_a}\right) & ; \quad n_a \le x \le n_a + r_a \end{cases}$$

$$v_A(x) = \begin{cases} 1 - L\left(\dfrac{m_a' - x}{l_a'}\right) & ; \quad m_a' - l_a' \le x \le m_a' \\ 0 & ; \quad m_a' \le x \le n_a' \\ 1 - R\left(\dfrac{x - n_a'}{r_a'}\right) & ; \quad n_a' \le x \le n_a' + r_a' \end{cases} \quad (2.4)$$

3.0 Generalised L-R Intuitionistic Fuzzy Numbers

This section will give some basic definitions such as the generalised L-R intuitionistic fuzzy numbers (GLRIFNs), alpha-cut and beta-cut of GLRIFNs, and Euclidean distance of generalised trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs).

3.1 Membership and Non-membership Functions of Generalised L-R Intuitionistic Fuzzy Numbers

Definition 4 *A generalised L-R intuitionistic fuzzy number is called a generalised trapezoidal L-R intuitionistic fuzzy number $A = (m_a, n_a; m_a', n_a'; l_a, r_a; l_a', r_a'; h_a; h_a')_{LR}$ defined by a membership and non-membership functions $A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\}$ with the condition $0 \le h_a + h_a' \le 1$ where*

$$\mu_A(x)=\begin{cases} h_a\cdot L\left(\dfrac{m_a-x}{l_a}\right) & ; \quad -\infty\le x\le m_a \\ h_a & ; \quad m_a\le x\le n_a \\ h_a\cdot R\left(\dfrac{x-n_a}{r_a}\right) & ; \quad n_a\le x\le +\infty \end{cases}$$

$$v_A(x)=\begin{cases} 1-(1-h_a')\cdot L\left(\dfrac{m_a'-x}{l_a'}\right) & ; \quad -\infty\le x\le m_a' \\ h_a' & ; \quad m_a'\le x\le n_a' \\ 1-(1-h_a')\cdot R\left(\dfrac{x-n_a'}{r_a'}\right) & ; \quad n_a'\le x\le +\infty \end{cases} \tag{3.1}$$

such that $m_a,n_a,m_a',n_a',l_a,r_a,l_a',r_a'\in\mathbb{R}$, $m_a\le n_a$, $m_a'\le n_a'$, $h_a\in(0,1]$, *and* $h_a'\in[0,1)$.

An L-R intuitionistic fuzzy numbers with confidence level is the GLRIFNs with triangular and/or trapezoidal form. The GTrLRIFNs is denoted as $A=(m_a,n_a;m_a',n_a';l_a,r_a;l_a',r_a';h_a;h_a')_{LR}$ where m_a and n_a are the core of membership degree, m_a' and n_a' are the core of non-membership degree, l_a and r_a are the left spread and right spread of the membership function μ_A respectively, while l_a' and r_a' are the left spread and right spread of the non-membership function v_A respectively. The functions L and R denote the left and right reference functions of μ_A and v_A respectively. Hence, the GTrLRIFNs also can be denoted as $A=(m_a-l_a,m_a,n_a,n_a+r_a;m_a'-l_a',m_a',n_a',n_a'+r_a';h_a;h_a')_{LR}$ and the graph of GTrLRIFNs is shown as Figure 1.

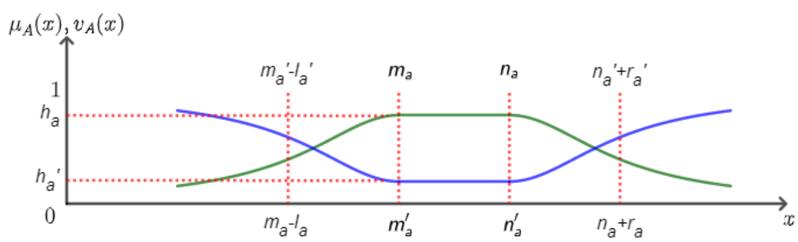

Figure 1. Graph of a Generalised Trapezoidal L-R Intuitionistic Fuzzy Number

The values of h_a and h_a' may represent the values of confidence level for membership and non-membership functions respectively, such that $h_a:X\to(0,1]$ and $h_a':X\to[0,1)$; $0\le h_a+h_a'\le 1$. The GLRIFNs is called a GTrLRIFNs if it satisfies the following properties:

i. μ_A and v_A are a continuous mapping from the universe of discourse X to $[0,1]$
ii. $\mu_A(x)=0$ for $x<m_a-l_a$ and $x>n_a+r_a$, $v_A(x)=1$ for $x<m_a'-l_a'$ and $x>n_a'+r_a'$
iii. $\mu_A^L(x)$ and $v_A^R(x)$ are monotonic increasing for $x\in[m_a-l_a,m_a;n_a',n_a'+r_a']$
iv. $\mu_A(x)=h_a$ for $x\in[m_a,n_a]$ and $v_A(x)=h_a'$ for $x\in[m_a',n_a']$
v. $\mu_A^R(x)$ and $v_A^L(x)$ are monotonic decreasing for $x\in[n_a,n_a+r_a;m_a'-l_a',m_a']$
vi. $L(0)=R(0)=1$ and $L(1)=R(1)=0$ for left and right reference functions
vii. $L(x)<1,\forall x>0$ and $R(x)<1,\forall x>0$
viii. $L(x)>0,\forall x<1$ and $R(x)>0,\forall x<1$.

For $m_a = n_a$ and $m_a' = n_a'$, GLRIFNs becomes a generalised triangular L-R intuitionistic fuzzy numbers (GTLRIFNs) denoted as $A = (m_a; m_a'; l_a, r_a; l_a', r_a'; h_a; h_a')_{LR}$. Choosing *L* and *R* reference functions is a subjective matter depends on the problem under investigation. Some examples of *L* and *R* reference functions are as follows [20]:

i. $y = \max(0, 1-|x|)$
ii. $y = \max(0, 1-x^2)$
iii. $y = e^{-|x|}$
iv. $y = e^{-x^2}$
v. $y = \dfrac{1}{1+|x|}$
vi. $y = \dfrac{1}{1+x^2}$
vii. $y = 1$ in $[-1, +1]$, $y = 0$ otherwise.

3.2 Alpha-cut and Beta-cut of Generalised L-R Intuitionistic Fuzzy Numbers

The concept of alpha-cut ($\alpha - \text{cut}$) and beta-cut ($\beta - \text{cut}$) plays a crucial part in the relationship between GLRIFNs and crisp numbers. It can be portrayed as the connection between GLRIFNs and crisp numbers [21]. The $\alpha - \text{cut}$ of a GLRIFNs is a crisp numbers of set A containing all elements of membership values, while the $\beta - \text{cut}$ of a GLRIFNs is a crisp numbers of set A containing all elements of non-membership values.

Proposition 1. The $\alpha - \text{cut}$ $A^{\alpha} = \left[A_L^{\alpha}, A_R^{\alpha}\right]$ and $\beta - \text{cut}$ $A^{\beta} = \left[A_L^{\beta}, A_R^{\beta}\right]$ of GTrLRIFNs can be expressed as follows:

$$A^{\alpha} = \begin{cases} A_L^{\alpha} = m_a - l_a L^{-1}\left(\dfrac{\alpha}{h_a}\right) & ; \quad \alpha \in (0, h_a] \\ A_R^{\alpha} = n_a + r_a R^{-1}\left(\dfrac{\alpha}{h_a}\right) & ; \quad \alpha \in (0, h_a] \end{cases} \tag{3.2}$$

$$A^{\beta} = \begin{cases} A_L^{\beta} = m_a' - l_a' L^{-1}\left(\dfrac{1-\beta}{1-h_a'}\right) & ; \quad \beta \in [h_a', 1) \\ A_R^{\beta} = n_a' + r_a' R^{-1}\left(\dfrac{1-\beta}{1-h_a'}\right) & ; \quad \beta \in [h_a', 1) \end{cases} \tag{3.3}$$

Proof. For $\alpha \in (0, h_a]$,

$$\begin{aligned} \mu_A(x) \geq \alpha &\Rightarrow h_a \cdot L\left(\frac{m_a - x}{l_a}\right) \geq \alpha, h_a \cdot R\left(\frac{x - n_a}{r_a}\right) \geq \alpha \\ &\Rightarrow x \geq m_a - l_a L^{-1}\left(\frac{\alpha}{h_a}\right), x \leq n_a + r_a R^{-1}\left(\frac{\alpha}{h_a}\right) \\ &\Rightarrow m_a - l_a L^{-1}\left(\frac{\alpha}{h_a}\right) \leq x \leq n_a + r_a R^{-1}\left(\frac{\alpha}{h_a}\right) \\ &\Rightarrow A^{\alpha} = \left[m_a - l_a L^{-1}\left(\frac{\alpha}{h_a}\right), n_a + r_a R^{-1}\left(\frac{\alpha}{h_a}\right)\right]. \end{aligned}$$

While, for $\beta \in [h_a', 1)$,

$$\begin{aligned}
v_A(x)\le\beta &\Rightarrow 1-(1-h_a')\cdot L\left(\frac{m_a'-x}{l_a'}\right)\le\beta, 1-(1-h_a')\cdot R\left(\frac{x-n_a'}{r_a'}\right)\le\beta\\
&\Rightarrow x\ge m_a'-l_a'L^{-1}\left(\frac{1-\beta}{1-h_a'}\right), x\le n_a'+r_a'R^{-1}\left(\frac{1-\beta}{1-h_a'}\right)\\
&\Rightarrow m_a'-l_a'L^{-1}\left(\frac{1-\beta}{1-h_a'}\right)\le x\le n_a'+r_a'R^{-1}\left(\frac{1-\beta}{1-h_a'}\right)\\
&\Rightarrow A^{\beta}=\left[m_a'-l_a'L^{-1}\left(\frac{1-\beta}{1-h_a'}\right), n_a'+r_a'R^{-1}\left(\frac{1-\beta}{1-h_a'}\right)\right].
\end{aligned}$$

Then, its $\alpha-\text{cut}$ and $\beta-\text{cut}$ are given by $A^{\alpha}=\left[A_L^{\alpha},A_R^{\alpha}\right]=\left[m_a-l_aL^{-1}\left(\frac{\alpha}{h_a}\right), n_a+r_aR^{-1}\left(\frac{\alpha}{h_a}\right)\right]$ and $A^{\beta}=\left[A_L^{\beta},A_R^{\beta}\right]=\left[m_a'-l_a'L^{-1}\left(\frac{1-\beta}{1-h_a'}\right), n_a'+r_a'R^{-1}\left(\frac{1-\beta}{1-h_a'}\right)\right]$, $\forall\alpha,\beta\in[0,1]$.

For $m_a=n_a$ and $m_a'=n_a'$, the $\alpha-\text{cut}$ and $\beta-\text{cut}$ of generalised triangular L-R intuitionistic fuzzy numbers are $A^{\alpha}=\left[A_L^{\alpha},A_R^{\alpha}\right]=\begin{bmatrix}m_a-l_aL^{-1}\left(\frac{\alpha}{h_a}\right),\\ m_a+r_aR^{-1}\left(\frac{\alpha}{h_a}\right)\end{bmatrix}$ and $A^{\beta}=\left[A_L^{\beta},A_R^{\beta}\right]=\begin{bmatrix}m_a'-l_a'L^{-1}\left(\frac{1-\beta}{1-h_a'}\right),\\ m_a'+r_a'R^{-1}\left(\frac{1-\beta}{1-h_a'}\right)\end{bmatrix}$, $\forall\alpha,\beta\in[0,1]$.

3.3 Euclidean Distance of Generalised Trapezoidal L-R Intuitionistic Fuzzy Numbers

Definition 5 *Let A and B be two generalized trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs) $A=(m_a,n_a;m_a',n_a';l_a,r_a;l_a',r_a';h_a;h_a')_{LR}$ and $B=(m_b,n_b;m_b',n_b';l_b,r_b;l_b',r_b';h_b;h_b')_{LR}$ or in the form $A=\begin{pmatrix}m_a-l_a,m_a,n_a,n_a+r_a;\\ m_a'-l_a',m_a',n_a',n_a'+r_a';h_a;h_a'\end{pmatrix}_{LR}$ and $B=\begin{pmatrix}m_b-l_b,m_b,n_b,n_b+r_b;\\ m_b'-l_b',m_b',n_b',n_b'+r_b';h_b;h_b'\end{pmatrix}_{LR}$. The distance between A and B can be calculated if $L_A(x)=L_B(x)$ and $R_A(x)=R_B(x)$ for the left and right reference functions respectively.*

$$d^{E}_{GTrLRIFNs}(A,B)=\sqrt{\frac{1}{8}\begin{bmatrix}\left|(m_b-l_b)-(m_a-l_a)\right|^2+\left(\left|m_b-m_a\right|^2+\left|h_b-h_a\right|^2\right)+\\ \left(\left|n_b-n_a\right|^2+\left|h_b-h_a\right|^2\right)+\left|(n_b+r_b)-(n_a+r_a)\right|^2+\\ \left|(m_b'-l_b')-(m_a'-l_a')\right|^2+\left(\left|m_b'-m_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\\ \left(\left|n_b'-n_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\left|(n_b'+r_b')-(n_a'+r_a')\right|^2\end{bmatrix}} \quad (3.4)$$

The distance measure of $d^{E}_{GTrLRIFNs}(A,B)$ between A and B satisfies the following propositions.

Proposition 2 If both A and B are GTrLRIFNs, then the distance measurement $d^{E}_{GTrLRIFNs}(A,B)$ is identical to the Euclidean distance.

Proof Suppose that both $A=(m_a-l_a,m_a,n_a,n_a+r_a;m_a'-l_a',m_a',n_a',n_a'+r_a';h_a;h_a')_{LR}$ and $B=(m_b-l_b,m_b,n_b,n_b+r_b;m_b'-l_b',m_b',n_b',n_b'+r_b';h_b;h_b')_{LR}$ are two GTrLRIFNs, then let $m_a-l_a=m_a=n_a=n_a+r_a=m_a'-l_a'=m_a'=n_a'=n_a'+r_a'=\overline{A}$, $m_b-l_b=m_b=n_b=n_b+r_b=m_b'-l_b'=m_b'=n_b'=n_b'+r_b'=\overline{B}$, $h_a=h_b=h=1$, and $h_a'=h_b'=h'=0$. The distance measurement $d^{E}_{GTrLRIFNs}(A,B)$ can be calculated as

$$
\begin{aligned}
d^{E}_{GTrLRIFNs}(A,B) &= \sqrt{\frac{1}{8}\begin{bmatrix} \left|(m_b-l_b)-(m_a-l_a)\right|^2+\left(\left|m_b-m_a\right|^2+\left|h_b-h_a\right|^2\right)+ \\ \left(\left|n_b-n_a\right|^2+\left|h_b-h_a\right|^2\right)+\left|(n_b+r_b)-(n_a+r_a)\right|^2+ \\ \left|(m_b'-l_b')-(m_a'-l_a')\right|^2+\left(\left|m_b'-m_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+ \\ \left(\left|n_b'-n_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\left|(n_b'+r_b')-(n_a'+r_a')\right|^2 \end{bmatrix}} \\
&= \sqrt{\frac{1}{8}\begin{bmatrix} \left|\overline{B}-\overline{A}\right|^2+\left(\left|\overline{B}-\overline{A}\right|^2+\left|h-h\right|^2\right)+ \\ \left(\left|\overline{B}-\overline{A}\right|^2+\left|h-h\right|^2\right)+\left|\overline{B}-\overline{A}\right|^2+ \\ \left|\overline{B}-\overline{A}\right|^2+\left(\left|\overline{B}-\overline{A}\right|^2+\left|h'-h'\right|^2\right)+ \\ \left(\left|\overline{B}-\overline{A}\right|^2+\left|h'-h'\right|^2\right)+\left|\overline{B}-\overline{A}\right|^2 \end{bmatrix}} \\
&= \sqrt{\frac{1}{8}\left[8\left|\overline{B}-\overline{A}\right|^2+2\left|1-1\right|^2+2\left|0-0\right|^2\right]} \\
&= \sqrt{\left|\overline{B}-\overline{A}\right|^2}=\left|\overline{B}-\overline{A}\right|=\left|\overline{A}-\overline{B}\right|.
\end{aligned}
$$

Proposition 3 Two GTrLRIFNs $A=B$ if and only if $d^{E}_{GTrLRIFNs}(A,B)=0$.

Proof Let $A=\begin{pmatrix} m_a-l_a, m_a, n_a, n_a+r_a; \\ m_a'-l_a', m_a', n_a', n_a'+r_a'; h_a; h_a' \end{pmatrix}_{LR}$ and $B=\begin{pmatrix} m_b-l_b, m_b, n_b, n_b+r_b; \\ m_b'-l_b', m_b', n_b', n_b'+r_b'; h_b; h_b' \end{pmatrix}_{LR}$ be two GTrLRIFNs. If $A=B$, then $m_a-l_a=m_b-l_b$, $m_a=m_b$, $n_a=n_b$, $n_a+r_a=n_b+r_b$, $m_a'-l_a'=m_b'-l_b'$, $m_a'=m_b'$, $n_a'=n_b'$, $n_a'+r_a'=n_b'+r_b'$, $h_a=h_b$, and $h_a'=h_b'$. Thus, the distance between A and B is

$$
\begin{aligned}
d^{E}_{GTrLRIFNs}(A,B) &= \sqrt{\frac{1}{8}\begin{bmatrix} \left|(m_b-l_b)-(m_a-l_a)\right|^2+\left(\left|m_b-m_a\right|^2+\left|h_b-h_a\right|^2\right)+ \\ \left(\left|n_b-n_a\right|^2+\left|h_b-h_a\right|^2\right)+\left|(n_b+r_b)-(n_a+r_a)\right|^2+ \\ \left|(m_b'-l_b')-(m_a'-l_a')\right|^2+\left(\left|m_b'-m_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+ \\ \left(\left|n_b'-n_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\left|(n_b'+r_b')-(n_a'+r_a')\right|^2 \end{bmatrix}} \\
&= \sqrt{\frac{1}{8}\begin{bmatrix} \left|(m_b-l_b)-(m_b-l_b)\right|^2+\left(\left|m_b-m_b\right|^2+\left|h_b-h_b\right|^2\right)+ \\ \left(\left|n_b-n_b\right|^2+\left|h_b-h_b\right|^2\right)+\left|(n_b+r_b)-(n_b+r_b)\right|^2+ \\ \left|(m_b'-l_b')-(m_b'-l_b')\right|^2+\left(\left|m_b'-m_b'\right|^2+\left|h_b'-h_b'\right|^2\right)+ \\ \left(\left|n_b'-n_b'\right|^2+\left|h_b'-h_b'\right|^2\right)+\left|(n_b'+r_b')-(n_b'+r_b')\right|^2 \end{bmatrix}} \\
&= \sqrt{\frac{1}{8}\begin{bmatrix} \left|0\right|^2+\left(\left|0\right|^2+\left|0\right|^2\right)+\left(\left|0\right|^2+\left|0\right|^2\right)+\left|0\right|^2+ \\ \left|0\right|^2+\left(\left|0\right|^2+\left|0\right|^2\right)+\left(\left|0\right|^2+\left|0\right|^2\right)+\left|0\right|^2 \end{bmatrix}} \\
&= \sqrt{0} \\
&= 0.
\end{aligned}
$$

Conversely, if $d^{E}_{GTrLRIFNs}(A,B)=0$, then the distance between A and B is

$$d^{E}_{GTrLRIFNs}(A,B)=\sqrt{\frac{1}{8}\left[\begin{array}{c}\left|(m_b-l_b)-(m_a-l_a)\right|^2+\left(\left|m_b-m_a\right|^2+\left|h_b-h_a\right|^2\right)+\\ \left(\left|n_b-n_a\right|^2+\left|h_b-h_a\right|^2\right)+\left|(n_b+r_b)-(n_a+r_a)\right|^2+\\ \left|(m_b'-l_b')-(m_a'-l_a')\right|^2+\left(\left|m_b'-m_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\\ \left(\left|n_b'-n_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\left|(n_b'+r_b')-(n_a'+r_a')\right|^2\end{array}\right]}$$
$$=0.$$

Implies that $m_a-l_a=m_b-l_b$, $m_a=m_b$, $n_a=n_b$, $n_a+r_a=n_b+r_b$, $m_a'-l_a'=m_b'-l_b'$, $m_a'=m_b'$, $n_a'=n_b'$, $n_a'+r_a'=n_b'+r_b'$, $h_a=h_b$, and $h_a'=h_b'$. Therefore, two GTrLRIFNs A and B are identical and the proposition has been proved.

Proposition 4 Let A and B be two GTrLRIFNs. Then, $d^{E}_{GTrLRIFNs}(A,B)=d^{E}_{GTrLRIFNs}(B,A)$.

Proof Let $A=\begin{pmatrix}m_a-l_a,m_a,n_a,n_a+r_a;\\ m_a'-l_a',m_a',n_a',n_a'+r_a';h_a;h_a'\end{pmatrix}_{LR}$ and $B=\begin{pmatrix}m_b-l_b,m_b,n_b,n_b+r_b;\\ m_b'-l_b',m_b',n_b',n_b'+r_b';h_b;h_b'\end{pmatrix}_{LR}$ be two GTrLRIFNs. The distance of $d^{E}_{GTrLRIFNs}(A,B)$ is equal to the distance of $d^{E}_{GTrLRIFNs}(B,A)$. It is obvious that $(m_a-l_a)-(m_b-l_b)\neq(m_b-l_b)-(m_a-l_a)$, $m_a-m_b\neq m_b-m_a$, $n_a-n_b\neq n_b-n_a$, $(n_a+r_a)-(n_b+r_b)\neq(n_b+r_b)-(n_a+r_a)$, $(m_a'-l_a')-(m_b'-l_b')\neq(m_b'-l_b')-(m_a'-l_a')$, $m_a'-m_b'\neq m_b'-m_a'$, $n_a'-n_b'\neq n_b'-n_a'$, $(n_a'+r_a')-(n_b'+r_b')\neq(n_b'+r_b')-(n_a'+r_a')$, $h_a-h_b\neq h_b-h_a$, and $h_a'-h_b'\neq h_b'-h_a'$.
But, $\left|(m_a-l_a)-(m_b-l_b)\right|=\left|(m_b-l_b)-(m_a-l_a)\right|$, $\left|m_a-m_b\right|=\left|m_b-m_a\right|$, $\left|n_a-n_b\right|=\left|n_b-n_a\right|$, $\left|(n_a+r_a)-(n_b+r_b)\right|=\left|(n_b+r_b)-(n_a+r_a)\right|$, $\left|(m_a'-l_a')-(m_b'-l_b')\right|=\left|(m_b'-l_b')-(m_a'-l_a')\right|$, $\left|m_a'-m_b'\right|=\left|m_b'-m_a'\right|$, $\left|n_a'-n_b'\right|=\left|n_b'-n_a'\right|$, $\left|(n_a'+r_a')-(n_b'+r_b')\right|=\left|(n_b'+r_b')-(n_a'+r_a')\right|$, $\left|h_a-h_b\right|=\left|h_b-h_a\right|$, and $\left|h_a'-h_b'\right|=\left|h_b'-h_a'\right|$.

Hence,

$$d^{E}_{GTrLRIFNs}(A,B)=\sqrt{\frac{1}{8}\left[\begin{array}{c}\left|(m_b-l_b)-(m_a-l_a)\right|^2+\left(\left|m_b-m_a\right|^2+\left|h_b-h_a\right|^2\right)+\\ \left(\left|n_b-n_a\right|^2+\left|h_b-h_a\right|^2\right)+\left|(n_b+r_b)-(n_a+r_a)\right|^2+\\ \left|(m_b'-l_b')-(m_a'-l_a')\right|^2+\left(\left|m_b'-m_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\\ \left(\left|n_b'-n_a'\right|^2+\left|h_b'-h_a'\right|^2\right)+\left|(n_b'+r_b')-(n_a'+r_a')\right|^2\end{array}\right]}$$
$$=\sqrt{\frac{1}{8}\left[\begin{array}{c}\left|(m_a-l_a)-(m_b-l_b)\right|^2+\left(\left|m_a-m_b\right|^2+\left|h_a-h_b\right|^2\right)+\\ \left(\left|n_a-n_b\right|^2+\left|h_a-h_b\right|^2\right)+\left|(n_a+r_a)-(n_b+r_b)\right|^2+\\ \left|(m_a'-l_a')-(m_b'-l_b')\right|^2+\left(\left|m_a'-m_b'\right|^2+\left|h_a'-h_b'\right|^2\right)+\\ \left(\left|n_a'-n_b'\right|^2+\left|h_a'-h_b'\right|^2\right)+\left|(n_a'+r_a')-(n_b'+r_b')\right|\end{array}\right]}=d^{E}_{GTrLRIFNs}(B,A).$$

Therefore, $d^{E}_{GTrLRIFNs}(A,B)=d^{E}_{GTrLRIFNs}(B,A)$.

Proposition 5 If A, B, and C are three GTrLRIFNs. Then, $d^{E}_{GTrLRIFNs}(A,C)\leq\begin{pmatrix}d^{E}_{GTrLRIFNs}(A,B)+\\ d^{E}_{GTrLRIFNs}(B,C)\end{pmatrix}$.

Proof Let $A=\begin{pmatrix}m_a-l_a,m_a,n_a,n_a+r_a;\\ m_a'-l_a',m_a',n_a',n_a'+r_a';h_a;h_a'\end{pmatrix}_{LR}$, $B=(m_b-l_b,m_b,n_b,n_b+r_b;m_b'-l_b',m_b',n_b',n_b'+r_b';h_b;h_b')_{LR}$, and $C=(m_c-l_c,m_c,n_c,n_c+r_c;m_c'-l_c',m_c',n_c',n_c'+r_c';h_c;h_c')_{LR}$ be three GTrLRIFNs. Since $A-C=(A-B)+(B-C)$, it is obvious that

$\left|(m_a-l_a)-(m_c-l_c)\right|^2 \le \left|(m_a-l_a)-(m_b-l_b)\right|^2+\left|(m_b-l_b)-(m_c-l_c)\right|^2$, $\left|m_a-m_c\right|^2 \le \left|m_a-m_b\right|^2+\left|m_b-m_c\right|^2$, $\left|n_a-n_c\right|^2 \le \left|n_a-n_b\right|^2+\left|n_b-n_c\right|^2$, $\left|(n_a+r_a)-(n_c+r_c)\right|^2 \le \left|(n_a+r_a)-(n_b+r_b)\right|^2+\left|(n_b+r_b)-(n_c+r_c)\right|^2$, $\left|(m_a'-l_a')-(m_c'-l_c')\right|^2 \le \left|(m_a'-l_a')-(m_b'-l_b')\right|^2+\left|(m_b'-l_b')-(m_c'-l_c')\right|^2$, $\left|m_a'-m_c'\right|^2 \le \left|m_a'-m_b'\right|^2+\left|m_b'-m_c'\right|^2$, $\left|n_a'-n_c'\right|^2 \le \left|n_a'-n_b'\right|^2+\left|n_b'-n_c'\right|^2$, $\left|(n_a'+r_a')-(n_c'+r_c')\right|^2 \le \left|(n_a'+r_a')-(n_b'+r_b')\right|^2+\left|(n_b'+r_b')-(n_c'+r_c')\right|^2$, $\left|h_a-h_c\right|^2 \le \left|h_a-h_b\right|^2+\left|h_b-h_c\right|^2$, and $\left|h_a'-h_c'\right|^2 \le \left|h_a'-h_b'\right|^2+\left|h_b'-h_c'\right|^2$.

Hence,

$$\sqrt{\frac{1}{8}\left[\begin{array}{c}\left|(m_a-l_a)-(m_c-l_c)\right|^2+\left(\left|m_a-m_c\right|^2+\left|h_a-h_c\right|^2\right)+\\ \left(\left|n_a-n_c\right|^2+\left|h_a-h_c\right|^2\right)+\left|(n_a+r_a)-(n_c+r_c)\right|^2+\\ \left|(m_a'-l_a')-(m_c'-l_c')\right|^2+\left(\left|m_a'-m_c'\right|^2+\left|h_a'-h_c'\right|^2\right)+\\ \left(\left|n_a'-n_c'\right|^2+\left|h_a'-h_c'\right|^2\right)+\left|(n_a'+r_a')-(n_c'+r_c')\right|^2\end{array}\right]}$$

$$\le\sqrt{\frac{1}{8}\left[\begin{array}{c}\left|(m_a-l_a)-(m_b-l_b)\right|^2+\left(\left|m_a-m_b\right|^2+\left|h_a-h_b\right|^2\right)+\\ \left(\left|n_a-n_b\right|^2+\left|h_a-h_b\right|^2\right)+\left|(n_a+r_a)-(n_b+r_b)\right|^2+\\ \left|(m_a'-l_a')-(m_b'-l_b')\right|^2+\left(\left|m_a'-m_b'\right|^2+\left|h_a'-h_b'\right|^2\right)+\\ \left(\left|n_a'-n_b'\right|^2+\left|h_a'-h_b'\right|^2\right)+\left|(n_a'+r_a')-(n_b'+r_b')\right|\end{array}\right]}$$

$$+\sqrt{\frac{1}{8}\left[\begin{array}{c}\left|(m_b-l_b)-(m_c-l_c)\right|^2+\left(\left|m_b-m_c\right|^2+\left|h_b-h_c\right|^2\right)+\\ \left(\left|n_b-n_c\right|^2+\left|h_b-h_c\right|^2\right)+\left|(n_b+r_b)-(n_c+r_c)\right|^2+\\ \left|(m_b'-l_b')-(m_c'-l_c')\right|^2+\left(\left|m_b'-m_c'\right|^2+\left|h_b'-h_c'\right|^2\right)+\\ \left(\left|n_b'-n_c'\right|^2+\left|h_b'-h_c'\right|^2\right)+\left|(n_b'+r_b')-(n_c'+r_c')\right|^2\end{array}\right]}.$$

Considering the above inequalities, the $d^E_{GTrLRIFNs}(A,C) \le d^E_{GTrLRIFNs}(A,B)+d^E_{GTrLRIFNs}(B,C)$ is obtained. Thus, the proposition 5 is satisfied and completes the proof.

4.0 Implementation

4.1 Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method was proposed by Hwang and Yoon [22], which is a method for solving multi-attribute decision-making problems, where the selected alternative should have the shortest distance from the positive ideal answer and the farthest from the negative ideal solution. The steps involved in obtaining the decision result is as follows:

Step 1: Construct an evaluation matrix with m alternatives and n criteria. Each alternative and criteria intersection should be expressed as x_{ij}, and the matrix should be $(x_{ij})_{m\times n}$.

Step 2: Normalise the matrix $(x_{ij})_{m\times n}$ into the matrix $(r_{ij})_{m\times n}$ using the formula for normalisation method as shown in Equation 4.1.

$$r_{ij}=\frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}^{\ 2}};i=1,2,...,m,\ j=1,2,...,n. \tag{4.1}$$

Step 3: Construct the weighted normalised decision matrix using Equation 4.2.

$$v_{ij}=r_{ij}\times wj;i=1,2,...,m;\ j=1,2,...,n. \tag{4.2}$$

Step 4: Determine the positive ideal solution (A^+) and negative ideal solution (A^-) using Equation 4.3

and Equation 4.4 respectively.

$$A^+ = \{v_1^+, v_2^+, ..., v_n^+\} = \{(\max_j v_{ij} | i \in I), (\min_j v_{ij} | i \in J)\} \quad (4.3)$$

$$A^- = \{v_1^-, v_2^-, ..., v_n^-\} = \{(\min_j v_{ij} | i \in I), (\max_j v_{ij} | i \in J)\} \quad (4.4)$$

where I indicates the benefit criteria, while J indicated the non-benefit criteria or cost criteria.

Step 5: Calculate the distance between target alternative and the positive ideal solution using Equation 4.5.

$$d_i^+ = \left\{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2\right\}^{1/2}; i = 1, 2, ..., m. \quad (4.5)$$

Similarly, calculate the distance between alternative and negative ideal solution using Equation 4.6.

$$d_i^- = \left\{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2\right\}^{1/2}; i = 1, 2, ..., m. \quad (4.6)$$

Step 6: Calculate the closeness coefficient to the ideal solution using Equation 4.7.

$$CC_i = \frac{d_i^-}{\left(d_i^+ + d_i^-\right)}; i = 1, 2, ..., m. \quad (4.7)$$

Step 7: Rank the preference order.

4.2 Criteria Importance Through Intercriteria Correlation (CRITIC)

Criteria Importance Through Intercriteria Correlation (CRITIC) is a method introduced by Diakoulaki and Mavrotas [23] to determine the objective weights of criteria in decision-making issues. The weights calculated using this method include the degree of conflict between the criteria and the severity of each contrast of criterion. The correlation coefficient and standard deviation are used, respectively, to measure the contrast and conflict intensities of the criterion.

Determining the criteria weight is the most crucial input in MCDM approaches such as TOPSIS since it indicates the volume of information accessible for each criterion [14]. Hence, this study used the CRITIC method to determine the objective weight of the river parameters used in TOPSIS evaluation. The objective weights calculation is calculated as follows [14]:

Step 1: Obtain the following criteria vectors by calculating the transformations of performance values:

$$x_{ijk}^T = \begin{cases} \dfrac{x_{ijk} - x_{jk}^-}{x_{jk}^* - x_{jk}^-} & ; j \in B \\[2ex] \dfrac{x_{jk}^- - x_{ijk}}{x_{jk}^- - x_{jk}^*} & ; j \in N \end{cases} \quad (4.8)$$

where x_{ijk}^T is the transformed value of k^{th} element of x_{ij}, x_{jk} denotes the k^{th} vector of j^{th} criterion, x_{jk}^* and x_{jk}^- are the ideal and anti-ideal values respectively. If $j \in B, x_{jk}^* = \max_i x_{ijk}$ and $x_{jk}^- = \min_i x_{ijk}$. While if $j \in N, x_{jk}^* = \min_i x_{ijk}$ and $x_{jk}^- = \max_i x_{ijk}$.

Step 2: Calculate the standard deviation for each criteria using Equation 4.9.

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{n} \left(x_{ij} - \bar{x}_j\right)^2}{n}}; \forall j \quad (4.9)$$

Step 3: Determine the symmetric matrix of $n \times n$ with element r_{jk}, which is the linear correlation coefficient between vectors x_j and x_k.

Step 4: Calculate the information measures of each criterion as follows:

$$C_j = \sigma_j \times \sum_{k=1}^{m} \left(1 - r_{jk}\right) \quad (4.10)$$

Step 5: Determine the objective weight of each criteria using Equation 4.11.

$$W_j = \frac{C_j}{\sum_{k=1}^{m} C_j} \quad (4.11)$$

4.3 TOPSIS Using Generalised Trapezoidal L-R Intuitionistic Fuzzy Numbers with CRITIC

This study proposed the TOPSIS method of generalised trapezoidal L-R intuitionistic fuzzy TOPSIS (GTrLRIF TOPSIS) with CRITIC to classify the river pollution for several rivers in Johor, Malaysia. Figure 2 shows the flowchart of the TOPSIS method using generalised trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs) with CRITIC. There are two phases to classify river water pollution. Phase 1 is where the GTrLRIF TOPSIS is being calculated, which consists of Step 1 until Step 9. While Phase 2 is where the CRITIC method is being calculated, which consists of Step 4.1 until Step 4.6. The CRITIC weight is calculated during the calculation of GTrLRIF TOPSIS. The procedure is depicted in Figure 2.

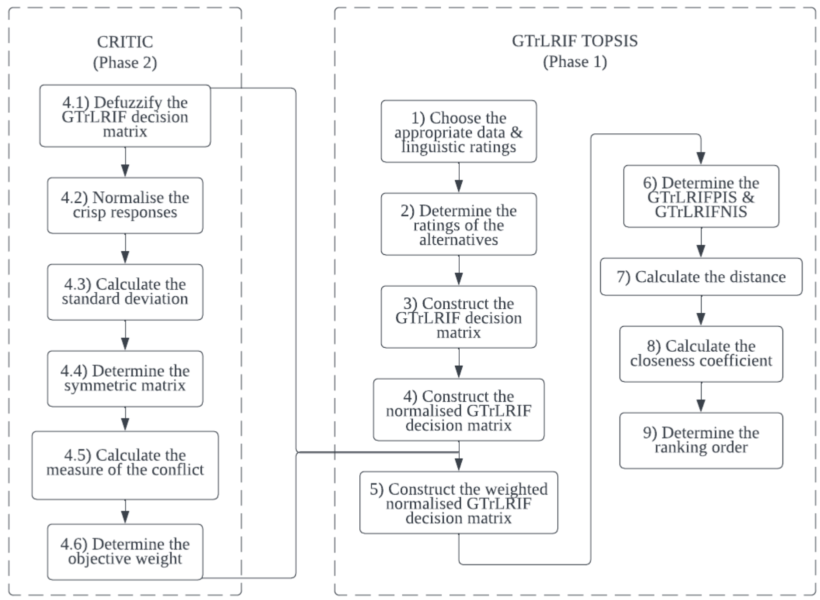

Figure 2. Flowchart of Generalised Trapezoidal L-R Intuitionistic Fuzzy TOPSIS with CRITIC

The steps involved in determining the river water pollution are as follows:

Step 1: Choose the appropriate data and linguistic ratings for alternatives with respect to criteria.

Step 2: Determine the ratings of the alternatives A_i under all criterion C_i. Table 1 gives a set of linguistic variables of all the elements used to classify the river water pollution.

Table 1. Linguistic Variables of Each Criterion [2]

Elements	Very Clean (VC)	Clean (C)	Slightly Polluted (SP)	Polluted (P)	Very Polluted (VP)
DO	>7	5-7	3-5	1-3	<1
BOD	<1	1-3	3-6	6-12	>12
COD	<10	10-25	25-50	50-100	>100
SS	<25	25-50	50-150	150-300	>300
pH	>7.0	6.0-7.0	5.0-6.0	<5.0	>5.0
AN	<0.1	0.1-0.3	0.3-0.9	0.9-2.7	>2.7

Step 3: Construct the generalised trapezoidal L-R intuitionistic fuzzy decision matrix (GTrLRIF) decision matrix by following the steps in Section 5.2.

Step 4: Construct the normalised GTrLRIF decision matrix, $R=\left[\tilde{r}_{ij}\right]_{m\times n}$, where B and C are the set of benefit criteria and cost criteria respectively, and

$$\tilde{r}_{ij} = \left\langle \begin{array}{c} \frac{(m-l)_{ij}}{(n+r)_j^*}, \frac{m_{ij}}{(n+r)_j^*}, \frac{n_{ij}}{(n+r)_j^*}, \\ \frac{(n+r)_{ij}}{(n+r)_j^*}; \frac{(m'-l')_j^-}{(n'+r')_{ij}}, \frac{(m'-l')_j^-}{n'_{ij}}, \\ \frac{(m'-l')_j^-}{m'_{ij}}, \frac{(m'-l')_j^-}{(m'-l')_{ij}}; h; h' \end{array} \right\rangle_{LR}, j \in B; \tag{4.12}$$

$$\tilde{r}_{ij} = \left\langle \begin{array}{c} \frac{(m-l)_j^-}{(n+r)_{ij}}, \frac{(m-l)_j^-}{n_{ij}}, \frac{(m-l)_j^-}{m_{ij}}, \\ \frac{(m-l)_j^-}{(m-l)_{ij}}; \frac{(m'-l')_{ij}}{(n'+r')_j^*}, \frac{m'_{ij}}{(n'+r')_j^*}, \\ \frac{n'_{ij}}{(n'+r')_j^*}, \frac{(n'+r')_{ij}}{(n'+r')_j^*}; h; h' \end{array} \right\rangle_{LR}, j \in C; \tag{4.13}$$

where,

$$(n+r)_j^* = \max_i (n+r)_{ij}; (m'-l')_j^- = \min_i (m'-l')_{ij} \text{ if } j \in B;$$
$$(m-l)_j^- = \min_i (m-l)_{ij}; (n'+r')_j^* = \max_i (n'+r')_{ij} \text{ if } j \in C.$$

Step 4.1: Defuzzify the GTrLRIF decision matrix using centroid method or centre of gravity method. This study used the centroid method due to its consideration of finding the area of the graph, which also considered the L and R function in the defuzzification process.

$$x^* = \frac{\int \mu_A(x) \cdot x\, dx}{\int \mu_A(x) dx} + \frac{\int \nu_A(x) \cdot x\, dx}{\int \nu_A(x) dx} \tag{4.14}$$

where $\int$ denoted an algebraic integration.

Step 4.2: Normalise the crisp responses. Each criterion is normalised based on the best and worst values of the normalised decision matrix. The normalisation expression of criteria is given as Equation 4.15.

$$x_{ij} = \frac{x_{ij} - x_j^{worst}}{x_j^{best} - x_j^{worst}} \tag{4.15}$$

where x_{ij} represents the normalised value of criterion i with respect to response j, x_j^{worst} represent the worst value of criterion with respect to response j, and x_j^{best} represent the best value of criterion with respect to response j. The x_{ij}, x_j^{worst}, and x_j^{best} are obtained based on the defuzzification values by using Equation 4.14.

Step 4.3: Calculate the standard deviation for each criteria using Equation 4.16.

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{n} \left(x_{ij} - \bar{x}_j\right)^2}{n}}; \forall j. \tag{4.16}$$

Step 4.4: Determine the symmetric matrix of $n \times n$ with element r_{jk}, which is the linear correlation coefficient between vectors x_j and x_k.

Step 4.5: Calculate the measure of the conflict created by criterion j with respect to the decision situation defined by the rest of the criteria and determine the quantity of the information with each criterion using Equation 4.17.

$$C_j = \sigma_j \times \sum_{k=1}^{m} \left(1 - r_{jk}\right). \tag{4.17}$$

Step 4.6: Determine the objective weight of each criteria using Equation 4.18.

$$W_j = \frac{C_j}{\sum_{k=1}^{m} C_j}. \tag{4.18}$$

Step 5: Construct the weighted normalised GTrLRIF decision matrix using the scalar multiplication of generalised trapezoidal L-R intuitionistic fuzzy numbers.
if $\forall \lambda > 0, \lambda \in \mathbb{R}$, then

$$\lambda \otimes \left(m,n;m',n';l,r;l',r';h;h'\right)_{LR} = \left(\lambda m,\lambda n;\lambda m',\lambda n';\lambda l,\lambda r;\lambda l',\lambda r';h;h'\right)_{LR} \tag{4.19}$$

if $\forall \lambda < 0, \lambda \in \mathbb{R}$, then

$$\lambda \otimes \left(m,n;m',n';l,r;l',r';h;h'\right)_{LR} = \left(\lambda m,\lambda n;\lambda m',\lambda n';-\lambda r,-\lambda l;-\lambda r',-\lambda l';h;h'\right)_{RL}. \tag{4.20}$$

Step 6: Determine the generalised trapezoidal L-R intuitionistic fuzzy positive ideal solution (GTLRIFPIS) and generalised trapezoidal L-R intuitionistic fuzzy negative ideal solution (GTLRIFNIS).

$$A^{+} = \left\{\mu_1^{+},\mu_2^{+},...,\mu_n^{+};v_1^{-},v_2^{-},...,v_n^{-};h_n^{+};h'^{-}_{n}\right\}$$

$$A^{+} = \left\{\begin{matrix} \left(\max_j \mu_{ij} \middle| i \in I\right),\left(\min_j \mu_{ij} \middle| i \in J\right); \\ \left(\min_j v_{ij} \middle| i \in I\right),\left(\max_j v_{ij} \middle| i \in J\right); \\ \left(\min_j h_{ij} \middle| i \in I\right),\left(\max_j h_{ij} \middle| i \in J\right); \\ \left(\max_j h_{ij}' \middle| i \in I\right),\left(\min_j h_{ij}' \middle| i \in J\right) \end{matrix}\right\} \tag{4.21}$$

$$A^{-} = \left\{\mu_1^{-},\mu_2^{-},...,\mu_n^{-};v_1^{+},v_2^{+},...,v_n^{+};h_n^{-};h'^{+}_{n}\right\}$$

$$A^{-} = \left\{\begin{matrix} \left(\min_j \mu_{ij} \middle| i \in I\right),\left(\max_j \mu_{ij} \middle| i \in J\right); \\ \left(\max_j v_{ij} \middle| i \in I\right),\left(\min_j v_{ij} \middle| i \in J\right); \\ \left(\max_j h_{ij} \middle| i \in I\right),\left(\min_j h_{ij} \middle| i \in J\right); \\ \left(\min_j h_{ij}' \middle| i \in I\right),\left(\max_j h_{ij}' \middle| i \in J\right) \end{matrix}\right\} \tag{4.22}$$

where I is associated with benefit criteria while J is associated with cost criteria.
Step 7: Calculate the distance of each alternative from GTLRIFPIS and GTLRIFNIS respectively. The distance of each alternative from A^{*} and A^{-} can be calculated as:

$$d_i^{*} = \sum_{j=1}^{n} d\left(v_{ij},v_j^{*}\right), \quad i = 1,2,...m, \tag{4.23}$$

$$d_i^{-} = \sum_{j=1}^{n} d\left(v_{ij},v_j^{-}\right), \quad i = 1,2,...m, \tag{4.24}$$

where $d(\cdot,\cdot)$ is the Euclidean distance between two generalised L-R intuitionistic fuzzy numbers using Equation 3.4.
Step 8: Calculate the closeness coefficient of each alternative. The relative closeness coefficient (CC_i) of the alternative A_i with respect to the generalised L-R intuitionistic fuzzy ideal solutions is defined as:

$$CC_i = \frac{d_i^{-}}{d_i^{*} + d_i^{-}}, \qquad i = 1,2,...,m. \tag{4.25}$$

Step 9: According to the closeness coefficient, the ranking order of all alternatives can be determined

5.0 Case Study of River Water Pollution Classification

This section discusses the case study of the classification of river water pollution starting from data collection, the determination of trapezoidal L-R intuitionistic fuzzy numbers with the confidence level, and the numerical example for the case study of the classification of river water pollution based on generalised trapezoidal L-R intuitionistic fuzzy TOPSIS (GTrLRIF TOPSIS) with CRITIC.

5.1 Data Collection

The data used in this study was obtained from the Department of Environment Malaysia and was taken from five rivers in Johor, Malaysia: Kim Kim River, Sayong River, Telor River, Pelepah River, and Bantang River from 2017 to 2021. The list of parameters in each river used in this article is shown in Table 2.

Due to insufficient river data, this study has used the bootstrap method to obey the Central Limit Theorem (CLT). CLT do not limit the sample size; however, it is recommended to have a minimum sample size of 30 according to the rules of thumb [24]. According to Chang and Wu [24], the distribution of the sample mean is assumed to be approximated to normal distribution since the criterion of the sample size is more than or equal than 30. Since then, this study has used the bootstrap method to solve this problem. The bootstrap method is a statistical method that was introduced by Efron [25]. In this study, the bootstrap method has been utilised due to its ability to repeatedly draw samples of the same size from the population of interest a large number of times [26].

Ultimately, this study has successfully simulated 100 river data using the bootstrap method. According to Gonçalves and Kilian [27], Hušková and Kirch [28], the bootstrap approach often gives better and accurate results in small samples. Therefore, this study only simulated 100 river data using bootstrap method.

Table 2. List of Parameters in River

Parameters	Description	Units
DO	Dissolved oxygen	*mg/l*
BOD	Biochemical oxygen demand	*mg/l*
COD	Chemical oxygen demand	*mg/l*
SS	Suspended solids	*mg/l*
pH	Potential of hydrogen	-
AN	Ammoniacal nitrogen	*mg/l*

5.2 Generalised Trapezoidal L-R Intuitionistic Fuzzy Numbers

This subsection discusses the development of generalised trapezoidal L-R intuitionistic fuzzy numbers, starting from determining trapezoidal L-R intuitionistic fuzzy numbers (TrLRIFNs), determination of confidence level, and selecting L and R reference functions.

5.2.1 Determine the Trapezoidal L-R Intuitionistic Fuzzy Numbers

In this study, TrLRIFNs for linguistic variables are modified based on the method used by Lee and Wang [29] that utilizes the data's minimum, maximum, mean, and standard deviation. This study has used 100 river data from the bootstrap method to determine the minimum (min), maximum (max), mean, and standard deviation (SD) used in Lee and Wang's [29] method. Table 3 shows the sample formulation of the membership function for DO. All elements of the river, which are DO, BOD, COD, SS, pH, and AN, have used the same formulation for the evaluation.

Table 3. Formulation of Membership Function for DO

Linguistic Variable	Linguistic Term	TrLRIFNs
DO	VC	$\left(Min, Mean-SD; Min, Mean-SD; 0, SD; 0, SD\right)_{LR}$
	C	$\left(\frac{Min+Mean}{2}, \frac{2Mean-SD}{2}; \frac{Min+Mean}{2}, \frac{2Mean-SD}{2}; \frac{SD}{2}, SD; \frac{SD}{2}, SD\right)_{LR}$
	SP	$\left(Mean, Mean; Mean, Mean; SD, SD; SD, SD\right)_{LR}$
	P	$\left(\frac{2Mean+SD}{2}, \frac{Mean+Max}{2}; \frac{2Mean+SD}{2}, \frac{Mean+Max}{2}; SD, \frac{SD}{2}; SD, \frac{SD}{2}\right)_{LR}$
	VP	$\left(Mean+SD, Max; Mean+SD, Max; SD, 0; SD, 0\right)_{LR}$

where VC = Very Clean, C = Clean, SP = Slightly Polluted, P = Polluted, VP = Very Polluted.

5.2.2 Determine the Confidence Level of L-R Intuitionistic Fuzzy Numbers

The height of the membership and non-membership functions are determined by the confidence level, which has been decided by the decision-maker [30]. It is crucial to include the confidence level since decision-makers usually come from various backgrounds, including those with different degrees of expertise, knowledge, and other characteristics that might influence the evaluation process [31].

Therefore, in this study, confidence level has been determined by the decision-makers based on their level of certainty towards the reliability of the dataset for each of the parameters of the rivers, which are DO, BOD, COD, SS, pH, and AN. This study has taken the average confidence level of four decision-makers from various backgrounds. This study has assumed the confidence level for all the classes of each parameter is the same and gives the exact height of membership and non-membership functions.

5.2.3 Determine the L and R Functions

This study used statistical distribution to choose which functions need to be used for L and R functions because it can provide a mathematical framework for describing the behaviour of random variables. Several distributions exist, such as normal, triangular, uniform, beta, gamma, Weibull, et cetera. This study has normalised 100 river data that have been simulated using the bootstrap method. Since all the parameters for all the rivers can be distributed using the normal distribution and according to Chang and Wu [24], the distribution of the sample mean is assumed to be approximated to normal distribution since the criterion of the sample size is more than or equal than 30, therefore this study used the L and R function suggested by Dubois and Prade [20] which is $y = \frac{1}{1+x^2}$.

5.3 River Water Pollution Classification Based on Generalised Trapezoidal L-R Intuitionistic Fuzzy TOPSIS With CRITIC

Suppose the Department of Environmental Malaysia (DOE Malaysia) wants to classify the river water pollution. Five rivers are selected, which are Kim Kim River (A_1), Sayong River (A_2), Telor River (A_3), Pelepah River (A_4), and Bantang River (A_5) as alternatives for further evaluation. The location of the five rivers is shown in Figure 3, where A_1 is the Kim Kim River, A_2 is the Sayong River, A_3 is the Telor River, A_4 is the Pelepah River, and A_5 is the Bantang River. The five possible alternatives can be evaluated under six criteria which are dissolve oxygen (DO) (C_1), biochemical oxygen demand (BOD) (C_2), chemical oxygen demand (COD) (C_3), suspended solid (SS) (C_4), potential of hydrogen (pH) (C_5), and ammoniacal nitrogen (AN) (C_6). This study has categorised BOD, COD, SS, and AN as benefit criteria while DO and pH as cost criteria. To cater this problem, the proposed approach is now being applied, and the computing is summarised as follows:

Figure 3. Location of the Five Rivers

Step 1: This study has averaged 100 river data for all five alternatives, as shown in Table 4.

Table 4. Average Data for Each Alternatives

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1.72	13.12	43.78	47.41	6.80	4.08
A_2	7.15	3.70	14.12	28.68	5.12	0.19
A_3	7.27	3.50	13.53	80.09	5.94	0.12
A_4	6.75	3.07	13.18	60.14	6.73	0.49
A_5	8.22	2.72	10.15	4.86	7.04	0.05

Step 2: Determine the ratings of the alternatives under all criteria as shown in Table 5.

Table 5. Ratings of the Five Rivers Under All Criteria

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	P	VP	SP	C	C	P
A_2	VC	SP	C	SP	SP	VC
A_3	VC	SP	C	SP	SP	VC
A_4	C	SP	C	SP	C	C
A_5	VC	C	C	VC	VC	VC

Step 3: Converting the linguistic evaluation from Table 5 to the GTrLRIF decision matrix as shown in Table 6 using steps in Section 5.2.

Table 6. Generalised Trapezoidal L-R Intuitionistic Fuzzy Decision Matrix

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	$(1.97,2.32;1.97,2.32;0.51,0.26;0.51,0.26;0.91;0.09)_{LR}$	$(15.63,18.9;15.63,18.9;2.51,0.00;2.51,0.00;0.89;0.11)_{LR}$	$(43.78,43.78; 43.78,43.78;5.97,5.97;5.97,5.97;0.91;0.09)_{LR}$	$(32.91,34.99;32.91,34.99;12.42,24.85;12.42,24.85; 0.91;0.09)_{LR}$	$(6.71,6.76;6.71,6.76;0.05,0.09;0.05,0.09;0.93;0.08)_{LR}$	$(4.91,6.20;4.91,6.20;0.83,0.00;0.83,0.00;0.91;0.09)_{LR}$
A_2	$(6.54,6.95;6.54,6.95;0.00,0.21;0.00,0.21;0.85;0.15)_{LR}$	$(3.70,3.70;3.70,3.70;0.69,0.69;0.69,0.69;0.83;0.17)_{LR}$	$(12.31,13.37;12.31,13.37;0.75,1.50;0.75,1.50;0.83;0.17)_{LR}$	$(28.68,28.68;28.68,28.68;7.70,7.70;7.70,7.70;0.76;0.24)_{LR}$	$(5.12,5.12;5.12,5.12;0.22,0.22;0.22,0.22;0.86;0.14)_{LR}$	$(0.13,0.16;0.13,0.16;0.03,0.06;0.03,0.06;0.85;0.15)_{LR}$
A_3	$(6.98,7.16;6.98,7.16;0.00,0.13;0.00,0.13;0.88;0.13)_{LR}$	$(3.49,3.49;3.49,3.49;0.66,0.66;0.66,0.66;0.88;0.13)_{LR}$	$(11.82,12.81;11.82,12.81;0.72,1.45;0.72,1.45;0.88;0.13)_{LR}$	$(80.09,80.09;80.09,80.09;24.96,24.96;24.96,24.96;0.85;0.15)_{LR}$	$(5.94,5.94;5.94,5.94;0.23,0.23;0.23,0.23;0.86;0.14)_{LR}$	$(0.08,0.09;0.08,0.09;0.03,0.06;0.03,0.06;0.88;0.13)_{LR}$
A_4	$(6.45,6.63;6.45,6.63;0.12,0.24;0.12,0.24;0.93;0.07)_{LR}$	$(3.07,3.07;3.07,3.07;0.46,0.46;0.46,0.46;0.93;0.07)_{LR}$	$(11,84,12.63;11.84,12.63;0.55,1.10;0.55,1.10;0.93;0.07)_{LR}$	$(60.14,60.14;60.14,60.14;22.32,22.32;22.32,22.32;0.93;0.07)_{LR}$	$(6.58,6.67;6.58,6.67;0.05,0.11;0.05,0.11;0.94;0.06)_{LR}$	$(0.49,0.49;0.49,0.49;0.11,0.11;0.11,0.11;0.93;0.07)_{LR}$
A_5	$(7.79,8.09;7.79,8.09;0.00,0.13;0.00,0.13;0.93;0.07)_{LR}$	$(2.28,2.49;2.28,2.49;0.24,0.47;0.24,0.47;0.93;0.07)_{LR}$	$(8.22,9.44;8.22,9.44;0.71,1.43;0.71,1.43;0.93;0.07)_{LR}$	$(2.29,3.62;2.29,3.62;0.00,1.23;0.00,1.23;0.91;0.09)_{LR}$	$(6.86,6.97;6.86,6.97;0.00,0.13;0.00,0.13;0.94;0.06)_{LR}$	$(0.01,0.02;0.01,0.02;0.00,0.04;0.00,0.04;0.93;0.07)_{LR}$

Step 4: By using Equation 4.12 and Equation 4.13, calculate the normalised GTrLRIF decision matrix as shown in Table 7.
Step 4.1: This step is the beginning of the CRITIC method by defuzzifying the GTrLRIF decision matrix using centroid method as shown in Table 8.
Step 4.2: Normalise the crisp responses based on the best and worst values of each criterion as shown in Table 9.
Step 4.3: Calculate the standard deviation for each criterion as shown in Table 10.
Step 4.4: Construct the symmetric matrix using linear correlation coefficient of normalised crisp responses of GTrLRIF decision matrix as shown in Table 11.
Step 4.5: Calculate the measure of the conflict using Equation 4.17 as shown in Table 12.

Table 7. Normalised Generalised Trapezoidal L-R Intuitionistic Fuzzy Decision Matrix

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	$(0.6293,0.7411; 0.2397,0.2822; 0.0634,0.2589; 0.0620,0.0316; 0.9100; 0.0900)_{LR}$	$(0.8270,1.0000; 0.1079,0.1305; 0.1328,0.0000; 0.0000,0.0250; 0.8900; 0.1100)_{LR}$	$(0.8800,0.8800; 0.1715,0.1715; 0.1200,0.1200; 0.0206,0.0271; 0.9100; 0.0900)_{LR}$	$(0.3133,0.3331; 0.0654,0.0696; 0.1182,0.2366; 0.0272,0.0422; 0.9100; 0.0900)_{LR}$	$(0.7249,0.7303; 0.9531,0.9602; 0.0095,0.0055; 0.0071,0.0128; 0.9300; 0.0700)_{LR}$	$(0.7919,1.0000; 0.0016,0.0020; 0.1339,0.0000; 0.0000,0.0004; 0.9100; 0.0900)_{LR}$
A_2	$(0.2101,0.2234; 0.7956,0.8455; 0.0062,0.0000; 0.0000,0.0255; 0.8500; 0.1500)_{LR}$	$(0.1958,0.1958; 0.5514,0.5514; 0.0365,0.0365; 0.0867,0.1264; 0.8300; 0.1700)_{LR}$	$(0.2474,0.2687; 0.5617,0.6101; 0.0151,0.0302; 0.0567,0.0396; 0.8300; 0.1700)_{LR}$	$(0.2730,0.2730; 0.0798,0.0798; 0.0733,0.0733; 0.0169,0.0293; 0.7600; 0.2400)_{LR}$	$(0.9570,0.9570; 0.7273,0.7273; 0.0394,0.0430; 0.0313,0.0313; 0.8600; 0.1400)_{LR}$	$(0.0210,0.0258; 0.0625,0.0769; 0.0048,0.0097; 0.0170,0.0231; 0.8500; 0.1500)_{LR}$
A_3	$(0.2039,0.2092; 0.8491,0.8710; 0.0036,0.0000; 0.0000,0.0158; 0.8800; 0.1200)_{LR}$	$(0.1847,0.1847; 0.5845,0.5845; 0.0349,0.0349; 0.0930,0.1363; 0.8800; 0.1200)_{LR}$	$(0.2376,0.2575; 0.5863,0.6354; 0.0145,0.0291; 0.0596,0.0412; 0.8800; 0.1200)_{LR}$	$(0.7624,0.7624; 0.0286,0.0286; 0.2376,0.2376; 0.0068,0.0129; 0.8500; 0.1500)_{LR}$	$(0.8249,0.8249; 0.8438,0.8438; 0.0308,0.0332; 0.0327,0.0327; 0.8600; 0.1400)_{LR}$	$(0.0129,0.0145; 0.1111,0.1250; 0.0048,0.0097; 0.0444,0.0750; 0.8800; 0.1200)_{LR}$
A_4	$(0.2202,0.2264; 0.7847,0.8066; 0.0077,0.0043; 0.0146,0.0292; 0.9300; 0.0700)_{LR}$	$(0.1624,0.1624; 0.6645,0.6645; 0.0243,0.0243; 0.0866,0.1171; 0.9300; 0.0700)_{LR}$	$(0.2380,0.2539; 0.5946,0.6343; 0.0111,0.0221; 0.0476,0.0309; 0.9300; 0.0700)_{LR}$	$(0.5725,0.5725; 0.0381,0.0381; 0.2125,0.2125; 0.0103,0.0225; 0.9300; 0.0700)_{LR}$	$(0.7346,0.7447; 0.9347,0.9474; 0.0119,0.0057; 0.0071,0.0156; 0.9400; 0.0600)_{LR}$	$(0.0790,0.0790; 0.0204,0.0204; 0.0177,0.0177; 0.0037,0.0059; 0.9300; 0.0700)_{LR}$
A_5	$(0.1805,0.1874; 0.9477,0.9842; 0.0029,0.0000; 0.0000,0.0158; 0.9300; 0.0700)_{LR}$	$(0.1206,0.1317; 0.8193,0.8947; 0.0127,0.0249; 0.1301,0.1053; 0.9300; 0.0700)_{LR}$	$(0.1652,0.1897; 0.7956,0.9136; 0.0143,0.0287; 0.1097,0.0864; 0.9300; 0.0700)_{LR}$	$(0.0218,0.0345; 0.6326,1.0000; 0.0000,0.0117; 0.1604,0.0000; 0.9100; 0.0900)_{LR}$	$(0.7091,0.7335; 0.9489,0.9815; 0.0131,0.0000; 0.0000,0.0185; 0.9400; 0.0600)_{LR}$	$(0.0016,0.0032; 0.5000,1.0000; 0.0000,0.0065; 0.3333,0.0000; 0.9300; 0.0700)_{LR}$

Table 8. Defuzzification Values of GTrLRIF Decision Matrix

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	8.3265	54.1563	197.2391	179.5290	30.2788	17.6154
A_2	21.2727	14.3285	46.8325	99.8291	20.8953	0.6082
A_3	24.8068	14.7900	49.8102	321.0263	24.2418	0.4171
A_4	29.1099	14.4581	55.5291	283.2280	29.9871	2.3076
A_5	29.8837	11.3830	40.6291	15.2836	27.7687	0.1724

Table 9. Normalise Crisp Responses

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.0000	0.0000	0.0000	0.4628	1.0000	0.0000
A_2	0.6006	0.9311	0.9604	0.7235	0.0000	0.9750
A_3	0.7645	0.9203	0.9414	0.0000	0.3566	0.9860
A_4	0.9641	0.9281	0.9049	0.1236	0.9689	0.8776
A_5	1.0000	1.0000	1.0000	1.0000	0.7325	1.0000

Table 10. Standard Deviation of Criterion

	C_1	C_2	C_3	C_4	C_5	C_6
Standard Deviation	0.4056	0.4238	0.4270	0.4141	0.4278	0.4319

Table 11. Symmetrix Matrix of Normalised Crisp Responses of GTrLRIF Decision Matrix

River	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1.0000	0.9324	0.9157	0.0233	-0.1396	0.8986
C_2	0.9324	1.0000	0.9989	0.0603	-0.4849	0.9947
C_3	0.9157	0.9989	1.0000	0.0680	-0.5244	0.9982
C_4	0.0233	0.0603	0.0680	1.0000	-0.1378	0.0567
C_5	-0.1396	-0.4849	-0.5244	-0.1378	1.0000	-0.5592
C_6	0.8986	0.9947	0.9982	0.0567	-0.5592	1.0000

Table 12. Measure of The Conflict

Conflict Measure	C_1	C_2	C_3	C_4	C_5	C_6
C_j	0.9610	1.0589	1.0861	2.0411	2.9290	1.1276

Step 4.6: Determine the objective weight of each criterion using Equation 4.18 as shown in Table 13.

Table 13. Objective Weight

Objective Weight	C_1	C_2	C_3	C_4	C_5	C_6
W_j	0.1044	0.1151	0.1180	0.2218	0.3182	0.1225

Step 5: Construct the weighted normalised GTrLRIF decision matrix. Step 4.1 until Step 4.6 has successfully calculate the CRITIC weight from GTrLRIF decision matrix. Hence, this study has used two types of weights which are the weight given by the Department of Environment Malaysia through the WQI formula (Equation 5.1) (after this being called WQI weight) and the objective weight using CRITIC (after this being called CRITIC weight). The result for WQI weight and CRITIC weight are shown in Table 14. Therefore, the weighted normalised GTrLRIF decision matrix using WQI weight and CRITIC weight were calculated using Equation 4.19, as shown in Table 15 and Table 16 respectively.

$$WQI = 0.22SiDO + 0.19SiBOD + 0.16SiCOD + \\ 0.16SiSS + 0.12SipH + 0.15SiAN \quad (5.1)$$

Table 14. Weight of WQI and CRITIC

Type of Weight	C_1	C_2	C_3	C_4	C_5	C_6
WQI Weight	0.2200	0.1900	0.1600	0.1600	0.1200	0.1500
CRITIC Weight	0.1044	0.1151	0.1180	0.2218	0.3182	0.1225

Table 15. Weighted Normalised GTrLRIF Decision Matrix Using WQI weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.1384,0.1630; 0.0527,0.0621; 0.0140,0.0570; 0.0136,0.0070; 0.9100; 0.0900)$_{LR}$	(0.1571,0.1900; 0.0205,0.0248; 0.0252,0.0000; 0.0000,0.0047; 0.8900; 0.1100)$_{LR}$	(0.1408,0.1408; 0.0274,0.0274; 0.0192,0.0192; 0.0033,0.0043; 0.9100; 0.0900)$_{LR}$	(0.0501,0.0533; 0.0105,0.0111; 0.0189,0.0378; 0.0043,0.0067; 0.9100; 0.0900)$_{LR}$	(0.0870,0.0876; 0.1144,0.1152; 0.0011,0.0007; 0.0009,0.0015; 0.9300; 0.0700)$_{LR}$	(0.1188,0.1500; 0.0002,0.0003; 0.0201,0.0000; 0.0000,0.0001; 0.9100; 0.0900)$_{LR}$
A_2	(0.0462,0.0491; 0.1750,0.1860; 0.0014,0.0000; 0.0000,0.0056; 0.8500; 0.1500)$_{LR}$	(0.0372,0.0372; 0.1048,0.1048; 0.0069,0.0069; 0.0165,0.0240; 0.8300; 0.1700)$_{LR}$	(0.0396,0.0430; 0.0899,0.0976; 0.0024,0.0048; 0.0911,0.0063; 0.8300; 0.1700)$_{LR}$	(0.0437,0.0437; 0.0128,0.0128; 0.0117,0.0117; 0.0027,0.0047; 0.7600; 0.2400)$_{LR}$	(0.1148,0.1148; 0.0873,0.0873; 0.0047,0.0052; 0.0038,0.0038; 0.8600; 0.1400)$_{LR}$	(0.0031,0.0039; 0.0094,0.0115; 0.0007,0.0015; 0.0026,0.0035; 0.8500; 0.1500)$_{LR}$
A_3	(0.0449,0.0460; 0.1868,0.1916; 0.0008,0.0000;	(0.0351,0.0351; 0.1111,0.1111; 0.0066,0.0066;	(0.0380,0.0412; 0.0938,0.1017; 0.0023,0.0047;	(0.1220,0.1220; 0.0046,0.0046; 0.0380,0.0380;	(0.0990,0.0990; 0.1013,0.1013; 0.0037,0.0040;	(0.0019,0.0022; 0.0167,0.0188; 0.0007,0.0015;

River	C_1	C_2	C_3	C_4	C_5	C_6
	0.0000,0.0035; 0.8800; 0.1200)$_{LR}$	0.0177,0.0259; 0.8800; 0.1200)$_{LR}$	0.0095,0.0066; 0.8800; 0.1200)$_{LR}$	0.0011,0.0021; 0.8500; 0.1500)$_{LR}$	0.0039,0.0039; 0.8600; 0.1400)$_{LR}$	0.0067,0.0113; 0.8800; 0.1200)$_{LR}$
A_4	(0.0484,0.0498; 0.1726,0.1774; 0.0017,0.0009; 0.0032,0.0064; 0.9300; 0.0700)$_{LR}$	(0.0309,0.0309; 0.1263,0.1263; 0.0046,0.0046; 0.0165,0.0223; 0.9300; 0.0700)$_{LR}$	(0.0381,0.0406; 0.0951,0.1015; 0.0018,0.0035; 0.0076,0.0049; 0.9300; 0.0700)$_{LR}$	(0.0916,0.0916; 0.0061,0.0061; 0.0340,0.0340; 0.0016,0.0036; 0.9300; 0.0700)$_{LR}$	(0.0882,0.0894; 0.1122,0.1137; 0.0014,0.0007; 0.0009,0.0019; 0.9400; 0.0600)$_{LR}$	(0.0119,0.0119; 0.0031,0.0031; 0.0027,0.0027; 0.0006,0.0009; 0.9300; 0.0700)$_{LR}$
A_5	(0.0397,0.0412; 0.2085,0.2165; 0.0006,0.0000; 0.0000,0.0035; 0.9300; 0.0700)$_{LR}$	(0.0229,0.0250; 0.1557,0.1700; 0.0024,0.0047; 0.0247,0.0200; 0.9300; 0.0700)$_{LR}$	(0.0264,0.0304; 0.1273,0.1462; 0.0023,0.0046; 0.0167,0.0138; 0.9300; 0.0700)$_{LR}$	(0.0035,0.0055; 0.1012,0.1600; 0.0000,0.0019; 0.0257,0.0000; 0.9100; 0.0900)$_{LR}$	(0.0851,0.0880; 0.1139,0.1178; 0.0016,0.0000; 0.000,0.0022; 0.9400; 0.0600)$_{LR}$	(0.0002,0.0005; 0.0750,0.1500; 0.0000,0.0010; 0.0500,0.0000; 0.9300; 0.0700)$_{LR}$

Table 16. Weighted Normalised GTrLRIF Decision Matrix Using CRITIC Weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.0657,0.0774; 0.0250,0.0295; 0.0066,0.0270; 0.0065,0.0033; 0.9100; 0.0900)$_{LR}$	(0.0952,0.1151; 0.0124,0.0150; 0.0153,0.0000; 0.0000,0.0029; 0.8900; 0.1100)$_{LR}$	(0.1038,0.1038; 0.0202,0.0202; 0.0142,0.0142; 0.0024,0.0032; 0.9100; 0.0900)$_{LR}$	(0.0695,0.0739; 0.0145,0.0154; 0.0262,0.0525; 0.0060,0.0094; 0.9100; 0.0900)$_{LR}$	(0.2306,0.2324; 0.3033,0.3055; 0.0030,0.0017; 0.0023,0.0041; 0.9300; 0.0700)$_{LR}$	(0.0970,0.1225; 0.0002,0.0002; 0.0164,0.0000; 0.0000,0.0001; 0.9100; 0.0900)$_{LR}$
A_2	(0.0219,0.0233; 0.0831,0.0883; 0.0006,0.0000; 0.0000,0.0027; 0.8500; 0.1500)$_{LR}$	(0.0225,0.0225; 0.0635,0.0635; 0.0042,0.0042; 0.0100,0.0145; 0.8300; 0.1700)$_{LR}$	(0.0292,0.0317; 0.0663,0.0720; 0.0018,0.0036; 0.0067,0.0047; 0.8300; 0.1700)$_{LR}$	(0.0606,0.0606; 0.0177,0.0177; 0.0163,0.0163; 0.0037,0.0065; 0.7600; 0.2400)$_{LR}$	(0.3045,0.3045; 0.2314,0.2314; 0.0125,0.0137; 0.0099,0.0099; 0.8600; 0.1400)$_{LR}$	(0.0026,0.0032; 0.0077,0.0094; 0.0006,0.0012; 0.0021,0.0028; 0.8500; 0.1500)$_{LR}$
A_3	(0.0213,0.0218; 0.0887,0.0909; 0.0004,0.0000; 0.0000,0.0017; 0.8800; 0.1200)$_{LR}$	(0.0213,0.0213; 0.0673,0.0673; 0.0040,0.0040; 0.0107,0.0157; 0.8800; 0.1200)$_{LR}$	(0.0280,0.0304; 0.0692,0.0750; 0.0017,0.0034; 0.0070,0.0049; 0.8800; 0.1200)$_{LR}$	(0.1691,0.1691; 0.0063,0.0063; 0.0527,0.0527; 0.0015,0.0029; 0.8500; 0.1500)$_{LR}$	(0.2625,0.2625; 0.2685,0.2685; 0.0098,0.0106; 0.0104,0.0104; 0.8600; 0.1400)$_{LR}$	(0.0016,0.0018; 0.0136,0.0153; 0.0006,0.0012; 0.0054,0.0092; 0.8800; 0.1200)$_{LR}$
A_4	(0.0230,0.0236; 0.0819,0.0842; 0.0008,0.0004; 0.0015,0.0030; 0.9300; 0.0700)$_{LR}$	(0.0187,0.0187; 0.0765,0.0765; 0.0028,0.0028; 0.0100,0.0135; 0.9300; 0.0700)$_{LR}$	(0.0281,0.0300; 0.0702,0.0748; 0.0013,0.0026; 0.0056,0.0036; 0.9300; 0.0700)$_{LR}$	(0.1270,0.1270; 0.0084,0.0084; 0.0471,0.0471; 0.0023,0.0050; 0.9300; 0.0700)$_{LR}$	(0.2338,0.2370; 0.2974,0.3015; 0.0038,0.0018; 0.0023,0.0050; 0.9400; 0.0600)$_{LR}$	(0.0097,0.0097; 0.0025,0.0025; 0.0022,0.0022; 0.0005,0.0007; 0.9300; 0.0700)$_{LR}$
A_5	(0.0188,0.0196; 0.0989,0.1027; 0.0003,0.0000; 0.0000,0.0017; 0.9300; 0.0700)$_{LR}$	(0.0139,0.0152; 0.0943,0.1030; 0.0015,0.0029; 0.0150,0.0121; 0.9300; 0.0700)$_{LR}$	(0.0195,0.0224; 0.0939,0.1078; 0.0017,0.0034; 0.0123,0.0102; 0.9300; 0.0700)$_{LR}$	(0.0048,0.0076; 0.1403,0.2218; 0.0000,0.0026; 0.0356,0.0000; 0.9100; 0.0900)$_{LR}$	(0.2256,0.2334; 0.3019,0.3123; 0.0042,0.0000; 0.0000,0.0059; 0.9400; 0.0600)$_{LR}$	(0.0002,0.0004; 0.0613,0.1225; 0.0000,0.0008; 0.0408,0.0000; 0.9300; 0.0700)$_{LR}$

Step 6: Determine the generalised trapezoidal L-R intuitionistic fuzzy positive ideal solution (GTrLRIFPIS) and generalised trapezoidal L-R intuitionistic fuzzy negative ideal solution (GTrLRIFNIS) using Equation 4.21 and Equation 4.22 respectively for WQI weight and CRITIC weight as shown in Table 17 and Table 18 respectively.

Step 7: The distance of each alternative from GTrLRIFPIS has been calculated for WQI weight and CRITIC weight, as shown in Table 19 and Table 20. In contrast, the distance of each alternative from GTrLRIFNIS has been calculated for WQI weight and CRITIC weight, as shown in Table 21 and Table 22.

Table 17. GTrLRIFPIS and GTrLRIFNIS Using WQI weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A^*	(0.0391,0.0397, 0.0412,0.0412; 0.2085,0.2085, 0.2165,0.2200; 0.9300; 0.0700)$_{LR}$	(0.1319,0.1571, 0.1900,0.1900; 0.0205,0.0205, 0.0248,0.0295; 0.8300; 0.1700)$_{LR}$	(0.1216,0.1408, 0.1408,0.1600; 0.0242,0.0274, 0.0274,0.0318; 0.8300; 0.1700)$_{LR}$	(0.0840,0.1220, 0.1220,0.1600; 0.0035,0.0046, 0.0046,0.0066; 0.7600; 0.2400)$_{LR}$	(0.0835,0.0851, 0.0876,0.0880; 0.1139,0.1144, 0.1178,0.1200; 0.9400; 0.0600)$_{LR}$	(0.0987,0.1188, 0.1500,0.1500; 0.0002,0.0002, 0.0003,0.0004; 0.8500; 0.1500)$_{LR}$
A^-	(0.1245,0.1384, 0.1630,0.2200; 0.0391,0.0527, 0.0621,0.0691; 0.8500; 0.1500)$_{LR}$	(0.0205,0.0229, 0.0250,0.0298; 0.1309,0.1557, 0.1700,0.1900; 0.9300; 0.0700)$_{LR}$	(0.0242,0.0264, 0.0304,0.0350; 0.1105,0.1273, 0.1462,0.1600; 0.9300; 0.0700)$_{LR}$	(0.0035,0.0035, 0.0055,0.0074; 0.0755,0.1012, 0.1600,0.1600; 0.9300; 0.0700)$_{LR}$	(0.1101,0.1148, 0.1148,0.1200; 0.0835,0.0873, 0.0873,0.0910; 0.8600; 0.1400)$_{LR}$	(0.0002,0.0002, 0.0005,0.0015; 0.0250,0.0750, 0.1500,0.1500; 0.9300; 0.0700)$_{LR}$

Table 18. GTrLRIFPIS and GTrLRIFNIS Using CRITIC Weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A^*	(0.0185,0.0188, 0.0196,0.0196; 0.0989,0.0989, 0.1027,0.1044; 0.9300; 0.0700)$_{LR}$	(0.0799,0.0952, 0.1151,0.1151; 0.0124,0.0124, 0.0150,0.0179; 0.8300; 0.1700)$_{LR}$	(0.0897,0.1038, 0.1038,0.1180; 0.0178,0.0202, 0.0202,0.0234; 0.8300; 0.1700)$_{LR}$	(0.1164,0.1691, 0.1691,0.2218; 0.0048,0.0063, 0.0063,0.0092; 0.7600; 0.2400)$_{LR}$	(0.2215,0.2256, 0.2324,0.2334; 0.3019,0.3033, 0.3123,0.3182; 0.9400; 0.0600)$_{LR}$	(0.0806,0.0970, 0.1225,0.1225; 0.0002,0.0002, 0.00025,0.0003; 0.8500; 0.1500)$_{LR}$
A^-	(0.0591,0.0657, 0.0774,0.1044; 0.0185,0.0250, 0.0295,0.0328; 0.8500; 0.1500)$_{LR}$	(0.0124,0.0139, 0.0152,0.0180; 0.0793,0.0943, 0.1030,0.1151; 0.9300; 0.0700)$_{LR}$	(0.0178,0.0195, 0.0224,0.0258; 0.0815,0.0939, 0.1078,0.1180; 0.9300; 0.0700)$_{LR}$	(0.0048,0.0048, 0.0076,0.0102; 0.1047,0.1403, 0.2218,0.2218; 0.9300; 0.0700)$_{LR}$	(0.2920,0.3045, 0.3045,0.3182; 0.2215,0.2314, 0.2314,0.2414; 0.8600; 0.1400)$_{LR}$	(0.0002,0.0002, 0.0004,0.0012; 0.0204,0.0613, 0.1225,0.1225; 0.9300; 0.0700)$_{LR}$

Table 19. Distance Measure for GTrLRIFPIS Using WQI weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.1436	0.0424	0.0566	0.1160	0.0073	0.0424
A_2	0.0610	0.1103	0.0842	0.0572	0.0636	0.0904
A_3	0.0392	0.1194	0.0937	0.0636	0.0583	0.0946
A_4	0.0273	0.1414	0.1123	0.1221	0.0031	0.1016
A_5	0.0000	0.1584	0.1315	0.1619	0.0002	0.1345

Table 20. Distance Measure for GTrLRIFPIS Using CRITIC Weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.0693	0.0424	0.0566	0.1244	0.0085	0.0424
A_2	0.0576	0.0668	0.0621	0.0793	0.0957	0.0738
A_3	0.0363	0.0776	0.0731	0.0636	0.0680	0.0782
A_4	0.0129	0.1025	0.0956	0.1239	0.0081	0.0891
A_5	0.0000	0.1112	0.1081	0.1999	0.0006	0.1146

Table 21. Distance Measure for GTrLRIFNIS Using WQI weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.0424	0.1445	0.1118	0.0930	0.0568	0.1229
A_2	0.1229	0.0814	0.0778	0.1483	0.0000	0.0920
A_3	0.1300	0.0502	0.0461	0.1347	0.0150	0.0752
A_4	0.1318	0.0253	0.0294	0.1075	0.0623	0.0784
A_5	0.1537	0.0000	0.0000	0.0141	0.0636	0.0000

Table 22. Distance Measure for GTrLRIFNIS Using CRITIC Weight

River	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.0424	0.0904	0.0830	0.1283	0.0888	0.1007
A_2	0.0583	0.0748	0.0747	0.1701	0.0000	0.0819
A_3	0.0645	0.0414	0.0415	0.1787	0.0397	0.0647
A_4	0.0800	0.0153	0.0217	0.1490	0.0896	0.0640
A_5	0.0883	0.0000	0.0000	0.0141	0.0955	0.0000

Step 8: The closeness coefficient of each alternative is obtained as shown in Table 23 for WQI weight while Table 24 for CRITIC weight.

Table 23. Separation Measures and The Relative Closeness Coefficient of Each Alternative Using WQI weight

River	d_i^*	d_i^-	CC_i
A_1	0.4083	0.5714	0.5832
A_2	0.4668	0.5224	0.5281
A_3	0.4689	0.4512	0.4904
A_4	0.5077	0.4347	0.4613
A_5	0.5865	0.2314	0.2830

Table 24 Separation Measures and The Relative Closeness Coefficient of Each Alternative Using CRITIC weight

River	d_i^*	d_i^-	CC_i
A_1	0.3437	0.5336	0.6082
A_2	0.4354	0.4598	0.5136
A_3	0.3969	0.4305	0.5203
A_4	0.4321	0.4196	0.4926
A_5	0.5345	0.1979	0.2703

Step 9: Determined the ranking order of all alternatives for determination of river water pollution using WQI weight and CRITIC weight.

6.0 Result and Discussion

Classifying river water pollution was a complex and challenging procedure since there were many aspects to consider simultaneously, as well as vagueness and subjectivity in the classification process. In this study, several rivers in Johor, Malaysia, have been evaluated, which are Kim Kim River, Sayong River, Telor River, Pelepah River, and Bantang River from 2017 to 2021. The criteria taken into consideration to determine river water pollution are dissolved oxygen (DO), biochemical oxygen demand (BOD), chemical oxygen demand (COD), suspended solid (SS), potential of hydrogen (pH), and ammoniacal nitrogen (AN). Using the generalised trapezoidal L-R intuitionistic fuzzy TOPSIS (GTrLRIF TOPSIS) method with WQI weight and CRITIC weight, the preferable solution was ranked based on the data obtained from DOE, Malaysia.

Table 25 summarises the comparison between the proposed and classical water quality index (WQI) methods. The result shows that Bantang River is the cleanest river. At the same time, Kim Kim River is the most polluted river for generalised trapezoidal L-R intuitionistic fuzzy TOPSIS (GTrLRIF TOPSIS) using WQI weight, GTrLRIF TOPSIS using CRITIC weight, and using the WQI method. The news in 2019 stated that the reports regarding the Kim Kim River incident resulted in dozens of students receiving medical treatment for respiratory issues and breathing difficulties caused by toxic chemical fumes [32]. The GTrLRIF TOPSIS using WQI weight and GTrLRIF TOPSIS using CRITIC weight show the slightest difference in rank at Sayong River and Telor River due to the different weight values. However, the GTrLRIF TOPSIS using WQI weight and GTrLRIF TOPSIS using CRITIC weight show the slightest different ordering of river water pollution with classical WQI method due to the consideration of confidence level in the evaluation process for the proposed methods of GLRIFNs. Since the proposed method of GLRIFNs considers the degree of confidence, it is more appropriate, flexible, and realistic because it can capture more uncertainty than the classical WQI method.

Table 25. Comparison Between Proposed Method and Water Quality Index (WQI) Method

River	GTrLRIF TOPSIS (WQI weight)		GTrLRIF TOPSIS (CRITIC Weight)		Water Quality Index		
	CC_i	Rank	CC_i	Rank	WQI	Rank	Class
A_1	0.5832	1	0.6082	1	44	1	IV (P)
A_2	0.5281	2	0.5136	3	83	3	II (C)
A_3	0.4904	3	0.5203	2	84	4	II (C)
A_4	0.4613	4	0.4926	4	82	2	II (C)
A_5	0.2830	5	0.2703	5	94	5	I (VC)
Order	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$		$A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5$		$A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_5$		

7.0 Conclusion

This study has discussed the TOPSIS and CRITIC method with generalised trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs) to classify the river water pollution for several rivers in Johor, Malaysia, namely Kim Kim River, Sayong River, Telor River, Pelepah River, and Bantang River from 2017 to 2021. At first, the river data obtained from DOE, Malaysia, was insufficient. Therefore, this study has simulated the river data using the bootstrap method and successfully simulated about 100 river data that later be used the Lee and Wang's [29] method to determine the trapezoidal L-R intuitionistic fuzzy numbers (TrLRIFNs) for five classes of river pollution. The height of membership and non-membership function of TrLRIFNs have been determined by the four decision-makers based on their level of certainty towards the reliability of the dataset for each of the parameters of the rivers, which are dissolved oxygen (DO), biochemical oxygen demand (BOD), chemical oxygen demand (COD), suspended solid (SS), potential of hydrogen (pH), and ammoniacal nitrogen (AN) and been assumed to be the same height for all five classes of river water pollution. The L and R functions in this study have been determined by the statistical distribution, which is normal distribution; therefore, this study used the L and R function suggested by Dubois and Prade [20]. Hence, GTrLRIFNs was developed for all classes of five rivers in Johor, Malaysia.

In the generalised trapezoidal L-R intuitionistic fuzzy TOPSIS (GTrLRIF TOPSIS) evaluation process, the rating of each alternative with respect to each criterion is extracted from the average data obtained from DOE, Malaysia, which has been bootstrapped due to insufficient river data. The normalised generalised trapezoidal L-R intuitionistic fuzzy decision matrix and weighted normalised generalised trapezoidal L-R intuitionistic fuzzy decision matrix are calculated. This study has used two weight types: the weight given by the Department of Environment Malaysia (DOE Malaysia) through the WQI formula (WQI weight) and the objective weight using the CRITIC method (CRITIC weight). In this study, the distance between each alternative from the generalised trapezoidal L-R intuitionistic fuzzy positive ideal solution (GTrLRIFPIS) and generalised trapezoidal L-R intuitionistic fuzzy negative ideal solution (GTrLRIFNIS) has also been calculated using Euclidean distance for both WQI and CRITIC weights. Therefore, the closeness coefficient of all alternatives is calculated, and the alternatives have ranked and classified.

The result shows that for generalised trapezoidal L-R intuitionistic fuzzy TOPSIS (GTrLRIF TOPSIS) using WQI weight, GTrLRIF TOPSIS using CRITIC weight, and using WQI method, Kim Kim River is the most polluted river, while Bantang River is the cleanest river. The GTrLRIF TOPSIS using WQI weight and GTrLRIF TOPSIS using CRITIC weight show the slightest different rank at Sayong River and Telor River due to the different weight values. The result also slightly differs between the proposed method (GTrLRIF TOPSIS using WQI weight and GTrLRIF TOPSIS using CRITIC weight) and classical WQI due to the consideration of confidence level in the evaluation process. Since the proposed procedure of GTrLRIF TOPSIS has the advantages of providing a comprehensive consideration of uncertainty of the data where the utilization of membership and the non-membership functions give a better representation of human evaluation process, it is more appropriate, flexible, and realistic compared to the classical WQI method for classifying river water pollution. Furthermore, the inclusion of confidence level values will give additional dimension of information in the evaluation process related to the judgment behaviour of the decision makers. The results proved that the GTrLRIF TOPSIS is a reliable method to classify river water pollution. Since its advantage is vast, the GTrLRIF TOPSIS is a useful method to classify river water pollution and may also be implemented in other fields.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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