MJFAS
Malaysian Journal of Fundamental and Applied Sciences

RESEARCH ARTICLE

An Improved Similarity-based Fuzzy Group Decision Making Model through Preference Transformation and K-Means Clustering Algorithm

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Abstract Group decision making plays a crucial role in organizational and community contexts, facilitating the exchange of expert opinions to arrive at effective decisions. The concept of preference, reflecting an individual's subjective evaluation of criteria or alternatives, forms a foundational element in this process. This study focuses on transforming non-fuzzy preferences, such as preference ordering and utility functions, into fuzzy preference relations (FPR) to address the uncertainty and uniformity inherent in expert preferences. To further enhance decision-making, we assess and visualize the similarity among the experts' uniform preferences. Integrating the K-means clustering algorithm into the fuzzy group decision making model allows for the predetermination of an appropriate number of groups based on the available alternatives. By aggregating individual preferences, we present a final ranking of alternatives. The enhanced methodology, as demonstrated through comparative analysis, showcases its ability to yield positive benefits when applied to decision-making applications.

Keywords: Transformation of Preferences, Fuzzy Preference Relation, K-Means Clustering, Similarity of Opinion, Fuzzy Group Decision Making.

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Received: 24 July 2023
Accepted: 7 Nov. 2023

Introduction

Group decision-making (GDM) model involves forming decisions as a team by gathering information and ideas from individuals with varied experiences, knowledge, and expertise to establish an agreement on a specific course of action. In GDM, it is important to consider not only the best alternative, but also the group consensus. Therefore, all experts involved in the process of making decisions must be satisfied with the final solution. This approach can be effective for problem-solving because it encourages diverse perspectives and fosters creativity. Fuzzy GDM is a process for making group decisions when fuzzy tools can be employed to convert imprecise and ambiguous information into fuzzy relationships, such as fuzzy objectives, constraints, and preferences. It can be used to design a decision-making process that considers how various individuals' preferences may interact with one another.

In decision-making, diverse preference representation formats are used, such as preference relations, preference ordering, utility functions and many more. Fuzzy preference relation, as proposed by Chiclana *et al*. [6] is a model used to make uniform information from various preference representation formats in multipurpose decision-making. Cabrerizo *et al*. [3] introduced the idea of information granularity as a method to approximate missing values and enhance the consistency of fuzzy preference relations. On the other hand, preference ordering involves process of arranging alternatives, indicating which ones are considered the best and which ones are considered the worst [5]. A utility function is a function used to determine the relationship between the fulfilment level of decision makers and the criterion employed in the decision outcome [13].

Social Network Analysis (SNA) in GDM is one of the recent concepts used to evaluate the similarity of perspectives among decision-makers or experts. This approach can help decision-makers to understand the network position and its impact on decision-making. The network structure shows the similarities of experts' opinion, the composition of clusters, and the way in which the clustering results are mapped to the experts' opinion similarities [8].

Clustering refers to the procedure of grouping a collection of items into subgroups based on their similarities, such that the items belonging to the same cluster exhibits a high level of resemblance to each other, while differing from items in other clusters [10]. Various clustering techniques have been developed and can be used in decision making. There are two main types of clustering algorithms, supervised and unsupervised. Researchers used different techniques of clustering methods including Hierarchical Clustering, Density-Based Clustering, K-Means clustering and many more. K-Means clustering algorithm is an unsupervised data analysis or data mining technique used for clustering data through a partitioning system [12]. The K-means technique endeavours to classify data into multiple groups, where the items within a group exhibit similar characteristics to each other, but differ from items in the other groups. Clustering algorithms have many applications including in grouping customers based on their purchasing behaviours or stock market companies based on their financial performance [9].

This study aims to address the uncertainty of human preferences by converting non-fuzzy preferences, which are utility function and preference ordering into Fuzzy Preference Relations (FPR). We integrate the K-means clustering algorithm incorporated into group decision-making model proposed by [8]. The K-means algorithm will determine an appropriate number of groups based on the number of alternatives or criteria. This improved methodology can help researchers in collecting opinions and map the clustering results based on their similarities of opinions. The advantage of utilization of K-Means algorithm in this research is the number of clusters can be pre-determined by considering the number of alternatives.

The paper follows a structured format, beginning with an introduction section that provides an overview of the topic. This is followed by subsequent sections on Methodology, Results and Discussion, and Comparative Analysis, which delve into further details of findings. Lastly, the Conclusion section is summarising the proposed work.

Methodology

In general, this study enhances the decision making model by Kamis *et al*. [8], and we have illustrated the framework of our proposed methodology in Figure 1. Initially, experts will discuss on the problem and provide their evaluations over alternatives in terms of non-fuzzy preferences, such as utility function and preference ordering. The different evaluation formats are intended to encompass a range of characteristics, aiming to standardize the information. The non-fuzzy evaluations are then transformed into a uniform fuzzy-based representation format, namely as Fuzzy Preference relations (FPR). The use of FPR as a base-format to convey an expert's viewpoint on a set of alternatives, emerges as a valuable tool for aggregating experts' preferences into a unified group preference. We employ the cosine similarity function to measure the degree of similarity among experts' preferences, which help experts to assess how closely aligned their preferences over alternatives. The experts with similar opinion need to be placed in a similar cluster, thus we implement the K-Means clustering algorithm. In this study, K-Means clustering algorithm is chosen because the number of clusters are depending on the number of alternatives in order to interpret the clustering results with ranking of alternatives. K-Means has capability to explicitly control over the desired number of clusters, therefore clusters provide prior knowledge or domain for understanding and making decisions. The uniform preferences from individual experts will be aggregated to a collective one and the final ranking of alternatives can be determined. The clustering results and the alternatives' ranking will be analysed and discussed. The coloured boxes represent the improvement works done in this study.

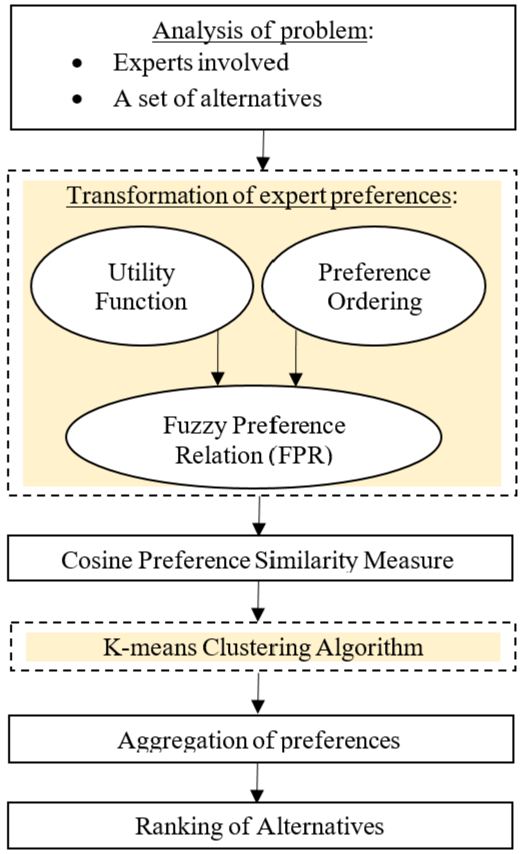

Figure 1. Framework of the proposed methodology

The specific methodology for this research is elaborated below. Let E represents a group of experts, where $E = \{e_{1,}e_{2,\dots,}e_m\}$ and A represents a set of alternatives, where $A = \{a_{1,}a_{2,\dots,}a_n\}$.

Transformation of Expert Preferences

The transformation step needs to be taken into account when different non-fuzzy preference representation formats are involved. In this case, each expert conveys his/ her preferences over alternatives in terms of preference ordering and utility functions. This different information might difficult to handle, especially in implementing decision making process, such as in measuring the preference similarities, clustering algorithm and aggregation phase. In order to carried out these processes, these different formats must be firstly in a uniform form.

The definitions of preference ordering and utility function are stated below, with their respective transformation functions to FPR context.

i. *Preferences Ordering* [7]

An expert denoted as e_k expresses their preferences on the alternatives as an individual preference ordering, $o^k = \{o^k(1), \dots, o^k(n)\}$ where $o^k(.)$ represents a permutation function over the index set denoted as $\{1, \dots, n\}$, for the expert, e_k.

The preference ordering, o^k is converted to FPR format, r_{ij}^k by applying the following transformation function:

$$r_{ij}^k = \frac{1}{2}\left(1 + \frac{o^k(j)}{n-1} - \frac{o^k(i)}{n-1}\right), \quad (1)$$

where n denotes the number of alternatives.

ii. *Utility Function* [7]

Let U^k be a set of preferences of expert e_k, $e_k \epsilon E$ on alternative A represented as utility values where $e_k U^k = \{u_i^k, i = 1, \ldots, n\}; u_i^k$ reflects the performance of alternatives based on expert e_k opinion.

It is possible to transform a utility function into FPR by obtaining the expert e_k preference value of alternative a_i over a_j, r_{ij}^k. The utility function, u^k is converted to FPR format, r_{ij}^k by applying the following transformation function:

$$r_{ij}^k = l\left(\frac{u_i^k}{u_j^k}\right) = \frac{\frac{u_i^k}{u_j^k}}{\frac{u_i^k}{u_j^k}+\frac{u_i^k}{u_j^k}} = \frac{\left(u_i^k\right)^2}{\left(u_i^k\right)^2+\left(u_j^k\right)^2}, i \neq j \qquad (2)$$

iii. *Fuzzy Preference Relations*

Definition 1 [4] A Fuzzy preference relation on A refers to a fuzzy binary relation R that captures the preference intensity of experts for alternative i over alternative j. It is required that $\mu_R(a_i, a_j) = r_{ij}$, and satisfy the conditions of $\mu_R(a_i, a_j) = 0.5\ \forall_{a_i} \in A$ and $r_{ij} + r_{ji} = 1, \forall_{a_i, a_j} \in A$.

Let represent $R_{n \times n}$ be the set of $n \times n$ matrices R that are generated from all fuzzy preference relations on A :

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \cdots \\ r_{n1} & r_{n2} & \cdots & r_{mn} \end{bmatrix} \qquad (3)$$

verify that: $0 \leq r_{ij} \leq 1$ and $r_{ij} + r_{ji} = 1$ for $i, j \in \{1,2, \ldots, n\}$.

Cosine Preference Similarity Measure

In order to measure the closeness of expert opinions, the similarity function is used. In this study, the cosine preference similarity measure is defined as:

Definition 2 [8] The measurement of cosine preference similarity between preference of expert e^r and e^s is:

$$CQ^{rs} = CQ(e^r, e^s) = \frac{\sum_{i=1}^{\frac{n(n-1)}{2}} (e^r, e^s)}{\sqrt{\sum_{i=1}^{\frac{n(n-1)}{2}} (e^r)} \cdot \sqrt{\sum_{i=1}^{\frac{n(n-1)}{2}} (e^s)}} \qquad (4)$$

K-Means Clustering Algorithm

The initial algorithm of the K-means clustering was presented by Lloyd [11]. The K-Means algorithm utilises distance-based metrics to evaluate the similarity among data points. It works by dividing a set of data into K clusters based on measurements, each represented by a cluster centre or centroid. K-Means algorithm has been widely used as a partitional algorithm [1] [2]. It commonly utilises the Euclidean distance measure, which is expressed as:

$$\sqrt{\sum_{i=1}^{d} (x_i - y_i)^2} \qquad (5)$$

where in a *d*-dimensional Euclidean space, the points x_i and y_i are cosidered. The objective function known as the Sum of Squared Error (SSE) is to be minimized. It can be expressed using the following equation:

$$SSE = \sum_{i=1}^{d} \sum_{X_i \in C_k} (X_i - c_k)^2 \qquad (6)$$

The cluster centroid c_k can be revised as:

$$c_k = \frac{\sum x_{i \in c_k} x_i}{c_k} \quad (7)$$

The K centroids are iteratively updated to minimize the average squared distance separating each data point and its nearest centroid [11].

Aggregation of Preferences

Individual preferences can be combined into a collective preference using an OWA-based aggregation operation.

Definition 3 [14]. An OWA operator of dimension n can be defined as a mathematical function $\emptyset: \mathbb{R}^n \rightarrow \mathbb{R}$ that is in addition to a weighting vector, $V = (v_1, \ldots, v_m)$. The operator satisfies the conditions $v_i \in [0,1]$ and $\sum_{i=1}^{n} v_i = 1$, which can be expressed as follows:

$$\emptyset(r_1, \ldots, r_m) = \sum_{i=1}^{m} v_i . r_{\sigma(i)}, \quad (8)$$

where $\sigma: \{1, \ldots, m\} \rightarrow \{1, \ldots, m\}$ be a permutation such that $r_{\sigma(i)} \geq r_{\sigma(i+1)}, \forall_i = 1, \ldots, m-1$.

In the case of a regular increasing monotone (RIM) quantifier, $Q(k): [0,1] \rightarrow [0,1]$ such that $Q(0) = 0$ and $Q(1) = 1$. The weights of the linguistic OWA operator influenced by quantifier are calculated using the following expression can be stated as:

$$v_i = Q\left(\frac{i}{m}\right) - Q\left(\frac{i-1}{m}\right), i = 1, \ldots, m. \quad (9)$$

The operator for induced ordered weighted averaging (IOWA) is additional capabilities of the OWA operator. The rearrangement of arguments in this operator is influenced by an additional mechanism an order-inducing variable, which based on the ordered position of the arguments.

Definition 4 [15]. The IOWA operator, denoted as a function of dimension n, $\emptyset_w: (\mathbb{R} \times \mathbb{R})^n \rightarrow \mathbb{R}$ is defined by a set of weighting vector, $V = (v_1, \ldots, v_m)$. It satisfies the conditions of $v_i \in [0,1]$ and $\sum_{i=1}^{n} v_i = 1$, and it aggregates the set of second arguments $\{\langle u_1, r_1 \rangle, \ldots, \langle u_n, r_n \rangle$ using the following function:

$$\emptyset_w\{\langle u_1, r_1 \rangle, \ldots, \langle u_n, r_n \rangle\} = \sum_{i=1}^{n} v_i . r_{\sigma(i)} \quad (10)$$

Ranking of Alternatives

The application of the Quantifier Guided Dominance Degree (QGDD), utilising the OWA operator as described in Definition 4 is used to rank the alternatives.

Definition 5 [7]. When considering a collective preference relation, $R^c = r_{ij}^c$ for a set of alternatives $A = \{a_1, a_2, \ldots, a_n\}$, the quantifier guided dominance degree, QGDD (a_i), is employed to assess the degree of dominance of one alternative a_i over the rest of the alternatives in a fuzzy majority as follows:

$$QGDD(a_i) = \phi_Q(r_{ij}^c, j = 1, \ldots, n, j \neq i) \quad (11)$$

where the value of ϕ_Q is determined based on the linguistic quantifier Q, which signifies the representation of fuzzy majority in relation to an OWA operator.

Results and Discussion

Hypothetically, a group of 30 experts, $e = \{e_1, e_2, \ldots, e_{30}\}$ provides their evaluations over the set of 6 alternatives involved, $A = \{A_1, A_2, \ldots, A_6\}$. Some experts give their evaluations in terms of preference ordering, such as expert 1 (O_1). The preference ordering is transformed into FPR using Equation (1), as shown in matrix $P1$.

$$O_1 = \{5,2,1,3,6,4\} \longrightarrow P1 = \begin{bmatrix} 1 & 0.2 & 0.1 & 0.3 & 0.6 & 0.4 \\ 0.8 & 1 & 0.4 & 0.6 & 0.9 & 0.7 \\ 0.9 & 0.6 & 1 & 0.7 & 1 & 0.8 \\ 0.7 & 0.4 & 0.3 & 1 & 0.8 & 0.6 \\ 0.4 & 0.1 & 0 & 0.2 & 1 & 0.3 \\ 0.6 & 0.3 & 0.2 & 0.4 & 0.7 & 1 \end{bmatrix}$$

Other expert, such as Expert 2 evaluates the alternatives in the form of utility function. Thus, U_2 can be transformed into FPR using Equation (2), as demonstrated in matrix $P2$.

$$U_2 = \{0.7,0.6,0.4,0.5,0.3,0.2\} \longrightarrow P2 = \begin{bmatrix} 1 & 0.6 & 0.8 & 0.7 & 0.8 & 0.9 \\ 0.4 & 1 & 0.7 & 0.6 & 0.8 & 0.9 \\ 0.2 & 0.3 & 1 & 0.4 & 0.6 & 0.8 \\ 0.3 & 0.4 & 0.6 & 1 & 0.7 & 0.9 \\ 0.2 & 0.2 & 0.4 & 0.3 & 1 & 0.7 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.3 & 1 \end{bmatrix}$$

From the similarity of experts' preferences measured using Equation (4), the K-Means clustering algorithm can be implemented. Given the presence of 6 alternatives, we have pre-determined the creation of 6 clusters. This decision is essential to ensure that the clustering outcomes correspond effectively to the alternative rankings. In our future research, we intend to establish a connection between the ranking results and the members within their respective clusters, thereby facilitating an in-depth analysis of the relationships among experts, their opinions, clusters, and rankings.

With the utilization of 6 clusters, the K-Means algorithm initiates the process by selecting an initial set of 6 cluster centroids. Data points are then assigned to the cluster whose centroid is in closest proximity, determined by the Euclidean distance metric (Equation (5)). Once all data points find their respective clusters, the centroids of these clusters are updated to reflect the mean of the data points within each cluster (Equation (6)). This iterative centroid adjustment process, outlined in Equation (7), aims to minimize the average squared distance between each data point and its closest centroid, ensuring that the centroids accurately represent the data within their clusters. The visualisation of clustering result is presented in Figure 2 below:

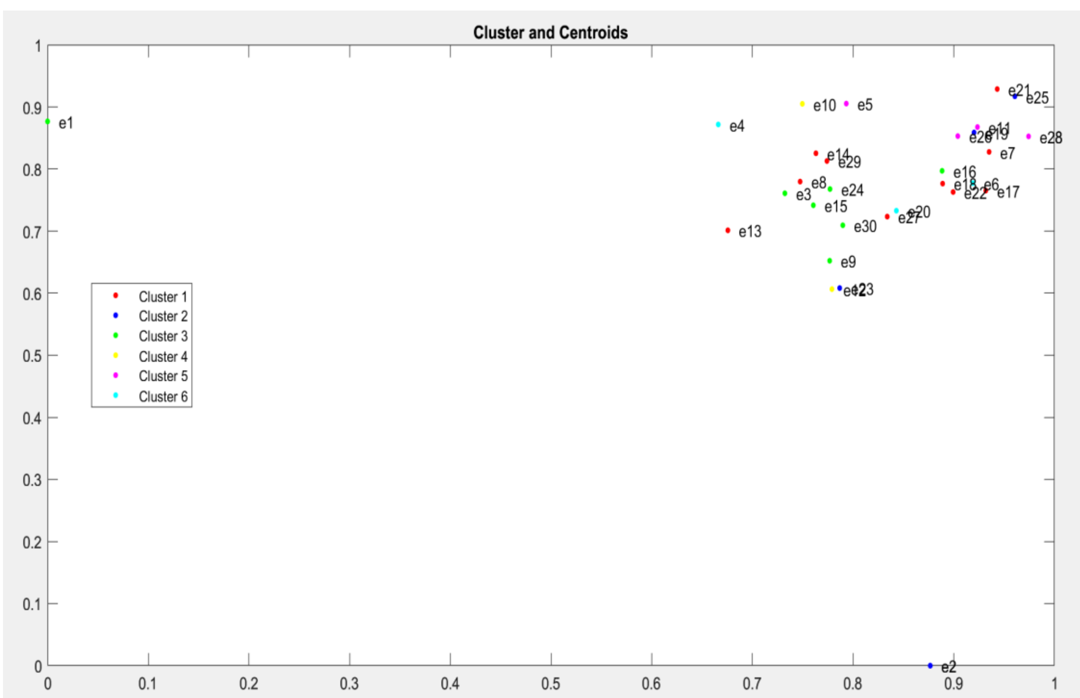

Figure 2. K-means clustering results based on experts' preference similarities

Figure 2 visually demonstrates the distinct placement of expert e_1 and e_2, who are positioned separately and are at a greater distance from the other experts. These two experts hold differing opinions compared to other experts in the network. Other experts are clustered closely to each other, meaning that their preferences are similar. In order to clearly interpret the clustering result, we list the associated experts cluster in Table 1.

Table 1. 6 clusters with their associated experts

Cluster	Cluster members (expert)
1	$e_7, e_8, e_{13}, e_{14}, e_{17}, e_{18}, e_{21}, e_{22}, e_{27}, e_{29}$
2	$e_2, e_{19}, e_{23}, e_{25}$
3	$e_1, e_3, e_9, e_{15}, e_{16}, e_{24}, e_{30}$
4	e_{10}, e_{12}
5	$e_5, e_{11}, e_{26}, e_{28}$
6	e_4, e_6, e_{20}

Using Equation (8-10), the collective preference relation, P^c is obtained as follows:

$$P^c = \begin{bmatrix} 1 & 0.469 & 0.467 & 0.505 & 0.592 & 0.495 \\ 0.531 & 1 & 0.502 & 0.538 & 0.621 & 0.530 \\ 0.533 & 0.498 & 1 & 0.538 & 0.626 & 0.533 \\ 0.495 & 0.462 & 0.462 & 1 & 0.587 & 0.488 \\ 0.407 & 0.378 & 0.373 & 0.413 & 1 & 0.404 \\ 0.505 & 0.469 & 0.467 & 0.512 & 0.596 & 1 \end{bmatrix}.$$

The alternatives are ranked (Equation 11) and arranged in descending order as follows:

$$A_3 \succ A_2 \succ A_6 \succ A_4 \succ A_1 \succ A_5.$$

Based on final ranking of alternative, the results indicate that the most preferred alternative is A_3, followed by A_2, A_6, A_4, and A_1. The least preferred alternative is A_5.

Comparative Analysis

The main aim of this paper is to improve the existing group decision making model, introduced by Kamis *et al*. [8]. Several elements have been compared between both methods, as depicted in Table 2 below:

Table 2. The comparisons of Kamis *et al*. [8] and the proposed work

Comparison elements	Original method (Kamis *et al*. [8])	Our proposed model
Consideration of non-fuzzy preference formats	No	Yes
Transformation of non-fuzzy to fuzzy preferences	No	Yes
Clustering algorithms	Agglomerative hierarchical clustering	K-Means
Predetermine number of clusters	No	Yes

For the purpose of achieving our main objective, several important factors have been taken into consideration. In this study, we incorporate expert evaluations in the form of non-fuzzy preferences, specifically preference ordering and utility function. This step is an addition to the original method proposed by Kamis *et al*. [8]. We find it essential to include this element in our work because not all individuals (experts) are well-versed in or familiar with the concept of fuzzy set theory. When people need to evaluate or rate something, they often prefer to use ordering or assign direct values to the evaluation set (utility function). Thus, this procedure offers flexibility to people with no knowledge on fuzzy set theory, enabling them to smoothly conduct the evaluation process.

Instead of its simplicity, non-fuzzy preferences unable to accommodate the inherent imprecision or vagueness of human preferences. Experts rely on making clear-cut distinctions and rankings, without acknowledging the existence of gradations or varying degrees of preference. For instance, when experts order criteria or alternatives, they may rank them from the best to the worst, without quantifying the difference in preference between them. This limitation can be addressed by incorporating fuzzy-based representation formats, such as fuzzy preference relations (FPR). In order to transform preference ordering and utility functions into FPR, specific transformation functions must be used. This procedure has been applied in our study but was not considered in Kamis *et al*. [8] decision-making model.

In the study by Kamis *et al*. [8], they employed the agglomerative hierarchical clustering algorithm to group experts based on their similarity of opinion. The resulting clusters were used to assess the consensus within the group and to establish a feedback mechanism when the consensus level was insufficient. The optimal number of clusters was determined by the highest degree of cluster consensus, meaning that the original decision-making model did not specify the number of clusters in advance. In our current study, we opted for the K-means clustering technique because it allows for the initial determination of the number of clusters, aligning with our aim to have the same number of clusters with respect to the number of alternatives.

Conclusion

This study modified the group decision-making model based on preference similarity network clustering proposed by Kamis *et al*. [8]. To accommodate different preference representation formats, we include a transformation procedure in the study. Experts are requested to express their preferences regarding a set of alternatives using preference ordering and utility function. In order to standardize the preference formats, a uniform context using FPR is presented. This step helps the researchers or experts to handle the uncertainty and vagueness of human preferences in decision making process.

The K-means clustering algorithm is widely used in partitioning objects into subgroups. Determining appropriate number of clusters for a data set is an essential process before adopting K-means. This study focuses on partitioning experts based on their preference similarities and mapping the clustering results with the pre-determined number of alternatives. Clusters with associated experts are clearly presented, the individual experts' preferences are successfully aggregated into a collective one and the ranking of alternatives are obtained.

This work can be extended, where the mapping of clusters with alternatives needs to be taken into account. Other clustering algorithms or aggregation operators can be explored further. Additionally, this proposed model can be used as an alternative tool in solving many decision making problems.

Acknowledgment

This work is financially supported by Geran Penyelidikan Khas (600-RMC/GPK 5/3 (162/2020)), Universiti Teknologi MARA (UiTM), Shah Alam, Selangor.

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