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RESEARCH ARTICLE

Type-2 Intuitionistic Interpolation Cubic Fuzzy Bézier Curve Modeling using Shoreline Data

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Abstract The notion of fuzzy sets is fast becoming a key instrument in defining the uncertainty data and has increasingly been recognised by practitioners and researchers across different disciplines in recent decades. The uncertainty data cannot be modeled directly and this causes hindrance in obtaining accurate information for analysis or predictions. Hence, this paper contributes to another approach in which an application of type-2 intuitionistic fuzzy set (T-2IFS) in geometric modeling onto complex uncertainty data where the data are defined using the type-2 fuzzy concept. T-2IFS is the generalized forms of fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and interval-valued intuitionistic fuzzy sets. Based on the concept of T2IFS, type-2 intuitionistic fuzzy point (T-2IFP) is defined in order to generate a type-2 intuitionistic fuzzy control point (T-2IFCP). Following, the T-2IFCP will be blended with the Bernstein blending function through the interpolation method, resulting to a type-2 intuitionistic interpolation cubic fuzzy Bézier curve. Shoreline data is used as the data and further verifies that the model can be conceivably accepted. In conclusion, the proposed methods are reliable and can be expanded to many other areas.

Keywords: Type-2 intuitionistic fuzzy set, Bézier curve, Real data.

Introduction

Both human and artificial intelligence for data analytics may offer great promise, but when they are used in situations with complex uncertainty, a variety of difficulties arise. These data that are collected from multiple sources such as sensors, satellite images, and others are fundamentally uncertain due to noise, inconsistency, and incompleteness [6]. For instance, shorelines are the most dynamic phenomenon in nature with a high level of inherent uncertainty and they are exposed to several complicated processes acting at various temporal and spatial scales over time [27]. Remarkably, the advancement in technology has helped in monitoring the shoreline changes such as using satellite images. However, there is an existence of complex uncertainty in the data since the shoreline changes are intensified by the impacts of wind, waves, and tidal [2]. Therefore, a different approach should be employed in handling such cases since traditional methods are incompetent in manipulating them.

The premise of fuzzy sets by Zadeh [19] that victoriously abandoned the concept of “two-valued logics” has accommodated a whole new viewpoint on the concept of classical set. Then, Zadeh [20] defined type-2 fuzzy sets or T-2FSs which is the extension of type-1 fuzzy set or T-1FS such that T-1FSs that have a crisp membership grade compared to T-2FS have a membership grade that is fuzzy [7]. Atannasov [1] introduced intuitionistic fuzzy sets (IFS) or type-1 intuitionistic fuzzy sets (T-1IFS) as a fuzzy sets generalization that explicated the membership, non-membership and hesitation degrees as the aggregate from the degrees of membership and non-membership is minus by one (1). As the basic concept in fuzzy set theory is the function should be between 0 and 1, the totality of these pivotal components in IFS should be less than or equal to one (1). Cuong *et al*. [5] explained on several

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computations of type-2 intuitionistic fuzzy sets (T-2IFS) and T-2IFS can deal with complex uncertainties similar to T-2FS since they are considering the non-membership function [3].

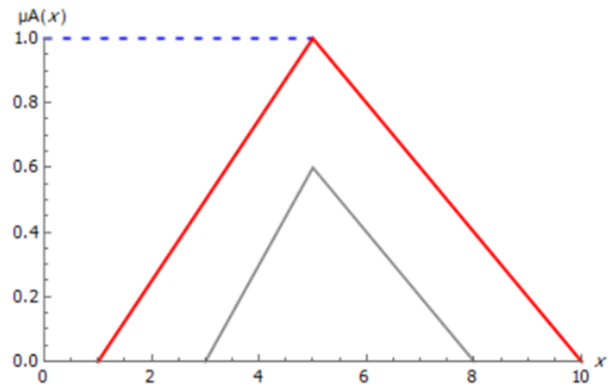

Figure 1. Triangular type-2 fuzzy number

Previously, Zakaria *et al*. have discussed on modeling the uncertainty or complex uncertainty data in the form of curve such as [22,25,26] or surface such as [24,21]. The implementation of the concept of fuzzy number or its extension to define the data is necessary to tackle the core issue and followed by fuzzification, type-reduction (if necessary) and defuzzification before they are blended with the curve or surface function in order to construct the model. Also, Wahab *et al*. [13,14,15,17] published some fuzzy geometric modeling and constructed one using interpolation Bézier curve in which this paper will be considering type-2 intuitionistic interpolation fuzzy Bézier curve modeling.

To highlight, the data may not be measured properly, and uncertainty is the quantitative measurement of data error and vague refers to something that is not clearly defined or applied. As a result, noisy data also contains some ambiguous predicates. Moreover, the traditional models have difficulty in quantifying complex uncertainty which means there is fuzziness in fuzziness and produce overconfident analysis or predictions [18]. Henceforth, one of the objectives of this study is to define the complex uncertainty data by using the type-2 intuitionistic fuzzy number concept. Next, the other objectives are to implement type-2 intuitionistic fuzzy Bézier model in curve and apply type-2 intuitionistic fuzzy Bézier curve model in modeling the shoreline data. Therefore, in this paper, the process will start with definition of the data using T-2IFSs that will create the type-2 intuitionistic fuzzy data points. Next, the fuzzification process will occur through an alpha-cut operation for the reason that the alpha value is computed from the Barycentric coordinate. Subsequently, membership functions will undergo total defuzzification by the deployment of mean operator to defuzzify into a single point and the points will be modeled using the Bézier curve function through interpolation method. The paper is organized, particularly, in the first section, a basic introduction and some related previous works are discussed. Then, in the next section, basic definitions are presented to grasp the idea of fuzzy sets and their extension together with the Bézier curve. The following section elaborates on the development of the curve model from defining the data to the defuzzification process before the next section will further illustrate them using shoreline data. Then, the discussion and conclusion will be shown before the final section that will conclude the research.

Preliminaries

Given that T-2IFS has a remarkable ability to simulate information with complex uncertainty and vagueness, in this section, some of the basic definitions of fuzzy set and its extensions along with the Bézier curve will be provided.

Definition 1

Fuzzy set G in X which is the objects cluster represented by x can be implied as

$$G = \{(x, \mu_G(x)) | x \in X\} \tag{1}$$

where $\mu_G(x)$ is the membership degree besides $\mu_G(x): X \to [0,1]$ [10].

Definition 2

A membership function in type-2, $\mu_{\overleftrightarrow{G}}(x,u)$ characterized a type-2 fuzzy set $\overleftrightarrow{G}$ whereas $u \in J_x \subseteq [0,1]$ and $x \in X$, therefore

$$\overleftrightarrow{G} = \left\{ \left((x,u), \mu_{\overleftrightarrow{G}}(x,u) \right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1] \right\} \tag{2}$$

Where $0 \le \mu_{\overleftrightarrow{G}}(x,u) \le 1$ and $\mu_{\overleftrightarrow{G}}(x,u)$ is a T-1FS and can be identified as the secondary set. Also, x is the primary membership and u is the secondary membership in $J_x \subseteq [0,1]$ [20,12].

Definition 3

Intuitionistic fuzzy set G^* in X is denoted as

$$G^* = \left\{ x, \mu_{G^*}(x), \nu_{G^*}(x) \middle| x \in X \right\} \tag{3}$$

where $\mu_{G^*}(x), \nu_{G^*}(x) \to [0,1]$ are the membership degree along with degree of non-membership, correspondingly, of an element $x \in X$ with $0 \le \mu_{G^*}(x) + \nu_{G^*}(x) \le 1$. Furthermore, there is a degree of indefiniteness or also termed as an intuitionistic fuzzy set index, $\pi_{G^*}(x) = 1 - \mu_{G^*}(x) - \nu_{G^*}(x)$ of $x \in X$ to G^* and $\pi_{G^*}(x) \in [0,1]$ [1].

Definition 4

Type-2 intuitionistic fuzzy set (T-2IFS) G in X is defined as

$$G = \left\{ \left\langle \left(x, \mu_G, \nu_G \right), f_x(\mu_G), \mathrm{t}_x(\nu_G) \right\rangle \middle| x \in X, \mu_G(x) \in j_x^1, \nu_G(x) \in j_x^2 \right\} \tag{4}$$

where in the domain element $\left(x, \mu_G\left(x\right), \nu_G\left(x\right) \right)$, $\mu_G\left(x\right)$ is the function of principal membership and $\nu_G\left(x\right)$ is the principal non-membership function of $x \in X$ while $f_x\left(\mu_G\right)$ and $\mathrm{t}_x\left(\nu_G\right)$ are the memberships for the function of principal membership (supplementary membership function) and principal non-membership (supplementary non-membership function), respectively where $\mu_G\left(x\right) \in j_x^1 \in [0,1]$, $\nu_G\left(x\right) \in j_x^2 \in [0,1]$ and j_x^1, j_x^2 are the principal membership function and principal of non-membership function of x, respectively [9].

Definition 5

P_i, $(i = 0,1,2,...,n)$, $P_i \in E^3$ be a set of points with degree of n. Hence, the general equation is denoted as

$$B(t) = \sum_{i=0}^{n} B_i^n(t) P_i, t \in [0,1] \tag{5}$$

in which $B_i^n(t) = \sum_{i=0}^{n} \binom{n}{i} (1-\mathrm{t})^{n-i} t^i$ is the function of Bernstein polynomial while P_i is the control points [11].

Methodology

In this sector, the process of data definition via T-2IFS prior to the defuzzification process will be further clarified. Foremost, the definition of a triangular type-2 intuitionistic fuzzy number is as shown below:

Definition 6

The type-2 intuitionistic fuzzy number (T-2IFN) in triangular form, G is denoted by

$$G = \left\langle \left(G^{*}{}_{1}, G^{*}{}_{2}, G^{*}{}_{3} \right); \mu_G, \nu_G \right\rangle \tag{6}$$

where $G^{*}{}_{1}$, $G^{*}{}_{2}$ together with $G^{*}{}_{3}$ portrays the T-1IFS in triangular form while μ_G is the membership degree while ν_G is the non-membership degree [8]. In this study, a perfectly normal of triangular T-2IFN is implemented and an example is shown in Figure 2 below.

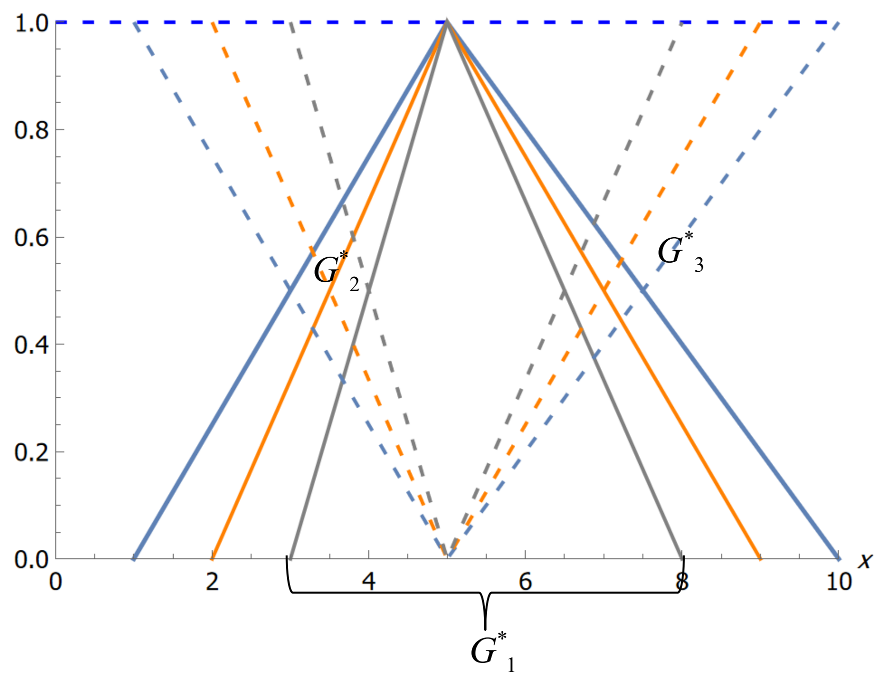

Figure 2. Perfectly normal of triangular T-2IFN

As in this study the value of $\mu_Q = 1$ and $\nu_Q = 0$, hence $Q^{*}{}_{1}$, $Q^{*}{}_{2}$ and $Q^{*}{}_{3}$ are the triangular fuzzy number (Q_1, Q_2, Q_3) [4]. The triangular fuzzy number Q_1 is defined as:

Definition 7

$$\mu_{Q_1} = \begin{cases} 0 \quad \text{if} \quad x \le m_1 \\ \dfrac{x - m_1}{n_1 - m_1} \quad m_1 < x \le n_1 \\ \dfrac{o_1 - x}{o_1 - n_1} \quad n_1 < x \le o_1 \\ 0 \quad \text{if} \quad x > o_1 \end{cases} \tag{7}$$

The groups of points that adjust the curvature of a spline curve are referred to as control points. Next, we will define the complex uncertainty data using T-2IFS. T-2IFS is an incredible extension of IFS henceforth, the definition of intuitionistic fuzzy data points (IFCP) is shown as below [16]:

Definition 8

In space S, the IFS of P^* is IFCP and a set of IFCPs, $P^* = \{P_i^*\}_{i=0}^{n}$ where $\mu_P : S \to [0,1]$ represents the function of membership, $\nu_P : S \to [0,1]$ is the function of non-membership and $\pi_P : S \to [0,1]$ is the uncertainty function with

$$\mu_P(\mathrm{P}_i^{*-}) = \begin{cases} 0 & \text{if} \quad P_i^- \notin S \\ c \in (0,1) & \text{if} \quad P_i^- \in S \\ 1 & \text{if} \quad P_i^- \in S \end{cases} \quad \text{and} \quad \mu_P(\mathrm{P}_i^{*+}) = \begin{cases} 0 & \text{if} \quad P_i^+ \notin S \\ k \in (0,1) & \text{if} \quad P_i^+ \in S \\ 1 & \text{if} \quad P_i^+ \in S \end{cases} \tag{8}$$

$$\nu_P(\mathrm{P}_i^{*-}) = \begin{cases} 0 & \text{if} \quad P_i^- \notin S \\ d \in (0,1) & \text{if} \quad P_i^- \in S \\ 1 & \text{if} \quad P_i^- \in S \end{cases} \quad \text{and} \quad \nu_P(\mathrm{P}_i^{*+}) = \begin{cases} 0 & \text{if} \quad P_i^+ \notin S \\ l \in (0,1) & \text{if} \quad P_i^+ \in S \\ 1 & \text{if} \quad P_i^+ \in S \end{cases} \tag{9}$$

$$\pi_P(\mathrm{P}_i^{*-}) = \begin{cases} 0 & \text{if} \quad \mu_P(P_i^-) \in S \cup \nu_P(P_i^-) \notin S \\ e \in (0,1) & \text{if} \quad \mu_P(P_i^{*-}) + \nu_P(P_i^{*-}) \in S \le 1 \\ 1 & \text{if} \quad \mu_P(P_i^-) \notin S \cup \nu_P(P_i^-) \in S \end{cases} \tag{10}$$

and

$$\pi_P(\mathrm{P}_i^{*+}) = \begin{cases} 0 & \text{if} \quad \mu_P(P_i^+) \in S \cup \nu_P(P_i^+) \notin S \\ m \in (0,1) & \text{if} \quad \mu_P(P_i^{*+}) + \nu_P(P_i^{*+}) \in S \le 1 \\ 1 & \text{if} \quad \mu_P(P_i^+) \notin S \cup \nu_P(P_i^+) \in S \end{cases} \tag{11}$$

where $\mu_P(\mathrm{P}_i^{*-})$, $\nu_P(\mathrm{P}_i^{*-})$ and $\pi_P(\mathrm{P}_i^{*-})$ are membership grade on the left while $\mu_P(\mathrm{P}_i^{*+})$, $\nu_P(\mathrm{P}_i^{*+})$ and $\pi_P(\mathrm{P}_i^{*+})$ are the grade of membership on the right. For respective i,

$\mathrm{P}^* = \langle \mathrm{P}_i^{*-}, \mathrm{P}_i, \mathrm{P}_i^{*+} \rangle$ with left, crisp and right control points, respectively and can be generally denoted as

$$\mathrm{P}^* = \mathrm{P}_i^* : i = \{0,1,2,...,n\} \tag{12}$$

Therefore, in the similar vein, the definition for type-2 intuitionistic fuzzy control points (T-2IFCP) can be referred from Definition 8 in accordance to Definition 6 with $\mu_P(\mathrm{q}_i^-)$, $\nu_P(\mathrm{q}_i^-)$, $\pi_P(\mathrm{q}_i^-)$ are left-left membership grade, $\mu_P(\mathrm{r}_i^-)$, $\nu_P(\mathrm{r}_i^-)$, $\pi_P(\mathrm{r}_i^-)$ are membership grade on the left including $\mu_P(\mathrm{s}_i^-)$, $\nu_P(\mathrm{s}_i^-)$, $\pi_P(\mathrm{s}_i^-)$ are membership grade on left-right meanwhile $\mu_P(\mathrm{s}_i^+)$, $\nu_P(\mathrm{s}_i^+)$, $\pi_P(\mathrm{s}_i^+)$ are membership grade for right-left, $\mu_P(\mathrm{r}_i^+)$, $\nu_P(\mathrm{r}_i^+)$, $\pi_P(\mathrm{r}_i^+)$ are membership grade on the right together with $\mu_P(\mathrm{q}_i^+)$, $\nu_P(\mathrm{q}_i^+)$, $\pi_P(\mathrm{q}_i^+)$ are membership grade on right-right.

Following, the fuzzification process will take place through alpha-cut operation to have smaller intervals. As mentioned earlier, since this study considered the value of $\mu_A = 1$ and $\nu_A = 0$, only alpha value will be computed using the Barycentric coordinate. Moving to another critical steps in this study, the formula of total defuzzification is shown as below:

Definition 9

Let W_i type-2 be the defuzzification value,

$$W_i = \frac{1}{3}\left(\sum\left(\frac{q_i^- + r_i^- + s_i^-}{3}\right), P_i, \left(\frac{q_i^+ + r_i^+ + s_i^+}{3}\right)\right) \tag{13}$$

where q_i^-, r_i^- and s_i^- are left membership grade and q_i^+, r_i^+ and s_i^+ are right membership grade. To summarize, Figure 3 below is the high-level processes that provides an overall summary in this study.

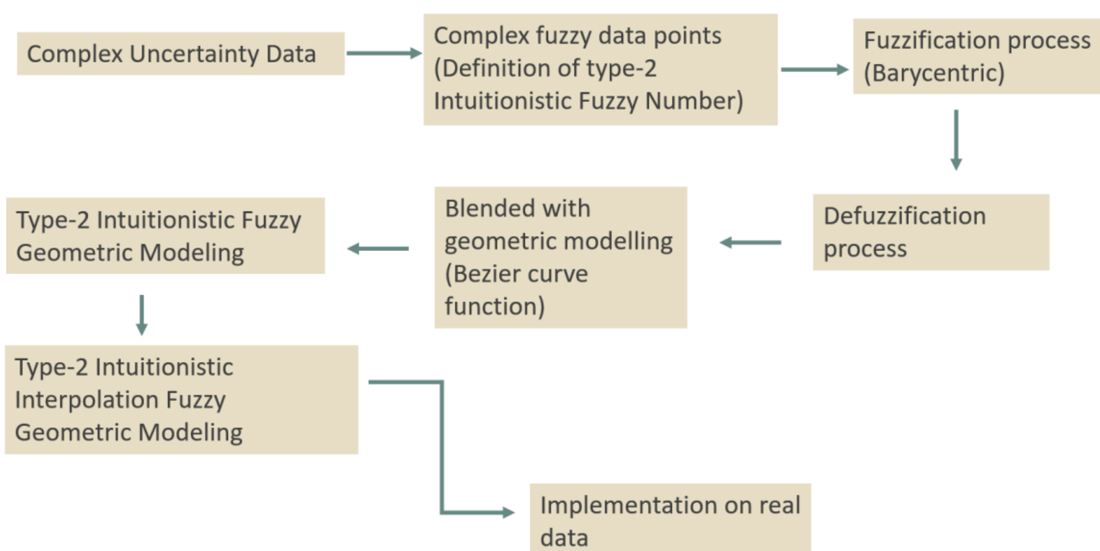

Figure 3. High-level process

Results and Discussion

The implementation and methodology of the proposed T-2IFS system for tackling the complex uncertainty data have already been discussed in previous sections. Hence, shoreline data from Mamutik will be utilized to model the type-2 intuitionistic interpolation cubic fuzzy Bézier curve. Also, one of the scopes in this research is the T-2IFCP generated are for the shoreline partially. Next, Figure 4 below illustrates the fuzzification curve which is the output after fuzzification process as per Definition 8 and blended together with Equation 5 while Figure 5 shows the defuzzification curve in which Equation 13 is also blended with Equation 5 by using the T-2IFN shoreline data in order to retrieve a single point and eventually a single curve is generated from each single points.

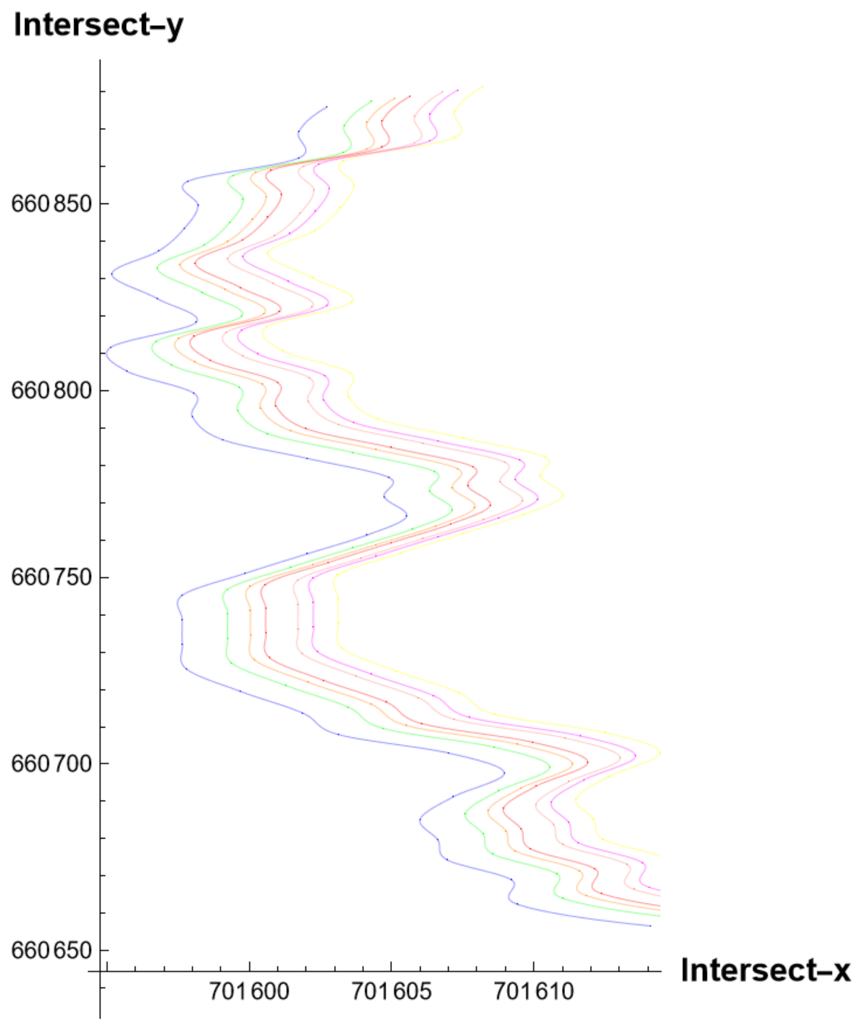

Figure 4. Type-2 intuitionistic interpolation cubic fuzzy Bézier defuzzification curve

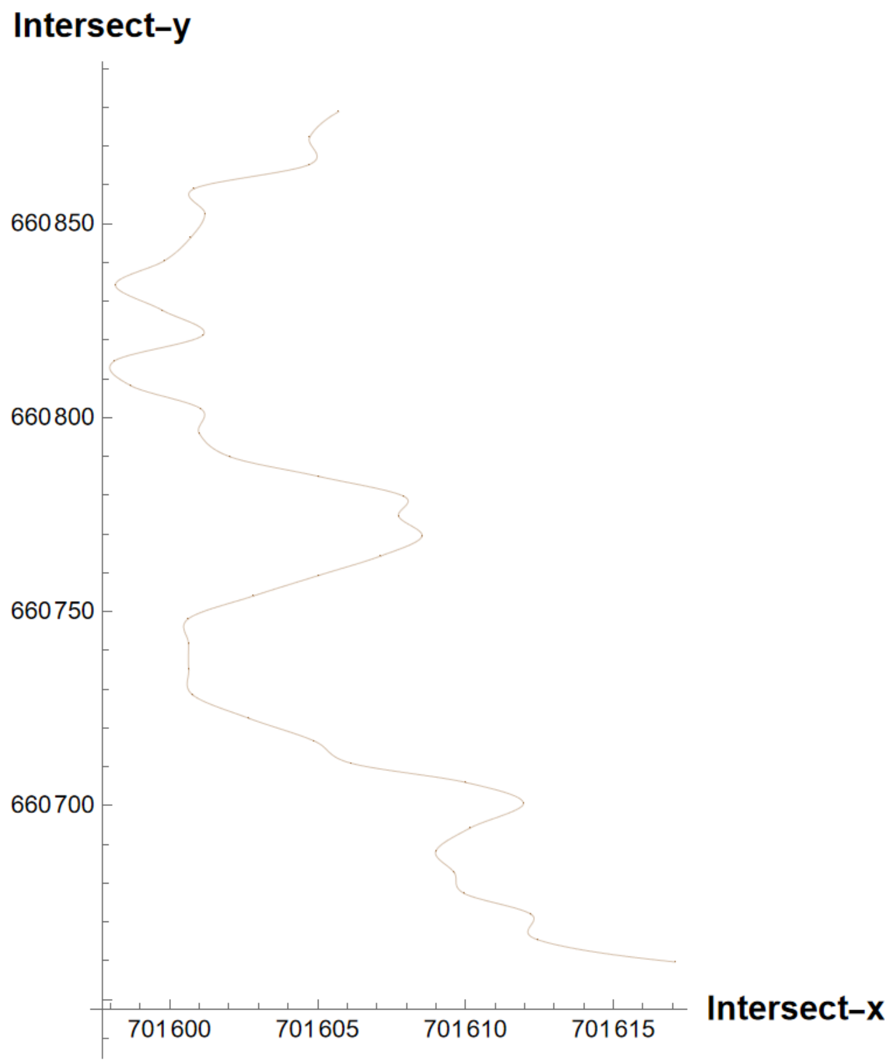

Figure 5. Defuzzification curve

As demonstrated in Figure 6 below, the distinction between the crisp curve and defuzzification curve is not dramatic and could hardly be seen. To rationalize this, the relative error method is used on the defuzzification values and conforming to the computation, the errors are 8.70069×10-8 and 9.23842×10-8 for both intersect x and intersect-y, respectively since the fuzzification is committed on both axes. Consequently, as the errors are smaller than 10% or 0.1, this shows that the type-2 intuitionistic cubic fuzzy interpolation Bézier curve model is reliably accepted [28].

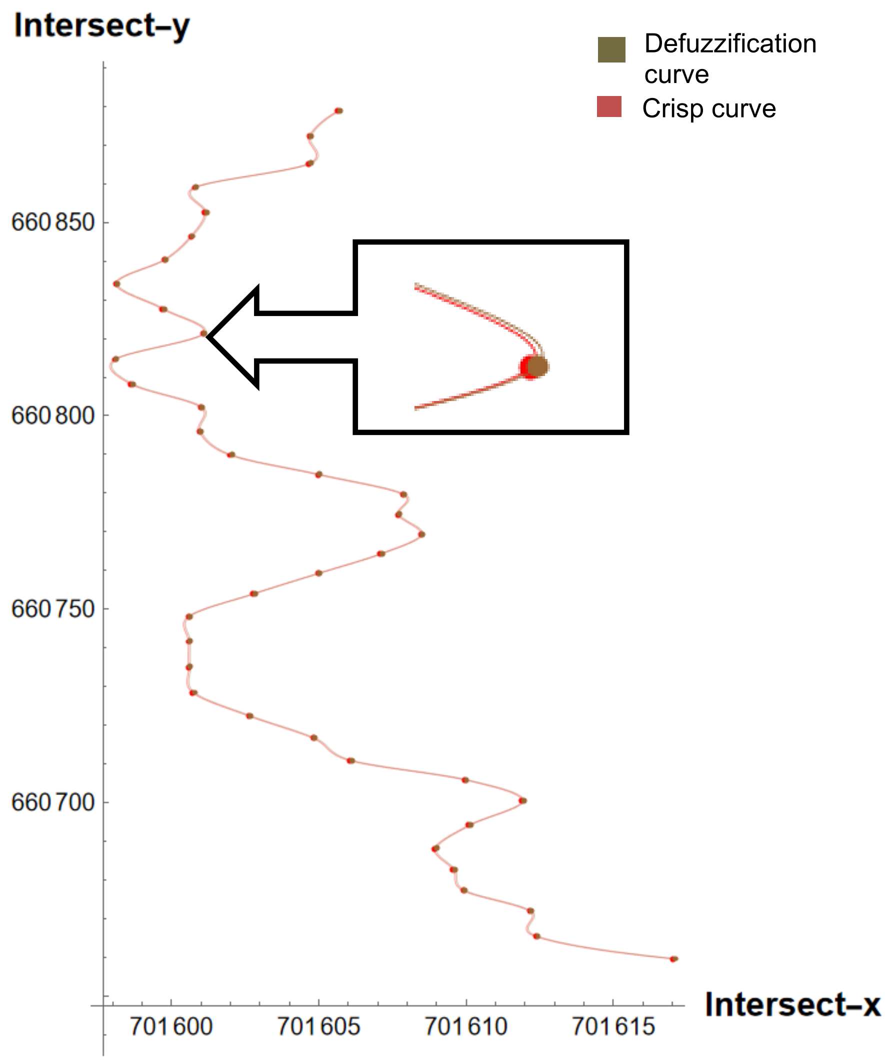

Figure 6. Comparison between crisp and defuzzification curve

The most striking result to arise from the outputs is that the outputs might look alike to T-2FS, nevertheless, there are decisive contradictions between T-2IFS and T-2FS. As both definitions very clearly demonstrate, T-2IFS is the extension of IFS which means it also embraces the membership, non-membership and indeterminacy function whereas T-2FS, as magnified in [23], only takes the membership degree that include lower and upper membership function into account. On the other hand, a perfectly normal triangular where the degree of lower and membership have the same value that is equals to one, is adjusted in this research, as mentioned earlier. Thus, since both lower and upper memberships degree are equal to one and the degree of non-memberships are equal to zero, they are now type-1 fuzzy numbers thus, causing them to be viewed as similar to the outputs from T-2FS.

Conclusions

In this study, we have defined the T-2IFCP and then presented a type-2 intuitionistic interpolation fuzzy Bézier curve. The Bézier curve function approach is used in this paper as the geometric data modelling in order to help the parties involved in data analysis and prediction visualise the complex uncertainty data. In detail, the complex uncertainty data points are converted into type-2 intuitionistic fuzzy data points using the type-2 intuitionistic fuzzy number notion. Then, using the alpha value acquired from the Barycentric coordinate, the alpha-cut operation is used to generate the fuzzification data points. The type-2 intuitionistic interpolation cubic fuzzy Bézier fuzzification curve model is produced as a result of the alpha-cut operation, and this operation helped to make the gap between crisp data points and type-

2 intuitionistic fuzzy data points smaller. The type-2 intuitionistic interpolation cubic fuzzy Bézier defuzzification curve model is then generated by computing the defuzzification data points in order to extract the crisp type-2 intuitionistic fuzzy data points.

Therefore, since the notion of type-2 intuitionistic fuzzy set further develops the idea of an intuitionistic fuzzy set that incorporates the degree of membership, degree of non-membership, and hesitancy degree, which in a sense contributes to managing the imprecise data, this model, in a way, is an ideal solution to modelling the complex uncertainty data. In order to test the efficiency of the developed model, Type-2 intuitionistic interpolation cubic fuzzy Bézier curve model is applied in modelling the Mamutik shoreline data. The result showed that the errors are smaller than 10% or 0.1 and this proved that the developed model is reliable. To sum up, the contributions of this study is the proposed method which is the type-2 intuitionistic fuzzy set theory to help in defining the data. Next, the second contribution is the development of type-2 intuitionistic interpolation cubic fuzzy Bézier curve model so that the expertise can apply this model to for data analysis and prediction, as mentioned earlier. For further research, this method can be expanded by utilising other values for both functions of membership and non-membership since this study is applying perfectly normal triangle. Bseides that, the approximation curve method or surface with interpolation method can be employed such as Spline, B-Spline and the most known one is Nonuniform rational B-Spline (NURBS) to model the complex uncertainty data.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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