

Fuzzy Intuitionistic Alpha Cut of B-Spline Curve Interpolation Modeling for Shoreline Island Data

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Abstract Traditional approaches are unable to handle the uncertainty data issue, which leads to inaccurate data analysis and prediction. Data with uncertainty are frequently collected during the data collection phase but cannot be directly used to create geometric models. Therefore, using intuitionistic alpha cuts for the uncertainty data, this paper discusses B-Spline curve interpolation modeling. To resolve the uncertain data and produce the mathematical model, fuzzy set theory, intuitionistic fuzzy sets, and geometry modeling are combined. Three main procedures are used in detail, the first of which is the application of fuzzy set theory to defined uncertainty data, followed by the use of an intuitionistic fuzzy set to take into account the membership, non-membership, and indeterminacy values of the alpha, and finally the fuzzification and defuzzification procedures. The B-spline curve interpolation function is used in geometric modeling to create mathematical geometry in the form of curves. As a result, several numerical examples are provided, along with their algorithms for producing the desired curve.

Keywords: Uncertainty data, fuzzy set theory, intuitionistic fuzzy set, geometric modeling.

Introduction

Assessing and expressing the nature of the data points received is challenging if the data are affected by ambiguity and errors made during data collection [1]. There are various methods for defining uncertainty, including fuzzy set theory and geometric modeling [2]. Besides, fuzzy geometric modeling refers to the fusion of fuzzy set theory with geometric modeling [3]. This approach uses judgements or analysis to address issues with ambiguous data [4]. The unpredictability of the data was caused by human and machine error, environmental defects, environmental problems, and other factors. Modeling cannot directly use this data on uncertainty [5]. In 1965, Zadeh used fuzzy set theory for the first time to describe problems with unclear data. Therefore, the uncertainty data will be described in a new form, incorporating measurement uncertainties for wise decision-making to make it relevant for analysis and modeling. Interpolation is a method for linking all data points to form a curve line, and curve modeling is a crucial technique for data representation [6,7].

In 1965, Lofti A. Zadeh proposed a fuzzy set theory to tackle this issue in scientific research and real-world applications [4]. The notion of fuzzy numbers and fuzzy sets can be used to tackle this data challenge. A group of related values, such as between 0 and 1, is referred to as a fuzzy number [8]. In mathematics, fuzzy numbers are often used to describe uncertain data [9]. According to previous studies, a large number of fuzzy curves were used to create geometry models to construct reliable curves from fuzzy data. Fuzzy B-spline curve interpolation for the Geography Information System (GIS) used the idea of fuzzy numbers to weed out unwanted data and gauge the level of uncertainty for each set of data collected [10].

The extension of fuzzy set theory is the intuitionistic fuzzy set (IFS) [11]. IFS was initially created by Atanassov, and it is effective at handling uncertainty [12]. When there is insufficient information to allow

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a regular fuzzy set to reflect an ambiguous idea, the IFS concept is a different way to define a fuzzy set. IFS considers the membership, non-membership, and indeterminacy functions, while fuzzy set theory solely takes into consideration the membership function [13]. IFSs are now being studied and used in several fields of science and mathematics [12]. Three functions that need equal summation are often used to characterise IFS: membership, non-membership, and indeterminacy [14].

The shoreline island was used as the source of uncertainty data for this research since it serves as the actual boundary between land and ocean [15]. In reality, the location of the shoreline is constantly shifting over time as a result of the cross-shore and alongshore movement of sediment in the littoral zone and in particular, as a result of the constantly fluctuating water levels at the coastal boundary which affects the surrounding terrestrial and aquatic-marine ecosystems [16].

This paper used intuitionistic alpha cuts for uncertainty data to produce and show B-spline curve interpolation (B-SCI) modeling. The following is a breakdown of the structure of this publication. The first section includes some background information and past studies on this topic. Next, basic definitions for fuzzy set theory and its characteristics, together with intuitionistic fuzzy sets and B-spline curve are presented in Section 2. The following section elaborates on the development of the B-SCI model from defining the data point to the defuzzification process. Then, the numerical examples will be shown before the conclusion of this research.

Materials and Methods

The uncertainty data for this study is based on shoreline island data. The uncertainty data will be defined using fuzzy set theory to get three fuzzy set data points. Besides, B-SCI is the geometric modeling used to model the data set. Next, the alpha value for the fuzzification process will be found using two methods: the centre of mass alpha cut and the intuitionistic alpha cut. Lastly, the defuzzification process will be applied to the fuzzified data set to obtain a single data set. Figure 1 below shows the workflow for the fuzzy intuitionistic alpha cut of B-spline curve interpolation for the shoreline island.

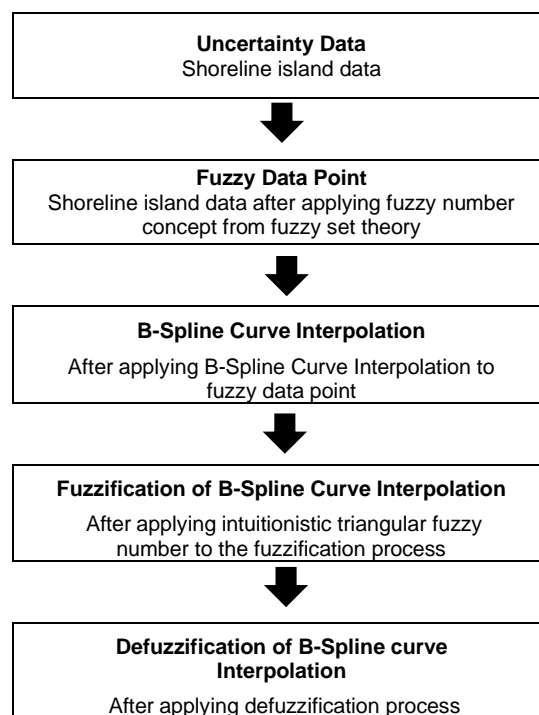


Figure 1. Workflow for the fuzzy intuitionistic alpha cut of B-spline curve interpolation modeling

Fuzzy Set Theory

Let x be an element in set X . If x has full membership of A , hence the membership function is one. For x that has half-membership, the membership function is in an open interval $(0,1)$. But, if x is not in A then the membership function is 0. Generally, the membership function of the fuzzy set A can be summarised as [4]:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \text{ (full membership)} \\ c \in (0,1) & \text{if } x \tilde{\in} A \text{ (non - full membership).} \\ 0 & \text{if } x \notin A \text{ (non membership)} \end{cases} \quad (1)$$

Triangular Fuzzy Number

One of the varieties of fuzzy set theory is the triangular fuzzy number as shown in Figure 2. Let a triangular fuzzy number A is defined as a triplet (f^-, f, f^+) . The membership function of this fuzzy number is interpreted as [3]:

$$\mu_A(x) = \begin{cases} \frac{x - f^-}{f - f^-} & \text{if } f^- \leq x \leq f, \\ \frac{f^+ - x}{f^+ - f} & \text{if } f \leq x \leq f^+, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

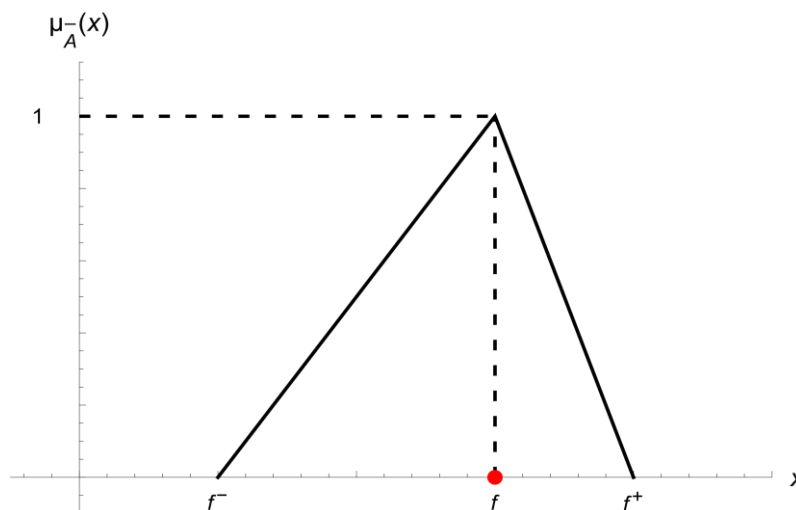


Figure 2. Triangular fuzzy number

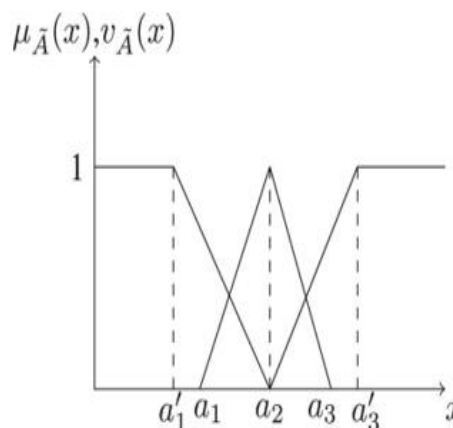


Figure 3. Triangular intuitionistic fuzzy number

Triangular Intuitionistic Fuzzy Number

There are several types of intuitionistic fuzzy numbers, including trapezoidal and triangular shapes. In this research, triangular intuitionistic fuzzy numbers will be employed. A subset of IFS in R is the

triangular intuitionistic fuzzy number \tilde{A} , as $\tilde{A} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$. The following is a description of its membership and non-membership function [17]:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

$$v_{\tilde{A}}(x) = \begin{cases} \frac{a_2 - x}{a_2 - a_1} & \text{if } a_3 \leq x \leq a_2 \\ \frac{x - a_2}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 1 & \text{otherwise.} \end{cases} \quad (4)$$

where, $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$. Figure 3 is the graphical representation of triangular intuitionistic fuzzy number.

Alpha Cut

To determine the alpha value, two procedures will be used. The first step in getting an alpha value involves utilising the centroid of a triangle polygon since a triangular fuzzy number is engaged. The intersection of the three medians on each side of a triangle determines its centroid, also known as a centre of mass. Figure 4 shows the example of coordinate centroid of triangle while Eq. (2) is the equation for the centroid of a triangle, C [18].

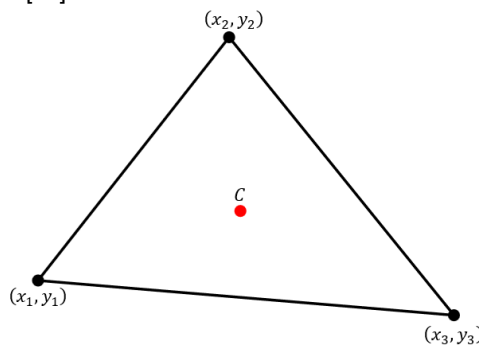


Figure 4. Coordinate centroid of triangle

$$C = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right). \quad (5)$$

Next, the alpha value from the centroid of a triangle will undergo a triangular intuitionistic fuzzy number. Let \tilde{A}^I be a triangular intuitionistic fuzzy number. Then the measure of membership value $\mu(\tilde{A}^I)$ is given by [19]:

$$\mu(\tilde{A}^I) = \frac{1}{4}(a_1 + 2a_2 + a_3). \quad (6)$$

Similarly, the measure of non-membership value $v(\tilde{A}^I)$ is given by:

$$v(\tilde{A}^I) = -\frac{1}{4}(a_1' + 2a_2' + a_3'), \tag{7}$$

and the degree of hesitancy is:

$$\pi_{\tilde{A}^I}(a_{ij}) = 1 - \mu(\tilde{A}^I) - v(\tilde{A}^I). \tag{8}$$

Lastly, the intuitionistic alpha value is:

$$\alpha_{\tilde{A}^I} = \mu(\tilde{A}^I) + \pi_{\tilde{A}^I}(a_{ij})\mu(\tilde{A}^I). \tag{9}$$

Fuzzification Process

After an intuitionistic alpha value is obtained from Eq. (10), the fuzzification process will take place using an intuitionistic alpha cut. The fuzzification process is a process used to narrow the fuzzy interval that maps the crisp value to the fuzzy set. This research uses a symmetric triangular fuzzy number for the fuzzification process. The output of this process depends on the value of the alpha. Fuzzification, D^{\leftrightarrow} defined as [20]:

Let D^{\leftrightarrow} be the set of fuzzy control points and $D^{\leftrightarrow}_{\alpha_{\tilde{A}^I}}$ after the intuitionistic alpha cut which is $\alpha_{\tilde{A}^I} \in (0,1]$ as follows:

$$\bar{D}_{\alpha_{\tilde{A}^I}} = \left\langle \bar{D}^-_{\alpha_{\tilde{A}^I}}, D, \bar{D}^+_{\alpha_{\tilde{A}^I}} \right\rangle \tag{10}$$

$$\bar{D}_{\alpha_{\tilde{A}^I}} = \left\langle \left[(\bar{D} - \bar{D}^-)\alpha_{\tilde{A}^I} + \bar{D}^- \right], D, \left[-(\bar{D}^+ - \bar{D})\alpha_{\tilde{A}^I} + \bar{D}^+ \right] \right\rangle \tag{11}$$

where $D^{\leftrightarrow-}_{\alpha_{\tilde{A}^I}}$ represents the intuitionistic alpha cut left fuzzy data point and $D^{\leftrightarrow+}_{\alpha_{\tilde{A}^I}}$ represent the intuitionistic alpha cut right fuzzy data point while D represents crisp data point for uncertainty data and $\alpha_{\tilde{A}^I}$ represent intuitionistic alpha value from Eq. (10).

Defuzzification Process

The defuzzification process is the opposite of fuzzification process. It converts the confidences in a fuzzy set of word descriptors into a real number. It is essential when the result is required as a crisp number by the user. The following is the formula for defuzzification process which is one of the most important phases in this research.

Let $D^{\dots}_{\alpha_{\tilde{A}^I}}$ be the defuzzification for $D^{\leftrightarrow}_{\alpha_{\tilde{A}^I}}$. $D^{\dots}_{\alpha_{\tilde{A}^I}}$ can be expressed as [20]:

$$\bar{D}^{\dots}_{\alpha_{\tilde{A}^I}} = \frac{\bar{D}^-_{\alpha_{\tilde{A}^I}} + D + \bar{D}^+_{\alpha_{\tilde{A}^I}}}{3}. \tag{12}$$

B-Spline Curve Interpolation Modeling

After all the uncertainty data undergo all the processes from defining uncertainty data using triangular fuzzy number until the defuzzification process, B-SCI modeling will be applied to the uncertainty data. The following is the definition of B-SCI model.

Crisp B-spline curve is defined by, let $P = \{P_0, P_1, P_2, \dots, P_n\}$ where P is a set containing $n + 1$ and denoted by $B(t)$ with the parametric function of degree $d = k - 1$, hence the crisp B-spline curve written as [21,22]:

$$B(t) = \sum_{i=0}^n P_i N_{i,k}(t) \tag{13}$$

with $t_{min} \leq t \leq t_{max}$ and $2 \leq k \leq n + 1$ where P_i are the position vectors of $n + 1$ control polygon, and $N_{i,k}(t)$ is defined as:

$$N_{i,0}(t) = \begin{cases} 1 & \text{if } u \in [t_i, t_{i+1}), \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

and

$$N_{i,0}(t) = \frac{t - t_i}{t_{i+k} - t_i} N_{i,k-1}(t) + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+k}} N_{i+1,k-1}(t). \tag{15}$$

Next, fuzzy B-SCI can be defined as [3]:

$$\text{Fuzzy B - SCI} = \left\{ \tilde{B}_\beta \mid \tilde{B}_\beta, \beta \in (0,1] \right\}, \tag{16}$$

where $B^{\leftrightarrow}_\beta = \langle B^{\leftrightarrow-}_\beta, B_\beta, B^{\leftrightarrow+}_\beta \rangle$. Left fuzzy B-SCI, crisp B-SCI, and right fuzzy B-SCI are represented by $B^{\leftrightarrow-}_\beta$, B_β , and $B^{\leftrightarrow+}_\beta$ respectively. The following are the definitions for all of these symbols:

$$\tilde{B}^-_\beta(t) = \sum_{i=0}^n \tilde{P}^-_i N_{i,k}(t) = \tilde{D}^-_i, \tag{17}$$

$$B_\beta(t) = \sum_{i=0}^n P_i N_{i,k}(t) = D_i, \tag{18}$$

$$\tilde{B}^+_\beta(t) = \sum_{i=0}^n \tilde{P}^+_i N_{i,k}(t) = \tilde{D}^+_i. \tag{19}$$

Numerical Example

The uncertainty that occurs to the shoreline island data that affect the shoreline island often changes due to climate change that caused global warming causing melting glaciers, rising sea levels and shoreline regression causing erosion [16,23]. Fifteen uncertainty data points, D of shoreline island data taken from Google Maps were used. Using Mathematica Software Application, we plot the model by applying the B-spline and B-SCI function from Eq. (13) to Eq. (19). Next, the algorithm step by step to obtain B-SCI is summarised as follows:

Step 1: Define uncertainty data using triangular fuzzy numbers to get triplet data points, (D^-, D, D^+) as shown in Figure 6(a).

Step 2: Determine the alpha value using two methods mentioned in Eq. (5) to Eq. (9).

Step 3: Next, find fuzzification data points using the fuzzification process through Eq. (10) and Eq. (11) to get $(\tilde{D}^-_{\alpha_{\bar{\lambda}}}, D, \tilde{D}^+_{\alpha_{\bar{\lambda}}})$ as shown in Figure 6(b).

Step 4: Finally, the defuzzification process uses Eq. (12) to get a single set of defuzzification data points, $\tilde{D}^-_{\alpha_{\bar{\lambda}}}$ as shown in Figure 7.

Results and Discussion

To demonstrate the B-SCI model developed from the methodology mentioned before, Figure 5 shows the crisp curve of the B-SCI model. Then, using the concepts of triangular fuzzy numbers from fuzzy set theory, Figure 6(a) shows the fuzzy curve of the B-SCI model. After that, Figure 6(b) shows the fuzzification curve of the B-SCI model using an intuitionistic alpha cut. Figure 7 shows the comparisons between the defuzzification curves of the B-SCI model using intuitionistic alpha cuts and craps B-SCI model.

Crisp B-Spline Curve Interpolation of Shoreline Island

By using Mathematica application, there are 15 uncertainty data points of shoreline island data used to plot using B-SCI model. The blue curve in Figure 5 shows the crisp B-SCI model of shoreline island data.

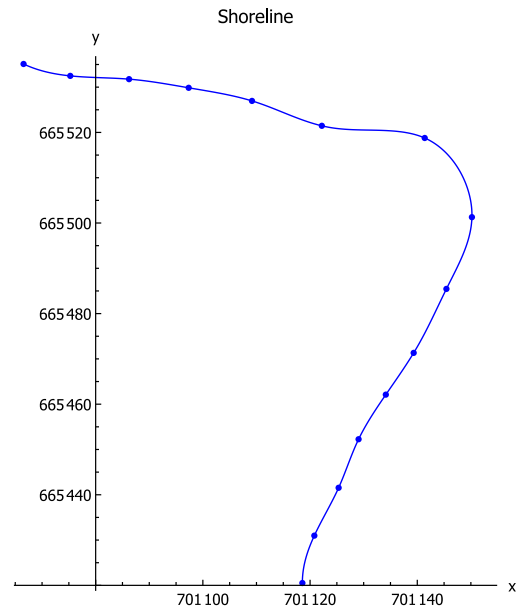


Figure 5. Crisp B-SCI model of shoreline island

Comparisons between Fuzzy and Fuzzification Model of B-Spline Curve Interpolation Model of Shoreline Island

Based on the Figure 6, which shows the comparisons between the fuzzy curve of the B-SCI model and the fuzzification model of the B-SCI model, Figure 6(a) shows Figure 6(a) was determined using Eq. (2) to get the three-value judgement which is left fuzzy curve, crisp curve, and right fuzzy curve. By using the alpha value determined from Eq. (6) to Eq. (11), fuzzification process will proceed to get the fuzzification curve of the B-SCI model, which is shown in Figure 6(b). The fuzzy interval in Figure 6(a) is decreasing after fuzzification using the alpha cut process applied to the B-SCI model in 6(a). For each model in both figures, the red, blue, and green curves represent the left B-SCI model, the crisp B-SCI model, and the right B-SCI model, respectively.

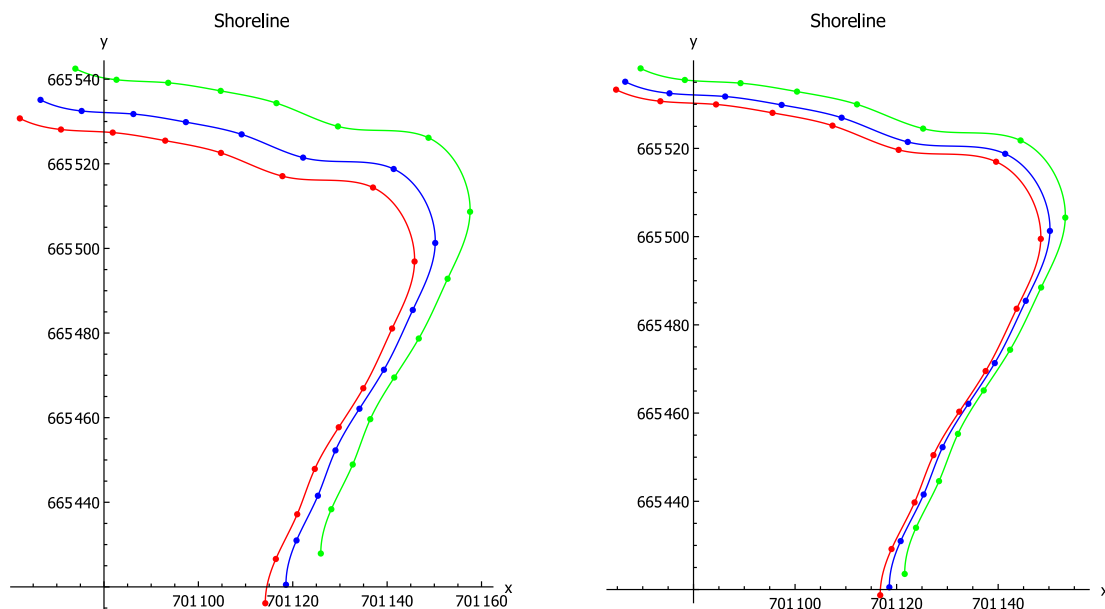


Figure 6. (a) Fuzzy curve for B-SCI model of shoreline island. (b) Fuzzification curve for B-SCI model of shoreline island using intuitionistic alpha cut

Comparisons between Defuzzification and Crisp Curve of B-Spline Curve Interpolation Model of Shoreline Island

Figure 7 shows the comparisons between the defuzzification curve and crisp curve of the B-SCI model. Eq. (12) is the equation used to determine the defuzzification curve of the B-SCI model. There is the slightest difference between the defuzzification curve and crisp curve of B-SCI model, as the defuzzification curve of B-SCI model is the exact B-SCI model for shoreline island. The orange curve represents the defuzzification curve of B-SCI model, while the blue curve represents the crisp curve of B-SCI model. The orange curve is the curve that uses an intuitionistic alpha cut, which means that three elements in IFS are considered to find the alpha value. So, the orange curve is more accurate than the blue curve since there is no specific formula to find alpha value but the more elements that are considered to find alpha value, the better the alpha value is.

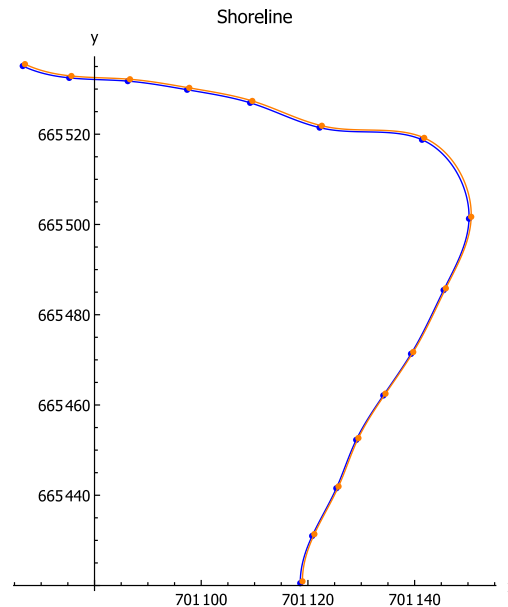


Figure 7. Comparisons between defuzzification curve for B-SCI model using intuitionistic alpha cut and crisp B-SCI model

Conclusions

This research proposed B-SCI modeling for uncertainty data using intuitionistic alpha cuts. The B-SCI model used in this study for geometric modeling is an optimal strategy because the generated curve goes across all the data points. The membership, non-membership, and indeterminacy degrees of intuitionistic play a vital role in determining the alpha value for the fuzzification process. This model aids the parties involved in data analysis and prediction. This approach, as well as the model it produces, will be able to contribute to the field of fuzzy modeling methods. This model can be extended to the Non-Uniform Rational B-Spline (NURBS) model since NURBS uses more complicated function that has more accurate and flexible curve.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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