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Contractual incompleteness in an extensive-form game under perfect information using Stackelberg model

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ABSTRACT

The paper presents the explanation of contractual commitments which are renegotiation-proof, based on "strategic default". Under this, financial contracts must provide incentives of their own so that the parties would honor the agreement. We investigates the reach of this type of commitment within the general class of extensive form games. The result is that a renegotiation-proof contract exists which commits against every deviation from the equilibrium which would induce a revenue acceleration. AMS Subj. Classification: 91A40, 91A20 .

| The Augmented model | Contractual commitments | Revenue acceleration | Standard debt contracts | Repeated games |

1. Introduction

The use of contracts for purposes of strategic commitment has attracted economist for sometime. Aghion and Bolton (1987), Fershtman and Judd (1987), have shown that contracts can have commitment value if decisions are delegated to an agent and her preferences are manipulated in accordance with the strategic needs.

Naturally, the question arises where a commitment against revenue accelerations is of practical importance. A revenue acceleration is a dynamic path of payoffs with a higher payment in the current period and a smaller value of payments in all future periods. The application considers industry exit decisions. If exit is linked to sales of remaining assets then it implies a revenue acceleration. Thus, a commitment arises to stay in an industry even if this is costly.

2. The Augmented Model

Suppose that F and L are two firms in an industry competing for a common standard. Both have heavily invested in their own system which they have patented and would like to impose as the standard. So here we have two players L (the Stackelberg leader) and F (the Stackelberg follower) competing in a multi-period game of perfect information.

(i) *Contracts*: Prior to the game, at t_0, *F* can sign a financial and governance contract with an outside agent. Throughout we need to consider only one such outside agent which we call agent *A*. We assume, however, that there is a competitive market for agents at t_0. The initial contract between *F* and *A* specifies a path of side payments between *A* and *F* and allocates control to either *A* or *F*. A contract *c* between *F* and an agent *A* can delegate the power to take the decision over $\tilde{f}$ to *A*. Thus, either *F* or *A* controls $\tilde{f}$. There is no cost for any of two managerial control options. We denote the control decision by a managerial control variable $m \in \{m_A, m_F\}$ which takes the value m_F if *F* remains in control of $\tilde{f}$, and m_A if control is delegated to *A*. Moreover, a contract may specify a path of side-payments $\{P_0, (P_1(i_1))_{i_1 \in I_1}, (P_2(i_2))_{i_2 \in I_2}, ...\}$ conditional on the history i_t, i.e. a contract can contain different side payments for up to *NI* revenue nodes. Let $C = \Re^{NI} \times \{m_A, m_F\}$ be the contracting space. We denote by $Q_t(i_t)$ the residual of *F*'s gross revenue $\Re_t^F$ which *F* retains for herself, i.e. $Q_t = \Re_t^F - P_t$.

(ii) *Strategic default*: In every but the last period *T* (at which time there is external liquidation). This is for simplicity. It can be shown that, in general, a commitment value arises also if strategic default is possible in the last period: then, obviously, *F* is to keep all of $\Re_t^F$. But *F* would lose $\Re_t^F$ by defaulting earlier, which produces the same effect as described here. *F* can decide to keep all of $\Re_t^F$ for herself, even if $P_t > 0$. However, if *F* defaults strategically in this way or if $\Re_t^F(i_t) > P_t(i_t)$, i.e. if the payment cannot be met by the current payoff (liquidity default), then A can take over control and keep all of $\Re_{t+1}^F, \Re_{t+2}^F, ...,$ etc. for herself.

(iii) *Renegotiation*: The contract c can be renegotiated at each period *t*, before *F* takes her action. Renegotiation will always lead to an efficient outcome whenever this is feasible. We specify renegotiation as a take-it-or-leave-it offer and impose that the equilibrium be robust irrespective of whether *A* or *F* is making the offer.

(iv) *Equilibrium*: Throughout, we are solving for sub game perfect equilibria.

(v) *Timing*: The timing of the game can be summarized as shown in Figure 1. At t_0, a contract $c \in C$ between *A* and *F* is signed and the up-front payment P_0 is made. Then t_1 comes along. First, *L* chooses l_1. Next, *F* and *A* can renegotiate. Then f_1 is chosen according to the (renegotiated) control variable. Payoffs $\Re_1^F$ are received; *F* decides whether to default or not. If she defaults, *A* is to get all of $\Re_2^F, \Re_3^F, ...$ If *F* does not default, then $\Re_1^F$ is split according to the contract in place. Then t_2 comes along, with the same timing as in t_1, etc.

t_0		t_1						
|__________	|__________	|__________	|__________	|__________	|__________	|__________	|_ ____	t
c signed	P_0 paid	l_1 chosen	c renegotiated	f_1 chosen	$\Re_1$ realized	Strategic default possible	assets seized (if default)	

Figure 1. Timing at t_0 and t_1

3. Revenue Accelerations

We denote by f a strategy of the party controlling f (F or A) in the augmented game from t_1 on (i.e. after contracting). Likewise, l denotes a strategy of L in the augmented game from t_1 on. Given strategy f^k, we denote by $f^k(h_t)$ and $l^k(h_t)$ the continuation strategy in the sub game h_t which is included by f^k and l^k, respectively. I say that $f'(h_t)$ is (c, l)-preferred to $f(h_t)$ if under contract c, the controlling party will prefer $f'(h_t)$ to f. Contract c is said to be a renegotiation-proof commitment against $f'(h_t)$ if $f(h_t)$ is(c, l)-preferred to $f'(h_t)$.

We call a revenue acceleration a shift of strategy such that a relatively larger fraction of total revenue of F and A would be earned in the current period. Formally:

Definition 1: Suppose (l, f) is played. Then $f'(h_t)$ is said to induce a revenue acceleration in sub game $h_t \in H$ if
- $\Re_t^F(l, f', h_t) \geq \Re_t^F(l, f, h_t)$ and
- $V_t^F(l, f', h_t) - \Re_t^F(l, f', h_t) \leq V_t^F(l, f, h_t) - \Re_t^F(l, f, h_t)$ and
- at least one inequality is strict.

In other words, a revenue acceleration increases the absolute amount of the spot revenue, but it does so at the expense of the value of future revenues. Note that the definition does not specify whether a profit acceleration is efficient or not. That is, it could be that $V_t^F(l, f', h_t) < V_t^F(l, f, h_t)$ or that $V_t^F(l, f', h_t) > V_t^F(l, f, h_t)$.

We confirm first an important insight as to the control structure:

Lemma 1: No contract with F-control can have commitment value.

Proof: F will default strategically in t if $Q_t > \sum_{\tau=1} \delta^\tau \Re_{t+\tau}^F - Q_{t+\tau}$

If the contract specifies m_F, then F will take the action such that her payoff is maximized, taking into account the possibility of strategic default. But then consider any renegotiation stage: F and A will always renegotiate to the efficiency frontier unless the No default constraint permits no interior solution of the joint renegotiation surplus. Recall that it is F who gains from a strategic default. If a profit acceleration is maximizing the joint surplus the F will choose it anyway. If a profit slow down were more profitable, A can always reduce her discounted total payoff *A* down to her actual payoff (taking into account strategic default) under the accelerated strategy, plus ε. Because a strategic default under the slowed path is less profitable for *F*, this proposal will always be possible for *L*, and it pays for both. Thus, a commitment in favor of profit accelerations is not possible.□

The intuition is quite simple: the commitment effect that we want to show depends on the impossibility to pay to *A* the contractual part of revenues if revenues are accelerated. Thus *A*'s veto the change of strategy is a crucial element in the mechanism. In other words, if *F* has power to default strategically and also the choice of strategy nothing will keep her from doing so. On the contrary, she would accelerate even if this was a costly thing to do, for coalition of *F* and *A*, unless renegotiation secures her a satisfactory piece of the pie (which would be the equilibrium outcome).

Thus, a commitment value can only arise if there is separation between the managerial decision right and the strategic default opportunity. In this respect, the model is different from most of the corporate control literature where both are unified in the hands of the agent in charge of "control". There are various possible interpretations for this separation. Note that all what is needed is that *A* has a veto right over the choice of S_F. Thus, as soon as the decision process involves both parties, separation is perfectly compatible with the received idea in the corporate control literature. Some examples how *A* may obtain a second key to the decisions are:

a. Monitoring by *A*. If *A* is a financier acquiring exclusive insider information then protective covenants may give *A* an effective veto.
b. Specific investments. Imagine that *A* is a supplier furnishing specialized equipment or a lender with access to exclusive information.
c. Governance. *F* could contractually commit to cede control to a manager whose interests are, via a compensation package, aligned to those of *A*.

The following Lemma provides the central building block of the theory:

Lemma 2: Suppose (*l, f*) is played. Then there exists a renegotiation-proof commitment against $f'(h_t)$ if $\exists h_t \in H$ such that $f'(h_t)$ induces a revenue acceleration in sub game h_t.

Proof: The proof is by construction. Because *f'* is a revenue acceleration, we know that $\exists h_t$ s.t $\Re_t^F(l,f',h_t) \geq \Re_t^F(l,f,h_t)$ and $V_t^F(l,f',h_t) \leq V_t^F(l,f,h_t)$. This proof considers the case where the second inequality is strict. The opposite case is proven analogously and is omitted.

One verifies that the incentive condition keeping *F* from defaulting strategically in period *T*-1 is (where arguments are omitted for simplicity-this must be true for all the equilibrium payoffs in all sub games):

$$P_{T-1} \leq \delta(\Re_T^F - P_T) \qquad (NDC_{T-1})$$

and

$$P_{T-2} \leq \delta(\Re_{T-1}^F - P_{T-1}) + \delta^2(\Re_T^F - P_T) \qquad (NDC_{T-2})$$

and in general:

$$\sum_{\tau=0} \delta^\tau P_{t+\tau} \leq \sum_{\tau=1} \delta^\tau \Re_{t+\tau}^F \qquad (NDC_t)$$

Consider the following contract of payments on the equilibrium path: $P_t = \delta\Re_{t+1}^F$ and $P_T = 0$. Also, $P_0 = \sum_{\tau=1}^{T-1} \delta^\tau \Re_\tau^F$.

Now recall that *f'* is a revenue acceleration. By construction of (P_t), there always will be strategic default under (*l, f'*). Further *c* is renegotiation-proof. To see this, note that for *A* to accept the offer, *A* must at least receive $\sum_{\tau=0} \delta^{\tau} P_{t+\tau}$ under the modified contract. By construction of path (P_t), in each sub game h_t is true that $\sum_{\tau=0} \delta^{\tau} P_{t+\tau}(l,f,h_t) = \sum_{\tau=1} \delta^{\tau} \Re^F_{t+\tau}(l,f,h_t) - \delta^{T-t} \Re^F_T(l,f,t) = V^F_t(l,f,h_t) - \Re^F_t(l,f,h_t)$. But then recall that under *f'*, $V^F_t(l,f',h_t) - \Re^F_t(l,f',h_t) < V^F_t(l,f,h_t) - \Re^F_t(l,f,h_t)$. Thus, in order to compensate *A*, P_t would have to increase, making a strategic default unavoidable.□

The intuition for this result is the following. We construct a payment path such that the no-default-condition is binding along the equilibrium path. Consider a revenue acceleration. This will inevitably lead to strategic default as the agent *F* can not be compensated sufficiently to make her honor the contract. For that to be the case, she would have to be promised an additional payment which exceeds the joint surplus from successful renegotiation. Otherwise she will prefer to default: under the equilibrium condition already, she was just indifferent between defaulting or not. Now default has become more attractive due to both the present increase and the future decrease of value. Thus, a renegotiation offer must lie outside the unit circle, making the equilibrium outcome preferable to *A*.

Following the same line of reasoning, one infers that a contract which commits, in a sub game h_t, against a single strategy of revenue acceleration, commits automatically against all strategies which induce a revenue acceleration in sub game h_t. Also, recall that nothing restricts from conditionalizing contracts on the history up to *t*. Thus, obviously, history-dependent commitments are feasible for all revenue accelerations in all sub games. We can moreover generalize by showing that a single contract suffices to commit against all revenue accelerations in all sub games. Putting the pieces together, our main result can be stated as:

Proposition 1: Suppose the strategy profile (*l, f*) is played. Then there exists a contract $c \in C$ which is a renegotiation-proof commitment against every $f'(h_t)$ which induces a revenue acceleration in some sub game h_t.

Proof: The proof again by construction. We construct the overall contract which commits against revenue accelerations in all sub games. Consider sub game h_t, and consider a contract $c(h_t)$ which is constructed as indicated in Lemma 2. Now $c(h_t)$ needs to contain only payments $P(l,f,h_t)$ in sub game h_t. Consider any revenue acceleration *f'* and h_t. By Lemma 2, $c(h_t)$ which commits against *f'* and h_t. Therefore, *f'* cannot be a sub game perfect equilibrium strategy in h_t. Recall that *c* can be history-dependent.

Next, we define the overall contract. Let $f^k(h_t)$ and $l^k(h_t)$ denote the strategies which are induced in h_t by strategies f^k and l^k. Note that this contract will have redundancies along the equilibrium path as all subsequent contracts $c(h_s)$ which are reached by the equilibrium path in sub game h_t (which may or may not be reached along the equilibrium path). Thus, a non redundant collection of contracts $c(h_t)$ is _|≡ $\{c(h_t) \mid h_t \in H$ and $\exists s.t.\ \ h_s \in H\}$ is reached by $(l(h_s), f(h_s))$. In equilibrium are already contained in the contract at the root, $c(h_0)$. Note that _| contains a contract $c(h_t)$ committing against revenue accelerations in every sub game $h_t \in H$. Thus, a strategy f' containing a revenue acceleration in any sub game h_t (whether h_t is reached in equilibrium or not) cannot be a profitable deviation if _| has been signed.□

In other words, we have demonstrated that a commitment value exists in as much as all revenue accelerations can be eliminated from *F'*s strategy set. On the other hand, one cannot show that a commitment against any other strategy of *F* is possible. Thus the concept of revenue accelerations, or of deviations which would yield a higher payoff in the current payoff but a lower present value in all future periods combined, encompasses a concise description of the commitment value of strategic default. Given this precision on the nature of the commitment, one is left to wonder what the practical importance of revenue accelerations is. Exemplary answers are presented in Application. Note that our result made abundant use of the assumption that contracts can depend on the history. This assumption permits to set contractual payments in every sub game to a level such that the inter temporal incentive constraint hold with equality. Then even the slightest revenue acceleration inevitably ends up in strategic default.

4. Reversal of The First Mover Advantage

We derive general conditions on the equilibrium that will obtain in the augmented game. We find that, in a certain sense, we get a reversal of the first mover advantage. We begin with a definition: let $\overline{S}_F(l,f)=\{f'\in S_F \,|\, \exists h_t \in H \text{ s.t. } \; f'(h_t)$ induces a revenue acceleration in $h_t\}$. In other words, $\overline{S}_F(l,f)\subset S_F$ is the subset of strategies of *F* which remain after all revenue accelerations, given that *(l, f)* is played, have been deleted. We get then the Proposition on the equilibrium outcome of the augmented game:

Proposition 2:

(1) *(l, f)* is an equilibrium in the augmented game if and only if *(l, f)* is an equilibrium in the truncated game $\overline{S}_F(l,f)\times S_L$.

(2) Let $\overline{S}=\{(l,f)\,|\,(l,f)$ is the equilibrium in the truncated game $\overline{S}_F(l,f)\times S_L\}$ and suppose that in each truncated game $\overline{S}_F(l,f)\times S_L$, the equilibrium is unique. Then the equilibrium in the augmented game will be such that $f=\arg\max_f \;\{V_0^F((l,f)\,|\,(l,f)\in\overline{S}\}$

Proof:

(1) By Lemma 2, a commitment is feasible whenever *f'* is a revenue acceleration. Hence there is a renegotiation-proof commitment against any *f* such that $f\in S_F,\, f\notin\overline{S}_F$.

(2) But clearly, *F* can choose any of the truncated sets $\overline{S}_F$ defined relative to a strategy profile *(l, f)*. Thus, as *F* has a first-mover advantage at the contracting stage, *F* will choose the truncation maximizing her payoff.□

Thus, we find the following sort of reversal of the first-mover advantage: *F* can effectively curtail her strategy set in the most profitable way. The important restriction, however, is that only revenue accelerations can be eliminated. Moreover, Proposition 2 provides a general idea of how to find an equilibrium in the augmented game: any strategy profile can be attained in this game provided that the strategy profile is an equilibrium in the truncated game that obtains after all revenue accelerations are eliminated. Thus, there are now in principle several outcomes obtainable, even though there is just a single Stackelberg outcome. Then *F* has the power to choose the truncation such that the resulting equilibrium profile is most in her favor. The latter point should be fairly intuitive: the power to choose the contract is in *F'*s hand, so one would expect to *F* to choose a contract which is most in her favor. But essentially, the power that commitment wield is to credibly eliminate unwanted strategies of one's own.

5. Standard Debt Contracts

Our results so far made extensive use of the fact that contracts can depend on history. The general result that a commitment contract exists against any revenue acceleration relies on this assumption. Nonetheless, one may also ask: under which circumstances can the transfer structure be simplified to a reasonable, familiar pattern? In particular, when can financial commitment contracts actually assume forms as they are found in practice?

We define a contract c to be a standard debt contract if transfer payments are a function of time only. Again, I am only able to specify a sufficient condition:

Proposition 3: Any equilibrium outcome in the augmented game can be replicated by a standard debt contract if the payoff structure is Markovian.

This is fairly straightforward, and needs no formal proof. Recall that history-dependent transfer rules where needed to reach the maximal transferable value for any possible continuation of the game. But if the payoffs of the game are a function of time only (and not of history), then it follows that the maximal transferable wealth in each sub game is a function of time only. But then a time-dependent transfer rule, in other words a standard debt contract, is sufficient.

6. Application

We start from the general description of the Stackelberg model. For simplicity, we assume that a choice of actions is made at t_1 only, and that both players have just two actions to choose from. Payoffs accrue over T periods thereafter. Omitting time subscripts form the actions taken at t_1, we assume that L decides first from a binary decision set, $l = \{l^A, l^E\}$. We suggest the following interpretation: l^E is the action of entrance. l^A amounts to abstaining. Then F counters with a choice from $f = \{f^A, f^E\}$. For F, the suggested interpreted is: f^A stands for acquiescing or for asset sales. f^E means expansion or staying in.

Next we assume the following payoff structure, in present value terms at t_0: For player L, $V^L(A,A) > V^L(A,E)$, $V^L(A,A) < V^L(E,A)$, $V^L(E,A) > V^L(E,E)$; hence also $V^L(A,A) > V^L(E,E)$. Likewise, for player F, $V^F(A,A) < V^F(A,E)$, $V^F(A,A) > V^F(E,A)$, $V^F(E,A) > V^F(E,E)$; hence also $V^F(A,A) > V^F(E,E)$. Let, moreover, present values be strictly positive, i.e. $V^L(.,.) > 0$ and $V^F(.,.) > 0$. As for the inter temporal distribution of payoffs, we introduce the following assumption:

Assumption 1: f^A is a revenue acceleration relative to (l^E, f^E).

Table 1. Present value payoffs in the exit game

		Player F	
		f^A	f^E
Player L	l^A	$V^L(A,A)$, $V^F(A,A)$	$V^L(A,E)$, $V^F(A,E)$
	l^E	$V^L(E,A)$, $V^F(E,A)$	$V^L(E,E)$, $V^F(E,E)$

The economic rationale behind this assumption runs as follows. For F, l^A is an exit strategy, coupled with divestitures (asset sales) in the present period. Thus, selling assets implies a large cash flow now, but a reduced revenue due to the reduced activity in the industry later. In other words, exit is tantamount to accelerating cash flows.

We then get the following result: the Stackelberg outcome of the game is easily identified as $(l^*, f^*) = (l^E, f^A)$, i.e. L exploits her first mover advantage to enter, and F adjusts by using the asset sales strategy f^A.

In the augmented game, one infers directly from Lemma 2 that F can launch a renegotiation-proof commitment against f^A, i.e. against the asset sales strategy. But then L will obviously reevaluate the situation: given that F will credibly reply with f^A, even after L has chosen l^E, L surely is better off by staying out. Thus the only equilibrium outcome in the augmented game is (l^A, f^E) which is tantamount to the outcome of the game if F were to be the leader. Which is just what Proposition 2 predicted: F will draw up a contractual commitment such that her payoff is maximixed, among all the equilibria which are in reach once the augmented game is considered.

A final word is in order to expand the model so as to allow for negative expected payoffs. Can there be a commitment to stay in even if F expects to lose? This seems to be quite an important case in industrial organization. The answer is: yes, so long as one is still confronted with a revenue acceleration in the sense of the definition, i.e. the future portion of losses is smaller under the stay in strategy than under the asset sales strategy. Then, for F not to default right away, she needs to compensated by future side payments from A's pocket. But these side payments would have to be larger under the strategy l^A, and so A would refuse her consent to a strategy of asset sales.

7. Conclusion

In this paper, it is shown that deficiencies in contract enforcement may lead to strategic commitment. The reach of the commitment is restricted to strategies which delay the payoff path. Application to exit decision, demonstrate that the commitment against revenue accelerations is of considerable practical relevance.

References

[1] Abreu, D, Econometrica 56 (1988) 383-396.
[2] Aghion, P. & Bolton, P, American Economic Review 77 91987) 488–501.
[3] Benoit, J. P. & Krishna, V, Econometrica 53 (1985) 905-922.
[4] Bensaid, B. & Gary-Bobo, R. J, Econometrica 61 (1993) 1423-1429.
[5] Berglof, E. & Von Thadden, E. L, Quarterly Journal of Economics 109 (1994) 1055-1084.
[6] Blanchard, O. J. Lopes de Silvanes, F. & Sheleifer, A, Journal of Financial Economics 36 (1994) 337-360.
[7] Bolton, P. & Scharfstein, D, American Economic Review 80 (1990) 93 – 106.
[8] Brander, J. A. & Lewis, T.R, American Economic Review 76 (1986) 956-970.
[9] Bulow, J. & Rogoff, K, Journal of Political Economy 97 (1989) 155-178.
[10] Dewatripont, M, Review of Economic Studies 55 (1988) 377-389.
[11] Fershtman, C. & Judd, K, American Economic Review 77 (1987) 927-940.
[12] Hart, O. D. & Moore, J. H, Quarterly Journal of Economics 109 (1994) 841-879.
[13] Hart, O. D. & Moore, J. H, American Economic Review 85 (1995) 567-585.
[14] Jensen, M. C, American Economic Review 76 (1986) 323-329.