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RESEARCH ARTICLE

Comparison of GBM, GFBM and MJD Models in Malaysian Rubber Prices Forecasting

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Abstract This research studies three mathematical models, namely geometric Brownian motion (GBM), geometric fractional Brownian motion (GFBM) model which was developed by adding the Hurst parameter to GBM to characterize the long-memory phenomenon, and Merton jump-diffusion (MJD) model which captures shocks via GBM. This study sets out to forecast Malaysia rubber prices for the six months period beginning in January 2022 and ending in June 2022, which involves four main steps; calculating the logarithmic return of rubber prices; estimating the parameters for forecasting the rubber prices using the three models; simulating the rubber prices using the GBM, GFBM and MJD models via Monte Carlo simulation; and computing the mean absolute percentage errors (MAPE) and forecast accuracy. Simulation results show that the MJD model is the most accurate model in forecasting the rubber prices.

Keywords: Geometric Brownian Motion, Geometric Fractional Brownian Motion, Merton Jump-Diffusion, Monte Carlo Simulation, Forecasting, Rubber Price.

Introduction

Thousands of products, including tires, medical gloves, bearings, and fenders, are made using natural rubber, which is a highly sought-after global commodity. Even though Malaysia used to export the most natural rubber in the world, we ranked fifth in 2021 according to Malaysian Rubber Council [4] shown in Figure 1. Capital markets provide trading platforms for a variety of long-term financial securities. Even though capital markets serve two purposes—first, as a factor in investor decisions or a means for businesses to raise money from investors, and second, as a means for society to invest in financial instruments, one of which is commodity—capital markets are essential for the operation of a nation's economy.

Following this, we began the process of researching to know which method is the most accurate in forecasting the rubber prices in Malaysia to ensure the findings of this study serve as one of the guidelines for other researchers as they gather the data required to make sure the sector is more sustainable and competitive. The most accurate strategy for predicting the direction of commodity prices is by forecasting. However, due to the imprecise forecast output, the rate of loss risk when employing this strategy is still quite significant [1].

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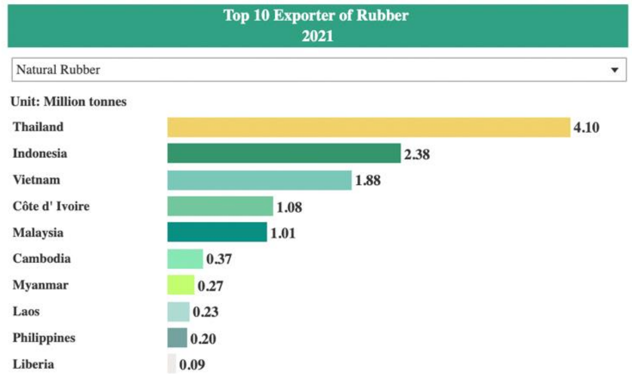

Figure 1. Rank of rubber exporter worldwide in 2021 [4]

The forecasting technique is used to predict future commodity closing prices for low-risk, short-term investments. The geometric Brownian motion (GBM) model, also referred to as the Wiener process, is one of the models that can be used to predict commodity price followed by the geometric fractional Brownian motion (GFBM) model, and Merton jump-diffusion (MJD) model. These models are continuous-time stochastic models in which the random variable moves in a Brownian fashion. Other than these models, the Markov-Fourier grey model could not be utilized because it is only appropriate for long-term investment [11], however the Clustering-Genetic Fuzzy System can only be used to predict the next-day investment [12]. In order for investors to make decisions right away and benefit after the maximum number of days it can reliably foresee, a more trustworthy forecast model that could provide prices for more than one day is required. As a result, even after a brief amount of time, the investors will make a larger profit.

Some studies that applied GBM for forecasting or simulating prices are [1], [2], and [6], while some applied GFBM, such as [13]. In reference [1], they forecast the stock close price for several small companies registered in Malaysia stock exchange. The forecast is limited to short term investment. In their research, it is proven that GBM model is accurate in forecasting the stock close price for two weeks period. It is proven by the small value of Mean Absolute Percentage Error (MAPE). In reference [2] the GBM model is used to predict the stock price of Jakarta Composite Index for January 2015, the daily stock closure price from January 2014 to December 2014 was used. The stages of stock price forecasting are return value calculation, parameter estimation, result collecting, stock price forecast calculation, and MAPE value calculation. Four forecasts are produced in this study utilizing the GBM model. Analysis and debate revealed that the MAPE value was $\leq 20\%$. Moreover, in [6], the study uses GBM to simulate the prices of rubber of various grades. Reference [13] compares both GBM and GFBM for modeling *Eucalyptus* wood prices.

The three models, GBM, GFBM and MJD are used in this study to predict the commodity price of the physical rubber SMR L Grade of the Malaysian Rubber Board within six months period. Using the daily commodity close price from January 2022 to June 2022, it is possible to determine which models are the most effective at predicting the rubber prices. The stages of rubber price forecasting are return value calculation, parameter estimation, result collecting and MAPE value calculation. Three forecasts are produced in this study utilizing GBM, GFBM and MJD models.

In this paper, we demonstrate the methodology of how to incrementally simulate a series of rubber prices given the set of data, and the estimation and calibration of the parameters for the three models in study – GBM, GFBM, and MJD. The results of the simulations using GBM, GFBM and MJD models and the conclusion are documented in this study.

Methodology

Data

The data used in this study is physical rubber SMR L Grade obtained from the official website of Malaysian Rubber Board [3] from January 2022 to June 2022 (118 trading days). The data is fitted to three types of stochastic differential equations, which are geometric Brownian motion (GBM), geometric fractional Brownian motion (GFBM) and Merton jump-diffusion (MJD) models. The models are then compared using Mean Absolute Percentage Error (MAPE) and forecast accuracy.

Geometric Brownian Motion

According to [1], a geometric Brownian motion (GBM) model is a continuous-time stochastic process in which the Wiener process or Brownian motion is used to describe the logarithm of the randomly fluctuating quantity. They add that GBM is crucial for mathematical modelling of the financial process. The short-term movement of stock prices can be forecast using the continuous model's derivative from the discrete model. A stochastic process at time t is *S(t)*, the random value at time t is *W(t)*, the volatility is σ and the drift is μ forming the stochastic model as follows:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t),$$

A stochastic process called a geometric Brownian motion (GBM) has the following properties, for any random initial value S_0 from the solution above,

$$S(t) = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) + \sigma W(t)}.$$

The commodity price *S(t)* at time *t* will be used. According to [6], the daily logarithmic return R_t of the commodity price over the time interval *t* for *t = 1, ..., n* is specified as:

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right),$$

Drift and volatility are estimated at this point. When predicting the commodity price, the constant commodity parameters of drift value and volatility are considered. Formulation for drift is defined as:

$$\mu = \frac{1}{N}\sum_{t=1}^{N} R_t \,,$$

From January 2022 to June 2022, the drift value must be calculated on a monthly average in order to be obtained. The volatility value is calculated next, after the drift value has been determined. Both the volatility common formula and the log volatility formulation are employed in this study. The volatility equation is as follows:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(R_t - \bar{R})^2}\,,$$

where $\bar{R}$ is the mean return of commodity price.

Geometric Fractional Brownian Motion

A generalization of the Brownian motion (BM) is the fractional Brownian motion (FBM). The critical difference between FBM and ordinary BM is that FBM increments are not independent while BM increments are real random function with independent Gaussian increments based on [8].

$$E[B_H(t)B_H(s)] = \frac{1}{2}(|t|^{2H} + |s|^{2H} - |t-s|^{2H}),$$

where H is a real number between 0 and 1, referred to as the Hurst index or Hurst parameter associated with FBM. The Hurst exponent characterizes the raggedness of the resulting motion, with a higher value indicating smoother motion according to [8]. A stochastic process at time *t* is *S(t)*, the FBM with $H \in (0,1)$ is $B_H(t)$, the volatility is *σ* and the drift is *μ* forming the stochastic model as follows:

$$dS(t) = \mu S(t)dt + \sigma S(t)W_H(t),$$

S_0 is given as follows for any initial value chosen at random, the geometric fractional Brownian motion (GFBM) model is,

$$S_H(t) = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_H(t)},$$

where $W_H(t)$ is used in place of $W(t)$. The GFBM model in financial mathematics is a generalization of the GBM model. The logarithmic returns for the GFBM model can be shown as,

$$R_H(t) = ln\left(\frac{S_H(t+1)}{S_H(t)}\right) = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma X_H(t),$$

where:

$$X_H(t) = W_H(t+1) - W_H(t),$$

is the growth in BM, also known as fractional Gaussian noise (FGN), during the period *t*. For the GFBM model, however, we must first estimate the Hurst parameter to get the sample mean and volatility. Three circumstances may be explained by the Hurst parameter's value according to [7]:

(i) The assumption that the discontinuous increments are positively correlated and display short memory dependency is made if $H \in \left(0, \frac{1}{2}\right)$.

(ii) This is the traditional Brownian motion if $H = \frac{1}{2}$.

(iii) The discontinuous increments are thought to be negatively associated and are said to demonstrate long memory dependency if $H \in \left(\frac{1}{2}, 1\right)$.

Rescaled range (R/S) analysis is used in this study to estimate the Hurst exponent to simulate the GFBM. R/S makes no assumptions about the underlying mechanism of the times series and employs elementary statistics. A time series $X_1, X_2, \ldots, X_N$ with a set of cumulative means is given according to [5].

$$m_n = \frac{1}{n}\sum_{t=1}^{n} X_t, \qquad n = 1, 2, \ldots, N,$$

Then, compute the series of cumulative deviations,

$$Z_n = \sum_{t=1}^{n} [X_t - m_n], \qquad n = 1, 2, \ldots, N,$$

with the range deviation series,

$$R_n = \max(Z_1, Z_2, \ldots, Z_n) - \min(Z_1, Z_2, \ldots, Z_n), \qquad n = 1, 2, \ldots, N,$$

and determine the standard deviation series as,

$$S_n = \sqrt{\frac{1}{n}\sum_{t=1}^{n} [X_t - m_n]^2},$$

Rescaled range deviation of a given time series is defined as a sequence (R_n/S_n). Rescaled range deviation should increase by *n* since R_n is expected to grow by *n* while S_n is anticipated to reach a number. The asymptotic behavior of the rescaled range can be represented by the Hurst exponent as a function of the period of a time series. Such a relation can be illustrated as follows for large *n*,

$$\lim_{n\to\infty} n^{-H} E\left[\frac{R_n}{S_n}\right] = C,$$

where *C* is a constant. For large *n*, it can be rewritten as,

$$E\left[\frac{R_n}{S_n}\right] \approx C \cdot n^H,$$

Hence, we have for a series of (R_n/S_n)

$$ln\left(\frac{R_n}{S_n}\right) \approx lnC + Hln(n), \qquad n = 1, 2, \ldots, N,$$

By performing a linear regression on $\ln(R_n/S_n)$ and $\ln(n)$, where *H* is the slope, the parameter *H* can be determined.

Merton Jump-Diffusion

For the Merton jump-diffusion (MJD) model, a Poisson process determines the jumps. As a result, the Poisson process is presented.

Definition 1. Let the series of exponential random variables in $\{\tau_i\}_{i\geq 1}$ have the parameter λ. Suppose $T_n = \sum_{t=1}^{N} \tau_i$. The Poisson process $\{N_t\}_{t\geq 1}$ is therefore defined as follows:

$$N_t = \sum_{n\geq 1} 1_{t\geq T_n},$$

where the estimated number of jumps per unit of time is the intensity λ. Both the jump times and the jump heights are random in the MJD model. A compound Poisson process, which is therefore presented here, controls the unpredictability.

Definition 2. Let $\{N_t\}_{t\geq 0}$ be a Poisson process with an intensity parameter λ, and let $\{Q_i\}_{i\geq 1}$ be a series of independent random variables with the same distribution. The compound Poisson process $\{Y_t\}_{t\geq 0}$ is thus defined as follows with jump intensity λ:

$$Y_t = \sum_{i=1}^{N_t} Q_i,$$

where Y_t is defined as $Y_t = \sum_{i=1}^{0} Q_i = 0$ if $N_t = 0$. Hence, the MJD model for the rubber price S_t is:

$$S_t = S_0 e^{\left(\mu_d - \frac{\sigma_d^2}{2}\right)t + \sigma_d W_t + \sum_{t=1}^{N} Q_t},$$

where $\sum_{t=1}^{N} Q_t$ is a compound Poisson process with normally distributed jumps $N(\mu_j, \sigma_j^2)$ and intensity λ, where μ_d is named as diffusion drift and σ_d is named as the volatility of diffusion, W_t is a standard Brownian motion. Here, the indices *d* and *j* stand for the MJD model's diffusion and jumps, respectively. The MJD model stock price logarithmic return R_t can be calculated as follows:

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right) = \left(\mu_d - \frac{\sigma_d^2}{2}\right)t + \sigma_d(W_t - W_{t-1}) + \sum_{i=N_{t-1}}^{N_t} Q_i,$$

where $W_t = W_t - W_{t-1}$ is a standard Brownian motion increment, Q_i are independent normal distributed variables having a mean of μ_j and a variance of σ_j^2, and $N_t = N_t - N_{t-1}$ is a Poisson random variable with a mean of λ_t. We notice that $\sum_{i=1}^{N_t} Q_i$ and $\sum_{i=N_{t-1}}^{N_t} Q_i$ have the same distribution, where once again $N_t = N_t - N_{t-1}$.

The MJD modeled rubber prices log-return expectation and variance are as follows:

$$E(R_t) = \left(\mu_d - \frac{\sigma_d^2}{2}\right)t + \mu_j\lambda t, \qquad Var(R_t) = \sigma_d^2 t + \left(\sigma_j^2 + \mu_j^2\right)\lambda t.$$

In summary, the expected and variance of the log-return are as follows when there is exactly one jump.

$$E\left(R_t^J\right) = \left(\mu_d - \frac{\sigma_d^2}{2}\right)t + \mu_j, \qquad Var\left(R_t^J\right) = \sigma_d^2 t + \sigma_j^2.$$

From that, parameters μ_j and σ_j are estimated,

$$\hat{\mu}_j = \hat{E}\left(R_t^J\right) - \left(\hat{\mu}_d - \hat{\sigma}_d^2\right)t, \qquad \hat{\sigma}_j^2 = \widehat{Var}\left(R_t^J\right) - \sigma_d^2 t.$$

where the sample mean and sample variance of the empirical log-returns are denoted by $\hat{E}\left(R_t^J\right)$ and $\widehat{Var}\left(R_t^J\right)$, respectively. The parameters μ_d and σ_d can be approximated as follows when there are no jumps,

$$\hat{\mu}_d = \frac{2\hat{E}(R_t^D) + \widehat{Var}(R_t^D)t}{2\Delta t}, \qquad \hat{\sigma}_d^2 = \frac{\widehat{Var}(R_t^D)}{\Delta t}.$$

where the sample mean and sample variance of the empirical log-returns are denoted by $\hat{E}(R_t^D)$ and $\widehat{Var}(R_t^D)$, respectively.

Mean Absolute Percentage Error

Due to its benefits of scale independence and interpretability, the Mean Absolute Percentage Error (MAPE) is one of the most often used indicators of forecast accuracy. In contrast to the actual values, which are zero or very close to zero, it produces endless or undefined values. The MAPE produces exceptionally high percentage errors or outliers if the real value is very small (less than one) according to [9]. The MAPE is described as:

$$MAPE = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{A_t - F_t}{A_t}\right| \times 100\%\,,$$

where A_t and F_t stand for the actual and forecast values at the given time t, respectively.

Table 1. A scale of Judgement of Forecast Accuracy [1]

MAPE	Judgement of Forecast Accuracy
$< 10\%$	Highly Accurate Forecast
11% - 20%	Good Forecast
21% - 50%	Reasonable Forecast
$> 51\%$	Inaccurate Forecast

The accuracy of forecasting models increased with a decrease in the MAPE value. Some evaluations of the models can be determined by applying the MAPE formula and the scale in Table 1.

Results and Discussion

This section discusses the findings that have obtained in the study of forecasting the rubber prices and provides a discussion on the findings using geometric Brownian motion (GBM), geometric fractional Brownian motion (GFBM) and Merton jump-diffusion (MJD) model simulation. The calibration of parameters using GBM and GFBM are documented in Table 2, where in addition, we calibrated the Hurst exponent for the GFBM and obtained $H = 0.82$ via the rescaled range (R/S) analysis. Table 3 provides the parameters calibration for MJD.

Table 2. The parameters estimation of drift and volatility for GBM and GFBM models

Drift, μ and Volatility, σ		
Model	μ	σ
GBM	0.00072	0.0046
GFBM	0.00064	0.0075

Table 3. The parameters estimation of sample mean and sample variance for MJD model

Sample Mean, μ and Sample Variance, σ of MJD model				
Model	μ_d	σ_d	μ_j	σ_j
MJD	-0.0765	0.0867	0.0974	0.1093

The graph pattern in Figure 2 plots the actual rubber prices with the simulated data using the GBM model. It is shown that the rubber prices are growing slow consistently when using the GBM model for forecasting within the 118 trading days which the ending price is the same with the actual data. The MAPE value achieved from the simulation using the GBM model is 18.0530% which is categorized as "good forecast" based on Table 1.

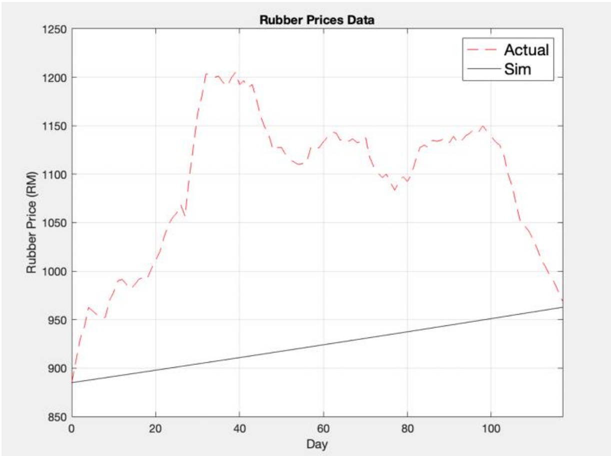

Figure 2. Simulation of Geometric Brownian Motion

Figure 3 is the graph pattern of plotting the actual data and simulated data of the rubber prices in 118 days which uses the GFBM model. The likelihood of the prices growing to at least RM950 at the end of 118 trading days with the MAPE values of 14.2486% which resulting in a "good forecast" based on the judgement scale provided in Table 1.

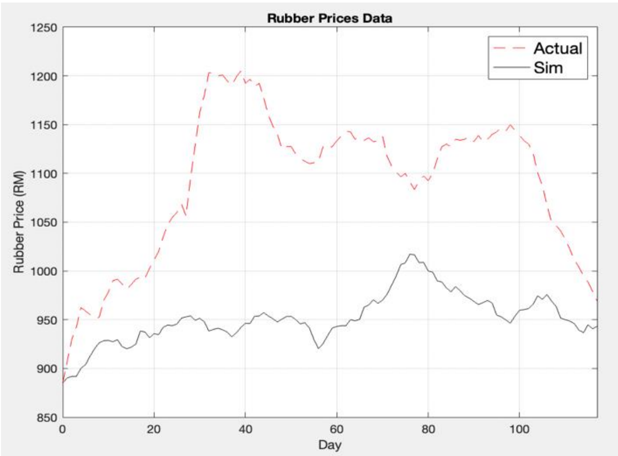

Figure 3. Simulation of Geometric Fractional Brownian Motion

The graph pattern of actual rubber prices and forecast prices after simulation using the MJD model is given in Figure 4. It is shown that the forecast price on the last day is higher than the actual price. The path of the actual and forecast rubber prices using the simulation of the MJD model is displayed. The MAPE value achieved from the simulation is 12.7696% which is categorized in "good forecast" based on Table 1.

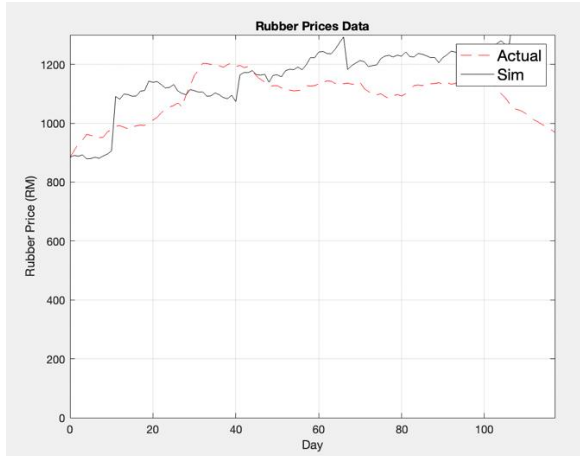

Figure 4. Simulation of Merton Jump-Diffusion

The MAPEs are calculated to evaluate the GBM, GFBM and MJD models' performance in predicting or simulating the price trajectory of rubber prices. The accuracy of forecasting the rubber prices is simulated with the usage of the GBM, GFBM and MJD models. Table 4 documents the MAPE values for all three models.

Table 4. The value of MAPE for each models used

MAPE values of Forecasting the Rubber Prices			
Period	GBM	GFBM	MJD
6 months	18.0530%	14.2486%	12.7696%

Overall, the GBM, GFBM and MJD models yield a "good forecast" predicted prices using the judgement scale provided in Table 1. The MJD model forecast, however, appears to be more accurate than the other two models, since the model obtained the lowest MAPE value, according to the data.

In addition to the MAPE values of the modelled prices, we also calculate the forecast accuracy (FA), which is a measure of the accuracy of the simulated prices over a 100% range.

$$FA = max\left(0, 100\% - \left(\left|\frac{A_t - F_t}{A_t}\right| \times 100\%\right)\right).$$

To determine further which model is accurate, FA is obtained for each model, as tabulated in Table 5.

Table 5. The values of MAPE and FA for each of models used

Model	MAPE	FA
GBM	18.0530	86.06
GFBM	14.2486	89.03
MJD	12.7696	91.77

Based on Table 5, MJD model is the most accurate model among the three models used in this study because not only it produces the lowest MAPE value among the three models, it also produces the highest forecast accuracy, which is 91.77%. This is then followed by GFBM at 89.03%, and GBM at 86.06%.

Conclusion

In conclusion, this study compares the accuracy of three mathematical models, which are geometric Brownian motion (GBM), geometric fractional Brownian motion (GFBM), and Merton jump-diffusion (MJD), to simulate Malaysia rubber price changes, by using past rubber prices in a period of six months (or 118 trading days). Unknown parameters were calibrated using historical prices to use these models. By calculating the mean absolute percentage error (MAPE), which measures the simulations' accuracy, we conclude that the three models can produce good accurate forecast prices. The MJD model, however, is more accurate than the GBM and GFBM, based on the simulations in this study.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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