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RESEARCH ARTICLE

LH-Moments of the Wakeby Distribution Applied to Extreme Rainfall in Thailand

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Abstract This article applies the Wakeby distribution (WAD) with high-order L-moments estimates (LH-moments) to annual extreme rainfall data obtained from 99 gauge stations in Thailand. The objective of this study is to investigate appropriate quantile estimates and return levels for several return periods, 2, 5, 10, 25, and 50 years. The 95% confidence intervals for the quantiles determined from the WAD are derived using the bootstrap technique. Isopluvial maps of estimated design values that correspond to selected return periods are presented. The LH-moments results are better than estimates from the more primitive L-moments method for a large majority of the stations considered.

Keywords: L-Moments, LH-Moments, Wakeby Distribution, Higher-Order Statistics, Bootstrap Resampling.

Introduction

The statistical modelling of extreme rainfall is important in the design of water-related structures such as agriculture. More generally, it provides us an information about weather modification and monitoring climate change (Huff and Angel, 1992). In particular, hydro-meteorologists can fit various statistical frequency distributions to historical rainfall data in order to estimate the magnitude of maximum rainfall at various recurrent intervals.

The rainfall between the months of August to October in Thailand is the time intereval that we are interested in this article. As elsewhere, extreme rainfall events can become natural disasters in Thailand, contributing to crop losses or property damage and general human misery (Deka et al., 2009). Khamkong (2012) and Khongthip et al. (2013) have previously modelled annual monthly maximum rainfalls in upper northern Thailand via generalised extreme value distributions (GEVD), Keawmun et al. (2015a), Keawmun et al. (2015b), and Busababodhin et al. (2015a) likewise modelled annual daily and monthly maximum rainfalls in northeastern Thailand.

To the best of our knowledge, no study has yet employed any other statistical distribution to model rainfall data in Thailand. However, empirical evidence related to the condition of separation (Matalas et al., 1975)

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suggests that flood distributions are well described by the Wakeby distribution (WAD) with $\beta > 1$ and $\gamma > 0$, i.e. the WAD provides a rather more plausible description of flood sequences to better represent the long stretched upper tail structures of flood distributions, as well as the tail structures of other hydrologic phenomena (Landwehr et al., 1980). Moreover, the WAD can be considered as a parent flood distribution quite widely. It successfully used in hydrology, especially in modelling extreme events. Another study by Wilks and McKay (1996) concluded that the WAD provides the best representation of extreme snowpack water equivalent. Its application to rainfall maxima was investigated by various authors. Park et al. (2001) used the WAD with the method of L-moments estimates for summer extreme rainfall data in South Korea. Oztekin (2007) measured upper right tail estimate performances for annual extreme and partial duration precipitation series at 31 stations in the northeastern and southeastern United States, and found that L-moments of the WAD mostly gave better results than beta-*K* and beta-*P* distributions and sometimes produced comparable results using bootstrap resampling to extrapolate the right tail behaviour. Zalina et al. (2002) and Su et al. (2009) simulated extreme precipitation over the Yangtze River Basin using the WAD, and concluded that it can adequately describe the probability distribution of precipitation extremes from both daily observational precipitation data at 147 stations in the Yangtze River Basin during 1960-2005 and projected daily data of 79 grid cells from the ECHAM5/ MPI-OM model.

In addition, Seckin et al. (2011), Soukissian (2013), Yao et al. (2013) and Rahman et al. (2015) employed the WAD for flood frequency and rainfall analyses. Busababodhin et al. (2015b) employed LH-moments to estimate WAD parameters. Simulations showed that LH-moments estimation performed better than L-moments estimation, and they proceeded to investigate annual maximum flood and wave height data.

Here we apply the WAD and the LH-moments method introduced by Busababodhin et al. (2015b) to extreme rainfall data in Thailand, to obtain reliable quantile estimates for several return periods that we define return period in the Section Quantile Estimation. The 95% confidence intervals for the quantiles are obtained by the bootstrap resampling technique. Isopluvial maps of the quantiles at selected return periods suitable for planners and meteorologists are presented. Section Climatology and Data in this article describes the climatology and descriptive statistics of the annual daily precipitation in Thailand. Section LH-Moments Estimation from the Wakeby Distribution presents LH-moments estimation of the WAD with our methodology and results. The quantile estimation by the bootstrap technique and isopluvial maps of the quantiles at the selected return periods are provided in Section Quantile Estimation, and Section Conclusions summarises our results.

Climatology and Data

Climatology

Thailand is tropical and the climate is also affected by the seasonal winds of the southwest and northeast monsoons. Rainfall in Thailand mainly occurs during the southwest monsoon, under the influence of the Inter-Tropical Convergence Zone (ITCZ) and tropical cyclones that produce heavy precipitation. The southwest monsoon typically starts in May, and brings a stream of warm moist air from the Indian Ocean towards Thailand producing abundant rain over the country, especially on the windward side of the mountains. The ITCZ first moves to the south during May, and then rapidly northwards to southern China around June or early July, when a dry spell occurs in northern Thailand. The ITCZ then again moves south to lie over northern and northeastern Thailand in August, and over the central and southern parts in September and October. The northeast monsoon typically starts in October, bringing cold and dry air from anticyclones over the Chinese mainland to much of Thailand, especially the north and northeast higher latitude areas but it causes milder weather and abundant rainfall along the southern-east coast, so the rainy season in southern Thailand generally differs from that in upper Thailand. During the southwest monsoon, heavy rainfall over the southern-west coast peaks in September, and at the southern-east coast in November until January. Tropical cyclones usually move across Thailand about 3 - 4 times a year, although not during January to March.

According to historical records, from 1951 to 2011 the northeast was hit by tropical cyclones 88 times and the south 53 times, with relatively higher frequencies in September and October. The heaviest cyclone damage usually occurs in the south e.g. tropical storm “HARRIET” hit Nakhon Si Thammarat province in October 1962, typhoon “GAY” hit Chumphon province in November 1989, and the latest was the typhoon “LINDA” that hit Prachuap Khiri Khan province in November 1997. The annual rainfall for most areas of the country is 1,200 - 1,600 mm. However, some areas on the windward side including Trat province in the east and Ranong province on the southwest coast receive more than 4,000 mm each year. In 2014 Thailand was not directly hit by any tropical cyclone and the total average rainfall was 1520.4 mm, 4% below normal. However, there were several tropical cyclones in the area that indirectly influenced the weather e.g. typhoon “RAMMASUN”, typhoon “KALMAEGI” and tropical storm “SINLAKU” that led to abundant rainfall and flash floods in late July, mid-September and late November, respectively, see Thai Meteorological (2014). In 2016, the majority of Thailand was warmer and larger rainfall than usual. The 1981-2010 normal annual rainfall averaged over the country is 1,718.1 mm, but in 2016 it was 130.4 mm (8%) above. Moreover, rainfall of Thailand was affected by 6 tropical cyclones with 2 tropical cyclones that hit Thailand when they were depressions, namely the tropical storm “Rai” on 13 September and the tropical storm “Aere” on 14 October, 2016. Besides, rainfall in Thailand was increasing by the indirect affected of the tropical depression in the middle Vietnam in late June, the tropical storm “MIRENAE” in late July, the tropical storm “DIANMU” in the middle of August, and the tropical depression in Cambodia in early November, see Thai Meteorological (2016). The weather of Thailand in 2017 was very unusual. The average annual rainfall was 27 percent higher than normal and the highest in 67 years (1951-2017). The total rainfall throughout the month and the total rainfall throughout the year were higher than the previous statistical measurements. In addition, there were many tropical cyclones, depressions and typhoons moved into Thailand such as tropical storm "TALAS", tropical storm "SONCA" and typhoon "DOKSURI" and so on, see Thai Meteorological (2017). In 2019, Thailand had less rainfall than it used to occur. Only January and August had more rainfall than usual, as a result of the influence of tropical cyclone that moved into Thailand. It started with tropical storm "PABUK", which was the first tropical cyclone that moved to Thailand in January in 69 years. In August, there were two storms entered in Thailand, that were tropical storm "WIPHA" moved into the northern region and tropical storm "PODUL" moved into the northeast region of the country. Because in 2019 there was less rain than normal, Thailand to had a higher temperature than usual every month in many areas with the highest temperatures breaking the previous record, see Thai Meteorological (2019).

Data

In this article, we consider the time series of annual maximum precipitation constructed from monthly rainfall data during 1984 to 2019, from each of 99 stations throughout Thailand, maintained by the Thailand Meteorological Administration, see Figure 1. In Table 1, we selected stations with the two highest and lowest maximum rainfalls in each region to show sample analysis. When divided by climate characteristics, Thailand can be separated into 6 regions as follows: northern, northeastern, central, eastern, southern-east coast and southern-west coast.

The station codes, names and locations, sample sizes (n), sample statistics including minimum, maximum, medians and interquartile ranges (IQR) of annual maximum precipitation (AMP), skewness and kurtosis of data computed from the time series are shown in Tables 1 and 2, respectively.

Table 1. Region, station code, station name, latitude and longitude for sample stations in Thailand

Region	Code	Name	Latitude	Longitude
Northern	328301	Lampang Agromet.	18.3166667	99.2833333
	330201	Phrae	18.1666667	100.1666667
	376401	Umphang	16.0247222	98.8644444
	380201	Kamphaeng Phet	16.4866667	99.5269444
Northeastern	431201	Nakhon Ratchasima	14.6419444	101.3213889
	354201	Udon Thani	17.3769444	102.8094444
	357301	Nakhon Phanom Agromet.	17.4430556	104.2736111
	436401	Nang Rong	14.5833333	102.8000000
Central	400301	Tak Fah Agromet.	15.3491667	100.5302778
	402301	Chai Nat Agromet.	15.1500000	100.1833333
	455201	Queen Sirikit National Convention Center	13.7263889	100.5600000
	455601	Don Muang	13.9072222	100.5966667
Eastern	423301	Chachoengsao Agromet.	13.5155556	101.4580556
	440201	Arunya Prathet	13.7000000	102.5833333
	480201	Chanthaburi	12.6166667	102.1133333
	501201	Trad	11.7802778	102.8780556
Southern-east coast	500301	Nong Plub Agromet.	12.5888889	99.7344444
	552301	Nakhon Si Thammarat Agromet.	8.3591667	100.0594444
	568501	Songkhla	7.1822222	100.6075000
	568502	Hat Yai	6.9180556	100.4333333
Southern-west coast	532201	Ranong	9.9833333	98.6166667
	564201	Phuket	7.8833333	98.4000000
	566201	Lun Ta (Krabi)	7.5333333	99.0500000
	570201	Satun	6.6500000	100.0833333

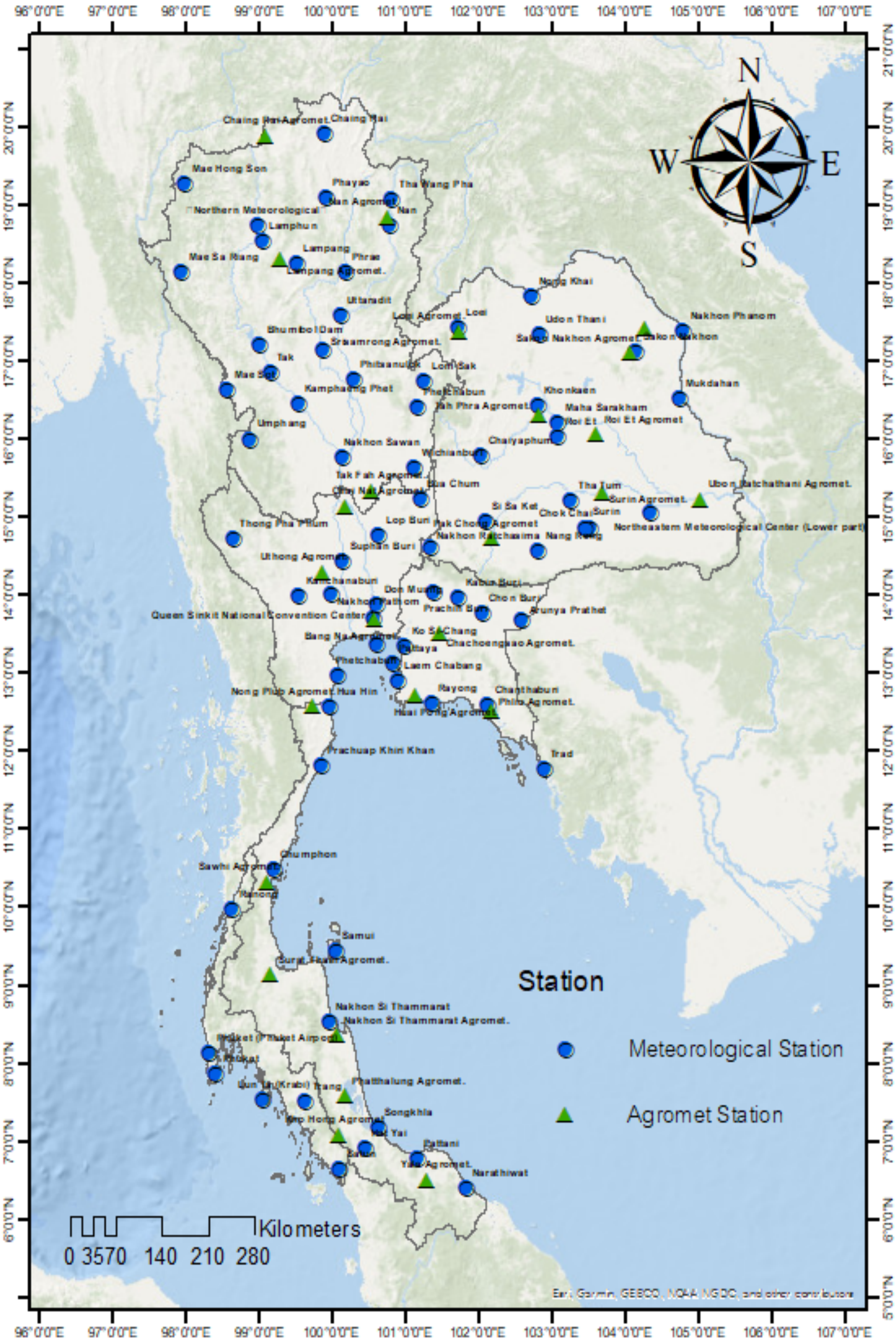

Figure 1. The location of weather stations in Thailand

Table 2. Station code, sizes, minimum, maximum, median, interquartile range (IQR) of AMP, skewness and kurtosis for sample stations in Thailand from monthly rainfall data during 1984 to 2019

Code	Size (*n*)	Minimum (unit; mm)	Maximum (unit; mm)	Median (unit; mm)	IQR (unit; mm)	Skewness	Kurtosis
328301	36	6.70	114.40*	20.73	70.25	-0.33	4.95
330201	36	43.30	218.20	22.75	80.10	2.07	8.15
376401	36	33.90	124.70	16.53	70.85	0.61	3.99
380201	36	49.20	248.90*	18.70	83.25	2.69	11.68
431201	36	50.10	129.70**	32.60	76.40	0.55	2.17
354201	36	54.20	274.50*	31.50	90.90	2.37	9.88
357301	36	71.50	272.60	53.00	124.00	1.18	3.96
436401	36	45.70	130.50	31.53	90.25	0.03	2.30
400301	36	52.90	116.70	28.23	81.35	0.14	1.92
402301	36	41.00	107.80**	21.55	69.90	0.35	2.53
455201	36	40.20	216.80*	52.58	93.95	0.82	3.19
455601	36	58.40	210.70	33.68	98.60	1.71	6.56
423301	36	49.00	144.60	32.33	79.05	0.95	3.22
440201	36	45.70	142.80**	17.88	74.80	1.18	5.09
480201	36	84.70	394.90	59.48	138.75	2.22	9.32
501201	36	127.50	445.30*	139.58	225.00	0.58	2.20
500301	36	45.20	226.00	37.73	83.60	1.36	4.20
552301	36	64.90	615.60*	104.60	146.60	2.08	8.15
568501	36	71.20	521.80	115.85	155.35	1.46	5.09
568502	36	51.00	219.40**	31.75	100.80	1.21	4.14
532201	36	116.50	249.70*	37.03	162.70	0.48	3.10
564201	36	70.00	180.70**	45.85	102.45	0.75	2.44
566201	36	73.00	241.60	41.25	122.10	1.33	4.55
570201	36	67.00	207.80	43.90	108.05	0.94	3.58

Results and discussion

LH-Moments Estimation from the Wakeby Distribution

LH-moments are expected to characterise the upper part of distributions well, as they generally emphasise high upper distribution quantiles rather than lower quantiles (Wang, 1997). LH-moments estimation procedure is more reliable than the classical method of moments estimation, particularly for small sample sizes, and is usually computationally more tractable than maximum likelihood estimation. The definition of LH-moment with order $\eta = 0,1,2,...$ for $r = 1,2,...$ as defined by Wang(1997) are

$$\lambda_r^\eta = \sum_{k=0}^{r-1} C_{r,k} E\left[X_{\eta+r-k:\eta+r}\right], \tag{1}$$

where $C_{r,k} = (-1)^k \binom{r-1}{k}$.

Furthermore, Park et al (2001) showed the use of linear combination moments instead of conventional product moments and the resistance to the presence of any outliers present in the sample, due to the occurrence of heavy rainfall and typhoon events, means the method is quite robust.

The Wakeby distribution quantile function has the form

$$x(F) = \xi + \frac{\alpha}{\beta}[1-(1-F)^{\beta}] - \frac{\gamma}{\delta}[1-(1-F)^{-\delta}] \tag{2}$$

where $F \equiv F(x) = P[X \le x]$, ξ is the location parameter and α, β, γ and δ are other parameters. The parameter α largely relates to the scale of the variable while β, γ and δ are exponential parameters defining the shape of the quantile function. The parameterisation explicitly exhibits the WAD as a generalisation of the generalised Pareto distribution for which $\alpha = 0$ or $\gamma = 0$ and provides estimates of the α and γ parameters that are more stable under small perturbations of the data.

The population LH-moments of the WAD, see Busababodhin et al. (2015b):

$$\lambda_1^\eta = \xi + \frac{\alpha}{\beta}[1-(\eta+1)T_1] - \frac{\gamma}{\delta}[1-(\eta+1)D_1], \tag{3}$$

$$\lambda_2^\eta = \frac{\eta+2}{2}\begin{bmatrix} \frac{\alpha}{\beta}\left((\eta+1)T_1-(\eta+2)T_2\right) \\ -\frac{\gamma}{\delta}\left((\eta+1)D_1-(\eta+2)D_2\right) \end{bmatrix}, \tag{4}$$

$$\lambda_3^\eta = \frac{\eta+3}{3!}\begin{bmatrix} \frac{\alpha}{\beta}\left(-W_3T_3+2W_2T_2-W_1T_1\right) \\ -\frac{\gamma}{\delta}\left(-W_3D_3+2W_2D_2-W_1D_1\right) \end{bmatrix}, \tag{5}$$

$$\lambda_4^\eta = \frac{\eta+4}{4!}\begin{bmatrix} \frac{\alpha}{\beta}\left(-G_4T_4+3G_3T_3-3G_2T_2+G_1T_1\right) \\ -\frac{\gamma}{\delta}\left(-G_4D_4+3G_3D_3-3G_2D_2+G_1D_1\right) \end{bmatrix}, \tag{6}$$

$$\lambda_5^\eta = \frac{\eta+5}{5!}\begin{bmatrix} \frac{\alpha}{\beta}\left(-Y_5T_5+4Y_4T_4-6Y_3T_3+4Y_2T_2-Y_1T_1\right) \\ -\frac{\gamma}{\delta}\left(-Y_5D_5+4Y_4D_4-6Y_3D_3+4Y_2D_2-Y_1D_1\right) \end{bmatrix}. \tag{7}$$

where $T_l = \mathrm{B}(\eta+l, \beta+1)$ and $D_l = \mathrm{B}(\eta+l, 1-\delta)$ for B been the beta function, $\eta = 0,1,2,3,4$, $l = 1, 2, 3, 4, 5$, and

$W_i = (\eta+i+1)(\eta+i)$ for $i = 1, 2, 3$,

$G_j = (\eta+j+2)(\eta+j+1)(\eta+j+1)(\eta+j)$ for $j = 1, 2, 3, 4$,

$Y_k = (\eta+k+3)(\eta+k+2)(\eta+k+1)(\eta+k)$ for $k = 1, 2, 3, 4, 5$.

When $\eta = 0$, the LH-moments reduce to L-moments. Here we consider the following constraints for the existence of LH-moments for the WAD, which are the same as the constraints for the WAD function. It is assumed that $\beta + \delta \geq 0$; and the range of x is $\xi \leq x \leq \infty$ if $\delta \geq 0$ and $\gamma \geq 0$, or $\xi \leq x \leq \xi + \dfrac{\alpha}{\beta} - \dfrac{\gamma}{\delta}$ if $\delta < 0$ or $\gamma = 0$. For $x(F)$ to be a valid quantile function, the conditionals $\gamma \geq 0$ and $\alpha + \gamma \geq 0$ must also hold (Hosking, 1986).

The sample LH-moments are obtained from the given observational data, see Busababodhin et al. (2015b). No explicit solution of the simultaneous equations is possible in the WAD, but the equations may be solved by an augmented Lagrangian adaptive barrier minimisation algorithm for optimising smooth nonlinear objective functions with nonlinear constraints. Varadhan (2012) introduced a nonlinear optimisation algorithm in the statistical software R for nonlinear constraints, involving the optimisation function “auglag” in the R package “alabama”. The elaborated form is an augmented lagrangian adaptive barrier minimisation algorithm for optimising smooth nonlinear objective functions with nonlinear constraints.

The order (η), parameter estimates (LH-moments) of the WAD, and the Kolmogorov-Smirnov’s goodness-of-fit statistic *D* (KS-*D*) at each station are given in Table 3. The p-values here are computed by the formula from Press et al. (1996, p.618), as if the parameters of the WAD are specified. If the estimated p-value is less than 0.1, then a more accurate way in the simulation is to compute the true p-value, see Ross (1990, section 9.2). It can be seen from Table 3 that, there are 2 stations with p-values less than 0.1 which are 0.054 and 0.052 at sites 455601 and 552301, respectively. There are only 3 stations (or 12.5%) with $\eta = 0$ and 21 stations (or 87.5%) with other values of η. Therefore, there is a strong evidence that the WAD modelling with LH-moments method approach is more accurate for each of the stations than the L-moments method. Figure 2 shows the order η of the WAD for weather stations in Thailand and Figure 3 shows the relative frequency histogram with various η values corresponding to LH-moments for the WAD at station 455201.

From Figure 3, solid black line is for eta = 0, red dashed for eta = 1, green dashed for eta = 2, blue dashed for eta = 3, and gray dashed for eta = 4. The result shows relative frequency histogram at station 455201 when using LH-moments ($\eta = 0, 1, 2, 3 \text{ and } 4$) for the WAD. We can see that $\eta = 0 \text{ and } 1$ are almost fit to the relative frequency histogram of real data set. However, the $\eta = 1$ is the best fit to the real data set that imply LH-moments method is better than L-moments method.

Quantile Estimation

The quantile or the design value corresponding to a return period of *T* years (*T* years return value) is defined by the magnitude *x*(*F*) with $F = 1 - 1/T$. The design values were computed via Eq. (2), and the confidence interval of the return value *q*(*T*) was obtained by the bootstrap resampling technique. The design values and 95% confidence intervals corresponding to 2, 5, 10, 25 and 50 years computed from the annual extreme rainfall at each station are presented in Table 4. We use the Percentile Bootstrap confidence interval with the number of replications 2000. For more discussion on bootstrap confidence intervals for predicted rainfall quantiles, see Dunn (2001). Isopluvial (rainfall frequency) maps of the estimated design values corresponding to selected return periods of 2, 5, 10, 25 and 50 years for the AMP using the values from the 99 stations are presented in Figures 4 to 8, respectively.

The highest return values were for the sites in the windward side of the eastern and southern-east coast parts of Thailand, due to the influence of typhoons or tropical storms. Thailand is subject to tropical depressions further inland, although some mountain ranges obstruct the wind, and in the southern-east coast there is also a relatively high risk of typhoons and tropical storms. In Table 4, it shows that, the site 501201 at Trad shows the highest return values of 2, 5, 10, and 25 years and the site 552301 at Nakhon Si Thammarat Agrome shows the highest return value of 50 year, with a large range of confidence intervals compared to the real data in Table 4. Moreover, both of these stations also show the top two highest rainfall. In general, the quantile estimation from our estimation and distribution approach is similar to the real rainfall data.

Table 3. Station code, order, parameter estimates (LH-moments) of the WAD, and KS-D's statistic computed from the 24 stations in Thailand.

Code	Order (η)	parameter estimate					KS-D	p-value
		ξ	α	β	γ	δ		
328301	2	45.087	40.841	3.019	13.364	0.011	0.126	0.617
330201	1	55.475	23.567	2.756	17.439	0.287	0.139	0.491
376401	1	42.860	65.516	3.639	11.052	0.121	0.150	0.392
380201	1	67.557	1.485	7.350	15.748	0.361	0.169	0.256
431201	2	60.356	-70.340	2.828	77.086	-0.751	0.139	0.491
354201	1	63.683	29.527	3.442	20.536	0.312	0.130	0.581
357301	2	83.181	1.088	15.983	58.388	-0.103	0.083	0.948
436401	1	60.640	5.796	11.305	53.303	-0.678	0.083	0.946
400301	0	52.885	-0.540	16.007	52.660	-0.885	0.078	0.980
402301	1	51.823	1.121	15.983	31.926	-0.449	0.095	0.903
455201	1	53.919	3.830	6.033	61.668	-0.241	0.076	0.985
455601	2	58.731	46.391	1.484	9.279	0.424	0.224	0.054
423301	0	52.422	117.628	18.364	30.301	-0.181	0.115	0.723
440201	1	50.598	49.984	3.166	11.127	0.196	0.181	0.188
480201	1	83.112	64.622	2.149	27.879	0.299	0.163	0.262
501201	3	98.637	1.056	15.981	236.555	-0.622	0.166	0.244
500301	1	50.432	7.767	14.230	46.070	-0.002	0.087	0.926
552301	0	54.940	83.433	2.397	68.550	0.209	0.225	0.052
568501	2	80.932	4.608	4.782	100.016	0.018	0.111	0.724
568502	3	62.592	1.021	15.996	46.588	-0.054	0.149	0.364
532201	1	131.508	3.505	6.241	46.952	-0.324	0.139	0.491
564201	3	77.520	1.233	7.386	45.015	-0.182	0.066	0.997
566201	1	81.869	42.181	4.407	34.990	0.034	0.070	0.995
570201	2	81.172	1.565	7.245	38.234	-0.101	0.125	0.625

Figure 2. The order, η , of the WAD for weather stations in Thailand.

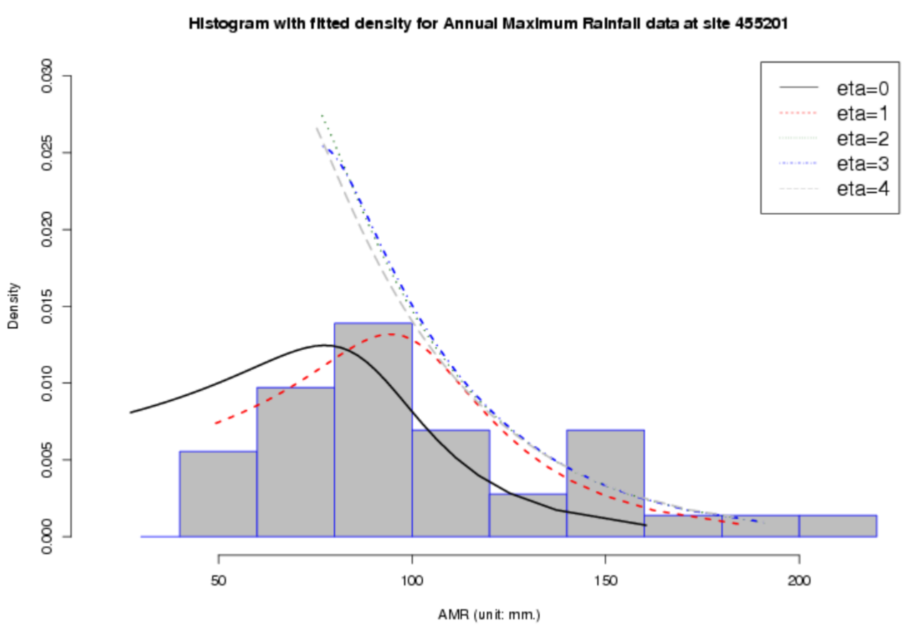

Figure 3. Relative frequency histogram and probability functions fitted for AMP data at station 455201 when using LH-moments for the WAD.

Table 4. Station code, return level (unit, mm) corresponding to various return periods (T) computed from the time series of annual extreme rainfall for 24 stations in Thailand.

Code	Order (η)	Return level (unit; mm)				
		T = 2	T = 5	T = 10	T = 25	T = 50
328301	2	66.25 (13.25, 72.34)	80.21 (30.27, 92.13)	89.77 (38.19, 103.97)	102.42 (46.77, 115.80)	112.06 (51.16, 122.75)
330201	1	76.13 (14.64, 84.53)	99.60 (38.66, 118.82)	120.92 (51.13, 145.06)	156.35 (64.26, 188.52)	190.05 (71.18, 226.76)
376401	1	67.41 (12.62, 72.55)	80.45 (32.31, 88.11)	90.21 (43.47, 99.19)	104.38 (53.80, 115.07)	116.18 (58.05, 127.48)
380201	1	80.16 (14.06, 107.33)	102.12 (38.41, 129.48)	124.30 (51.45, 156.10)	163.56 (65.23, 204.14)	203.20 (72.10, 255.07)
431201	2	80.65 (20.02, 83.78)	107.76 (43.33, 110.44)	119.98 (55.87, 122.67)	129.02 (66.07, 132.47)	132.73 (72.53, 136.98)
354201	1	87.36 (10.55, 96.71)	115.16 (37.59, 136.67)	141.45 (54.50, 170.15)	186.13 (71.38, 224.56)	229.51 (80.68, 271.58)
357301	2	122.32 (4.47, 133.91)	169.88 (51.09, 197.81)	203.00 (72.38, 234.36)	243.32 (91.94, 271.71)	271.40 (103.19, 294.51)
436401	1	90.63 (30.52, 96.69)	113.36 (52.01, 119.30)	123.25 (64.36, 127.59)	130.88 (75.78, 132.70)	134.20 (82.40, 135.86)
400301	0	80.14 (23.01, 87.81)	98.04 (41.59, 104.35)	104.61 (47.91, 108.53)	108.92 (52.76, 112.05)	110.50 (54.48, 114.68)
402301	1	70.91 (13.53, 82.56)	88.49 (29.89, 101.14)	97.73 (39.33, 111.64)	106.27 (50.27, 121.03)	110.76 (57.00, 125.19)
455201	1	93.91 (3.28, 105.17)	136.83 (42.97, 151.73)	163.54 (60.91, 178.22)	192.66 (74.61, 209.32)	210.78 (80.73, 229.84)
455601	2	86.29 (8.57, 99.17)	108.53 (32.85, 123.50)	125.16 (45.32, 146.41)	153.48 (57.52, 181.60)	182.89 (66.24, 213.66)
423301	0	78.56 (2.04, 83.37)	101.12 (24.61, 108.23)	115.86 (37.57, 123.56)	132.70 (50.76, 141.36)	143.71 (58.32 154.76)
440201	1	72.89 (15.70, 79.60)	87.35 (36.05, 100.02)	98.76 (46.98, 115.97)	116.32 (56.80, 136.24)	131.86 (62.05, 151.88)
480201	1	127.88 (7.48, 140.73)	169.89 (51.84, 193.48)	205.39 (75.84, 238.32)	264.13 (100.10, 305.39)	320.44 (112.68, 375.12)
501201	3	231.90 (26.79, 248.30)	339.25 (121.51, 356.28)	388.19 (171.37, 399.65)	427.64 (215.88, 432.81)	445.62 (236.95, 446.37)
500301	1	82.88 (7.25, 96.68)	124.98 (42.78, 154.32)	156.76 (55.11, 186.60)	198.69 (66.83, 225.28)	230.35 (75.55, 253.84)
552301	0	134.26 (8.55, 169.12)	220.13 (80.81, 280.89)	292.26 (114.23, 368.94)	404.31 (142.29, 501.47)	504.40 (154.72, 624.23)
568501	2	151.62 (8.98, 173.14)	245.22 (80.60, 297.34)	317.04 (113.13, 371.88)	413.37 (139.11, 468.84)	487.30 (154.08, 536.93)
568502	3	94.35 (3.27, 103.99)	134.47 (48.64, 154.10)	163.53 (62.57, 188.15)	200.30 (77.45, 220.43)	226.94 (85.78, 239.23)
532201	1	161.21 (30.40, 163.24)	190.97 (63.83, 187.72)	208.28 (85.03, 203.32)	225.94 (106.66, 224.25)	236.23 (120.21, 240.90)
564201	3	107.00 (4.87, 110.65)	140.48 (40.63, 147.66)	162.34 (58.42, 170.75)	187.31 (74.99, 195.61)	203.61 (83.26, 213.38)
566201	1	113.14 (9.67, 124.17)	147.89 (44.88, 168.79)	174.21 (61.83, 199.82)	211.13 (76.42, 236.99)	240.97 (86.07, 261.96)
570201	2	106.98 (7.12, 113.85)	138.19 (40.89, 147.24)	159.96 (56.44, 169.95)	186.50 (70.46, 194.99)	205.00 (78.21, 212.38)

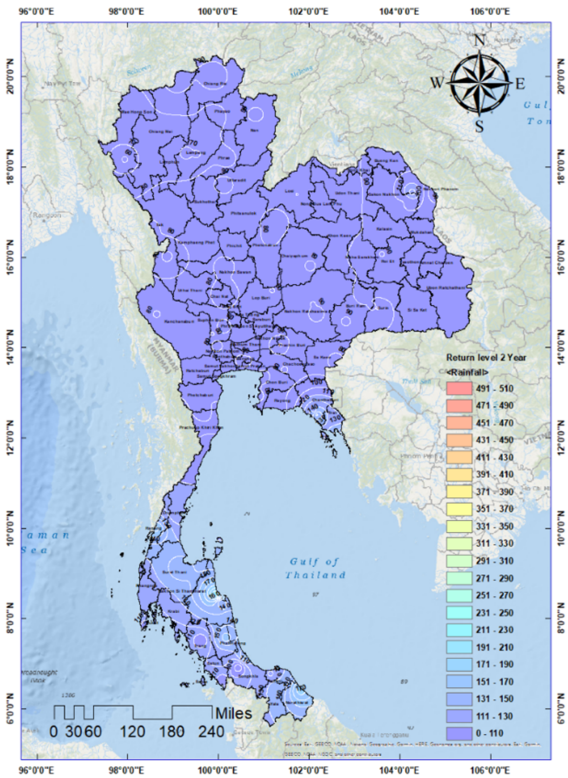

Figure 4. Isopluvial map of the estimated design value (unit; mm) corresponding to 2-year return period

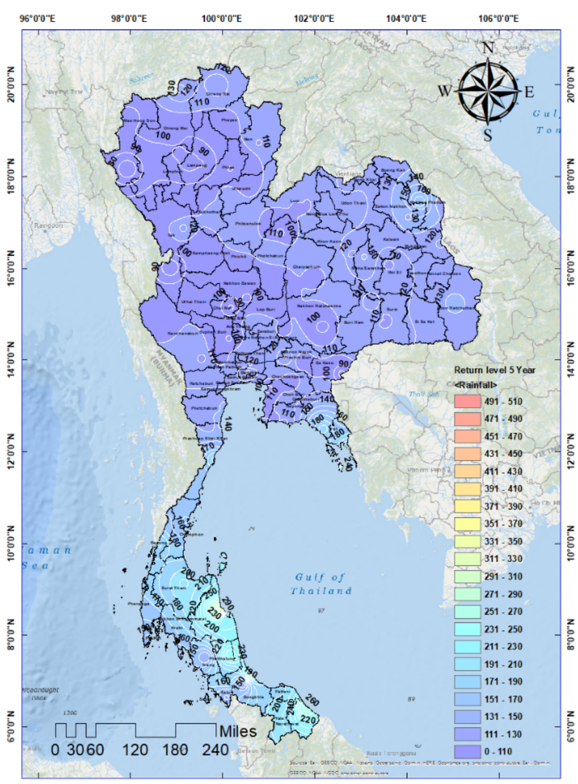

Figure 5. Isopluvial map of the estimated design value (unit; mm) corresponding to 5-year return period

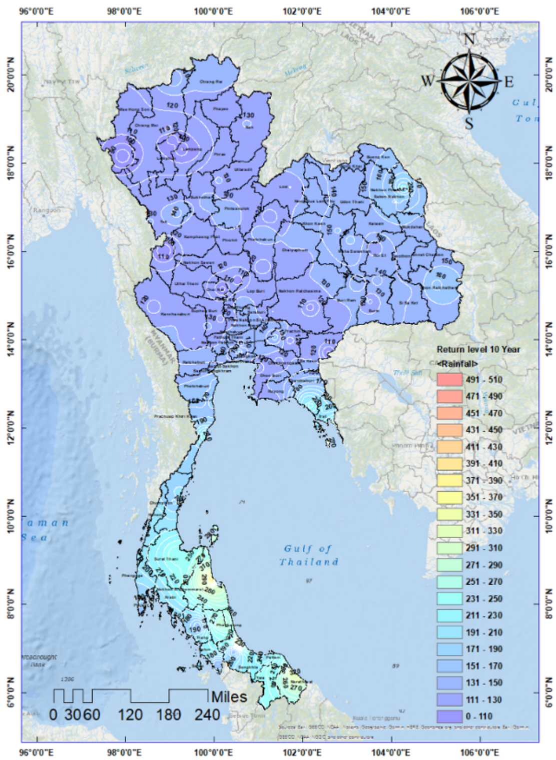

Figure 6. Isopluvial map of the estimated design value (unit; mm) corresponding to 10-year return period

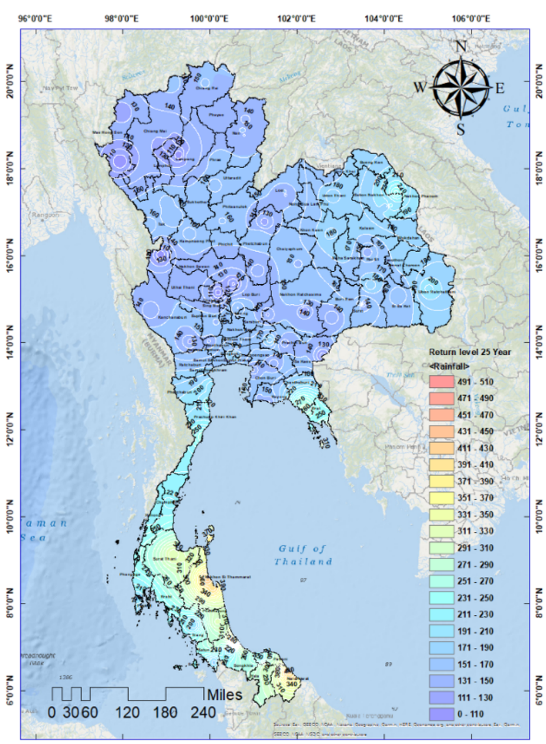

Figure 7. Isopluvial map of the estimated design value (unit; mm) corresponding to 25 year return period.

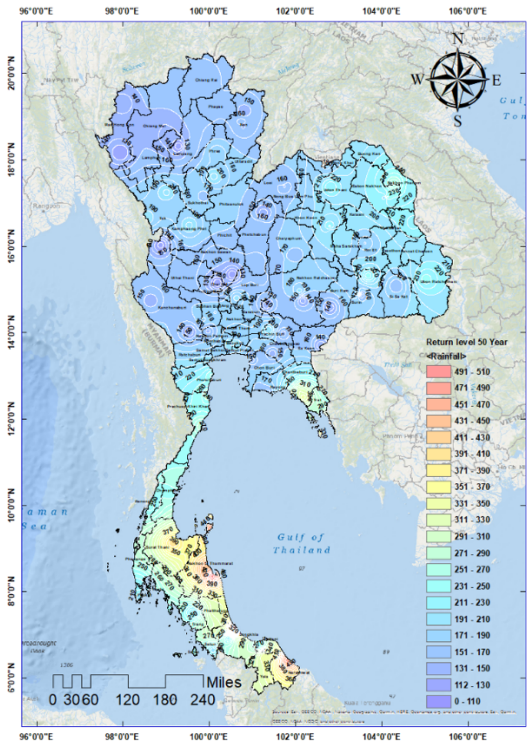

Figure 8. Isopluvial map of the estimated design value (unit; mm) corresponding to 50-year return period.

Conclusions

Extreme rainfall has posed a significant challenge for statistical modelling. Recent research on the GEVD using several methods of estimation (L-moment, MLE, and probability weighted moments) to model extreme rainfall involved three distribution parameters. Park et al. (2001) used the WAD instead of the K4D to avoid problems of fit when the shape parameter is above −1, in modelling summer extreme rainfall over the Korean peninsula via L-moments estimates from 61 gauging stations. They obtained reliable quantile estimates for several return periods, although a study comparing these estimates from the WAD is ongoing. Here we have modelled extreme rainfall in Thailand by the Wakeby distribution and the method of LH-moments. Design values corresponding to various return periods and their confidence intervals have been obtained for annual maximum rainfalls on a day to day basis, and isopluvial maps of the design values have been derived. Based on the η values, we conclude that the LH-WAD estimates are better than those obtained using the L-moment WAD for 87.5% of the chosen stations. Stations experiencing the greatest rainfalls, in southern-east coast Thailand and the Trat station in the eastern part, correspond to the top two return values. Moreover, confidence intervals from the bootstrap resampling technique also largely represent the true values for 24 sample gauge stations. We therefore conclude that our approach can be applied to model extreme rainfall in Thailand, although we do not claim that our method is the best possible and further consideration of skewness family distributions is ongoing.

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