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RESEARCH ARTICLE

MHD mixed convective flow of power-law nanofluid in a lid-driven cavity with heat generation and chemical reaction effects: Buongiorno's Model

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Abstract

The two-dimensional laminar MHD mixed convective flow of non-Newtonian power-law nanofluid in a lid-driven cavity with the presence of heat generation and chemical reaction effects is analyzed. The nanofluid incorporates the effects of Brownian motion and thermophoresis using Buongiorno's model. The governing equations in a vector form are transformed into non-dimensional form. Then, the dimensionless equations are solved numerically by finite element method (FEM) using automated solution technique which is FEniCS. The effects of different physical parameters on velocity, temperature, and nanoparticles volume fraction profiles are presented and discussed in this paper.

Keywords: MHD, power-law nanofluid, cavity, Buongiorno, FEniCS

INTRODUCTION

Nanofluid has been used to increase the rate of heat transfer in many applications [1]. There are two popular approaches in studying nanofluid which are Buongiorno's model [2] and Tiwari and Das' model [3]. Our research focuses on flow in square cavity using Buongiorno's model. Brownian motion and thermophoresis effects are considered in this model.

Since we focus on mixed convective flow in lid-driven cavity, there are several papers that investigated this problem. Sheremet et al. [4] mentioned that different Richardson number affects the heat and mass transfer mechanisms in cavity. Besides, Yu et al. [5] stated that no significant changes on streamlines, isotherms and isoconcentrations when Brownian motion and thermophoresis parameter increased.

Currently, magnetohydrodynamics (MHD) has been considered in the study of convective flow filled with nanofluid in cavity for example wavy-walled cavity [6], square cavity [7] and lid-driven cavity [8,9]. Elshehaby et al. [8] performed a numerical study on MHD convective flow of nanofluid in lid-driven cavity with sinusoidal temperature distribution on both vertical walls while Alsabery et al. [9] investigated nanofluid in a doubled lid-driven cavity. Both of them considered inclined magnetic field effect and found that increasing Hartmann number leads to decrease fluid motion.

Chemical reaction is important since there is no pure air or water without foreign mass in it [10]. However, very limited literature is available on chemical reaction effects in cavity since it consider mass transfer problem. Rashad et al. [11] reported the study on the unsteady natural convective flow in a porous cavity with thermal radiation and chemical reaction effects. They found that chemical reaction can decrease Nusselt number and decrease Sherwood number. Mondal & Sibanda [12] extended [11] by adding the inclined magnetic fields in inclined cavity. Zhuang et. al [13] studied effects of chemical reaction in porous cubic cavity filled with power-law fluid. They summarized that chemical reaction effecs is more significant on the shear-thinning fluids rather than shear thickening fluids. Aly [14] solved porous cavity filled with nanofluid with chemical reaction effects.

Several papers considered heat generation/absorption effect in mixed convective flow of nanofluid using Tiwari and Das' model [15,16] and Buongiorno's model [17]. Rashad et al [15] demonstrated MHD mixed convective flow in a porous lid-driven cavity filled with nanofluid and presented that increment in heat generation/absorption parameter increases the velocity and temperature distributions. Hussain et al. [16] focused on MHD mixed convective flow of nanofluid in double lid-driven cavity and obtained the result where an enhancement of heat transfer occurred by increasing the heat generation/absorption parameter. Shekar et al. [17] investigated natural convection for nanofluid in a porous square cavity and found that local Nusselt number and skin friction decrease with the increasing of heat generation /absorption parameter.

All papers mentioned above used Newtonian fluid as a base fluid. In this study, we focus on non-Newtonian fluid. Some papers have explored convective flow of non-Newtonian nanofluid in a cavity

[18,19,20]. Kefayati [19] proposed mixed convective flow of non-Newtonian nanofluid in a lid-driven cavity. The results exhibited that when power-law index increases, heat and mass transfer decrease at $Ri = 1$. Besides, heat transfer decreases and mass transfer increases with the enhancement of Brownian motion and thermophoresis effects. Kefayati & Tang [20] performed MHD natural convective flow in cavity and showed that heat and mass transfer decline as Hartmann number rises.

The effects of magnetic field, heat generation/absorption and chemical reaction on mixed convective flow of power-law nanofluid using Buongiorno's mathematical model in a lid-driven cavity has not been considered yet. In this study, this problem is solved by finite element method using automated solution technique which is FEniCS [21]. FEniCS is an open source package that enable us to solve partial differential equations problem in vector form using Python language.

MATHEMATICAL FORMULATION

Governing equations

Steady two-dimensional MHD mixed convective flow of power-law nanofluid in a lid-driven cavity is explored. Heat generation/absorption and chemical reaction effects are taken into account and Buongiorno's model for nanofluid is considered. Let Ω be a bounded domain. The flow of configuration is shown through diagram below.

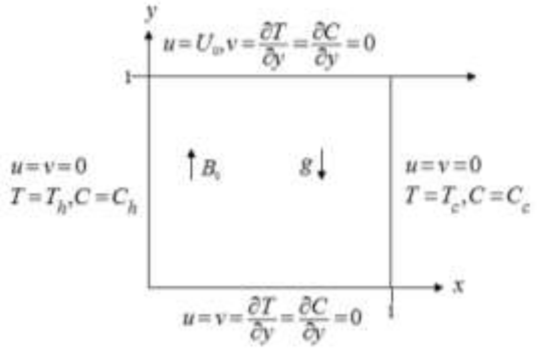

Fig. 1 Flow configuration with $\Omega = [0,1]\times[0,1]$.

Following [2,22,23], the governing equations for this problem are:

Continuity Equation for Nanofluid:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

Continuity Equation for Nanoparticles:

$$\mathbf{u}\cdot\nabla C = D_B \nabla^2 C + \left(\frac{D_T}{T_\infty}\right)\nabla^2 T + \kappa_0\left(C - C_\infty\right) \tag{2}$$

Momentum Equation:

$$\rho_f\left(\mathbf{u}\cdot\nabla\right)\mathbf{u} = \nabla\cdot\tau_{ij} - \nabla p - \sigma B_0{}^2\mathbf{u} + \left[\left(\rho_p - \rho_f\right)(C - C_\infty) - (1 - C_\infty)\rho_f \beta\left(T - T_\infty\right)\right]\mathbf{g} \tag{3}$$

Energy Equation:

$$\mathbf{u}\cdot\nabla T = \alpha\nabla^2 T + \tau\left[D_B \nabla C\cdot\nabla T + \left(\frac{D_T}{T_\infty}\right)\nabla T\cdot\nabla T\right] + \frac{Q_0}{\left(c\rho\right)_f}(T - T_\infty) \tag{4}$$

where $\mathbf{u} = (u, v)$ is the fluid velocity, C is the nanoparticle volume fraction, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, T is the fluid temperature, $\kappa_0\left(C - C_\infty\right)$ represents chemical reaction where κ_0 is the reaction rate, τ_{ij} is a viscous stress for power-law fluid with $\tau_{ij} = K\left(\frac{e_{ij}e_{ij}}{2}\right)^{\frac{n-1}{2}}\nabla\cdot e_{ij}$ and $e_{ij} = \left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)$, σ is the electrical conductivity of the fluid, B_0 is the magnetic field strength, ρ_f is the density of the base fluid, ρ_p is the density of the nanoparticles and $\mathbf{g} = \left(0, -g_y\right)$ is gravity acceleration applied to the flow, $\alpha = \frac{k}{\left(c\rho\right)_f}$ with k is thermal conductivity of nanofluid and c_f is the heat capacitance of the base fluid, $\tau = \frac{\left(c\rho\right)_p}{\left(c\rho\right)_f}$ with c_p is the heat capacitance of the nanoparticles and Q_0 is the dimensional heat generation/absorption coefficient.

The governing equations are subjected to initial and boundary conditions:

$$\begin{aligned}
&u = 0,\ v = 0,\ C = C_h,\ T = T_h, && \text{for } x = 0,\ 0 \le y \le 1\\
&u = 0,\ v = 0,\ C = C_c,\ T = T_c, && \text{for } x = 1,\ 0 \le y \le 1\\
&u = 0,\ v = 0,\ \frac{\partial C}{\partial y} = 0,\ \frac{\partial T}{\partial y} = 0, && \text{for } y = 0,\ 0 \le x \le 1\\
&u = U_0,\ v = 0,\ \frac{\partial C}{\partial y} = 0,\ \frac{\partial T}{\partial y} = 0, && \text{for } y = 1,\ 0 \le x \le 1
\end{aligned} \tag{5}$$

Next, Equations (1) – (4) are converted to non-dimensional form, using the following dimensionless parameters [2, 24]:

$$\mathbf{x} = \frac{\bar{\mathbf{x}}}{L},\ \mathbf{u} = \frac{\bar{\mathbf{u}}}{U},\ p = \frac{\bar{p}}{\rho U^2},\ \nabla = L\bar{\nabla},$$

$$\theta = \frac{T - T_\infty}{T_W - T_\infty}, \phi = \frac{C - C_\infty}{C_W - C_\infty},\ t = \frac{U}{L}\bar{t}$$

The dimensionless governing equations can be written as follows:

$$\nabla \cdot \mathbf{u} = 0 \tag{6}$$

$$\mathbf{u} \cdot \nabla \phi = \frac{1}{\mathrm{Re}\, Le \,\mathrm{Pr}} \left(\nabla^2 \phi + \frac{Nt}{Nb} \nabla^2 \theta \right) + \kappa \phi \tag{7}$$

$$\left(\mathbf{u} \cdot \nabla \right) \mathbf{u} = \frac{1}{\mathrm{Re}} \left(\frac{e_{ij} e_{ij}}{2} \right)^{\frac{n-1}{2}} \nabla \cdot e_{ij} - \nabla p - M\mathbf{u} + Ri \left(\theta - Nr\phi \right) \tag{8}$$

$$\mathbf{u} \cdot \nabla \theta = \frac{1}{\mathrm{Re}} \frac{1}{\mathrm{Pr}} \nabla^2 \theta + Nb \nabla \phi \cdot \nabla \theta + Nt \nabla \theta \cdot \nabla \theta + Q\theta \tag{9}$$

with corresponding dimensionless parameters defined as [24]:

$$\begin{aligned}
&\mathrm{Re} = \frac{\rho L^n U^{2-n}}{K}, \mathrm{Pr} = \frac{K}{\rho \alpha U^{1-n} L^{n-1}}, Le = \frac{\alpha}{D_B}, \\
&Nb = \frac{\tau D_B \left(C_w - C_\infty \right)}{UL}, Nt = \frac{\tau D_T \left(T_w - T_\infty \right)}{T_\infty UL}, \\
&Nr = \frac{\left(\rho - \rho_{f\infty} \right) \left(C_w - C_\infty \right)}{\rho \left(1 - C_\infty \right) \beta \left(T_w - T_\infty \right)}, M = \frac{\sigma B^2 L}{\rho U}, \\
&Ri = \frac{L}{U^2} \left(1 - C_\infty \right) \beta g \left(T_w - T_\infty \right), \kappa = \frac{L}{U} \kappa_0, Q = \frac{Q_0 L}{c \rho U}
\end{aligned} \tag{10}$$

is

where Re is Reynolds number, Pr is Prandtl number, Le is Lewis number, Nb is Brownian motion parameter, Nt is thermophoresis parameter, Nr is buoyancy ratio, Ri is Richardson number, M magnetic parameter, and κ is dimensionless chemical reaction parameter. It is called destructive reaction if $\kappa > 0$ and generative reaction if $\kappa < 0$. Chemical reaction involves endothermic and exothermic process where heat is generated or absorbed [25]. It is called heat generation if $Q > 0$ and heat absorption if $Q < 0$ where Q is dimensionless heat generation/absorption parameter. However, if the level of species concentration is very low, heat generation/absorption is ignored [26].

The dimensionless equations are subjected to dimensionless initial and boundary conditions:

$$\begin{aligned}
&u = 0,\ v = 0,\ \phi = 1,\quad \theta = 1, \quad &&\text{for } x = 0, 0 \le y \le 1 \\
&u = 0,\ v = 0,\ \phi = 0,\quad \theta = 0, \quad &&\text{for } x = 1, 0 \le y \le 1 \\
&u = 0,\ v = 0,\ \frac{\partial \phi}{\partial y} = 0,\ \frac{\partial \theta}{\partial y} = 0, \quad &&\text{for } y = 0, 0 \le x \le 1 \\
&u = 1,\ v = 0,\ \frac{\partial \phi}{\partial y} = 0,\ \frac{\partial \theta}{\partial y} = 0, \quad &&\text{for } y = 1, 0 \le x \le 1
\end{aligned} \tag{11}$$

Weak formulation

In order to solve this problem using finite element method, variational form is constructed using galerkin weighted residual approach for the dicretization of governing equations. Taylor Hood element is considered to mesh the domain for the Navier-Stokes equation where V is continuous quadratic polynomial function and P is continuous linear polynomial function. Meanwhile, continuous quadratic polynomial function L is used for the continuity equation for nanoparticles and heat transfer equation. The space should be mixed function space $W = V \times P \times L \times L$. Equations (5) – (8) are multiplied by the test functions and integrated over the domain $\Omega = \left[0,1\right] \times \left[0,1\right]$. We have $\mathbf{u}, p, \theta$ and ϕ as functions and $\mathbf{v}, q, s$ and r as test functions such that $\mathbf{u}, \mathbf{v} \in V,\ p, q \in P,\ \theta, s \in L$ and $\phi, r \in L$.

Weak form can be written as

$$\int_\Omega \left(\nabla \cdot \mathbf{u} \right) q = 0 \tag{12}$$

$$\int_\Omega \left(\mathbf{u} \cdot \nabla \phi \right) r + \frac{1}{\mathrm{Re}\, Le \,\mathrm{Pr}} \nabla \phi \nabla r + \frac{1}{\mathrm{Re}\, Le \,\mathrm{Pr}} \frac{Nt}{Nb} \nabla \theta \nabla r - \kappa \phi r = 0 \tag{13}$$

$$\int_\Omega (\mathbf{u} \cdot \nabla) \mathbf{u} \cdot \mathbf{v} + \frac{1}{\mathrm{Re}} \left(\frac{e_{ij} e_{ij}}{2} \right)^{\frac{n-1}{2}} e_{ij} \cdot \nabla \mathbf{v} + p \cdot \nabla \mathbf{v} + M \mathbf{u} \cdot \mathbf{v} + Ri Nr \phi \cdot \mathbf{v} - Ri \theta \cdot \mathbf{v} = 0 \tag{14}$$

$$\int_\Omega \left(\mathbf{u} \cdot \nabla \theta \right) s + \frac{1}{\mathrm{Re}} \frac{1}{\mathrm{Pr}} \nabla \theta \nabla s - Nb \left(\nabla \phi \cdot \nabla \theta \right) s - Nt \left(\nabla \theta \cdot \nabla \theta \right) s - Q \theta s = 0 \tag{15}$$

Then, Equations (10) – (13) are solved using fully coupled approach and expressed in Python programming. Finally, the solution is obtained using FEniCS package [27].

RESULTS AND DISCUSSION

In this section, we present the investigation of mixed convective flow in a lid-driven cavity that is filled with power-law nanofluid by considering magnetic field, chemical reaction and heat generation/absorption effects. Equations (10) – (13) are solved numerically using FEniCS and the post-processing of solutions obtained is interpreted using matplotlib. The dimensionless parameter $\mathrm{Pr} = 1, \mathrm{Re} = 10, Ri = 1, Nb = Nt = Nr = 0.1$ and $Le = 1$ is fixed throughout the computation. We demonstrate velocity, temperature and nanoparticle volume fraction profiles at four cases which are:

1) different power-law indexes $(n = 0.6, 1, 1.6)$
2) different magnetic parameters $(M = 0, 5, 10)$
3) different heat generation/absorption parameters $(Q = -0.2, 0, 0.2)$
4) chemical reaction parameters $(\kappa = -0.2, 0, 0.2)$

The present code is validated by the results obtained with some earlier studies on the mixed convective flow of Newtonian fluid in a lid-driven cavity and considered isothermal top moving lid and isothermal bottom unmoving lid.

Table 1 Comparison of present velocity at the middle of the cavity for different Re $(Gr = 100, \mathrm{Pr} = 0.71)$.

	Iwatsu et.al [2]	Khanafer & Chamkha [24]	Bakar et al. [25]	Present
		$\mathrm{Re} = 100$		
$u_{\min}$	-0.2037	-0.2122	-0.2049	-0.2078
$u_{\max}$	1.0000	1.0000	1.0000	1.0000
$v_{\min}$	-0.2448	-0.2506	-0.2328	-0.2477
$v_{\max}$	0.1699	0.1765	0.1673	0.17482
		$\mathrm{Re} = 400$		
$u_{\min}$	-0.3197	-0.3099	-0.3023	-0.3123
$u_{\max}$	1.0000	1.0000	1.0000	1.0000
$v_{\min}$	-0.4459	-0.4363	-0.4219	-0.4335
$v_{\max}$	0.2955	0.2866	0.2802	0.2873

The grid independence test has been conducted and the results are presented in Table 2. It is shown that grid size of 200×200 gives insignificant difference with grid size of 250×250. Hence, we use 200×200 grid size to execute all results.

Table 2 Grid Independence Test $(Ri = 1, \mathrm{Pr} = 1, n = 1, M = 0, Nr = Nb = Nt = 0.1, Le = 1, Q = 0, \kappa = 0)$

Mesh	$u_{\min}$	$v_{\min}$	$T_{\min}$	$C_{\min}$
50×50	-0.21075	-0.43177	0	-0.10442
100×100	-0.21237	-0.43844	0	-0.10325
150×150	-0.21293	-0.4407	0	-0.10286
200×200	-0.21320	-0.44186	0	-0.10269
250×250	-0.21336	-0.44255	0	-0.10260
300×300	-0.21347	-0.44298	0	-0.10253

Fig. 2 – 3 display the velocity, temperature and nanoparticle volume fraction profiles for different power-law indexes at constant $M = 5$, $Q = -0.2$ and $\kappa = 0.2$. We demonstrate three types of fluid which are shear thinning fluid $(n = 0.6)$, Newtonian fluid $(n = 1)$ and shear thickening fluid $(n = 1.6)$. It can be seen that in Fig 3, where we focus the profiles in the middle of cavity $(y = 0.5)$, increasing power-law index will lead to small decreasing pattern for temperature and nanoparticle volume fraction profiles. Meanwhile, two trends occur for velocity profile where it enhances when $x < 0.5$ and declines when $x > 0.5$. However, it is observable for velocity and temperature profiles in Fig. 2, where both of them show the increasing pattern at the top lid as power-law index increases due to the effects of external forces by the moving lid .

Besides, Fig. 4 – 5 illustrate the velocity, temperature and nanoparticle volume fraction profiles for different magnetic parameter M at constant $n = 0.6$, $Q = -0.2$ and $\kappa = 0.2$. Fig. 5 presents that by increasing magnetic parameter, velocity profiles decreases at $x < 0.5$ and increases at $x > 0.5$, while temperature and nanoparticle volume fraction profiles show slight increasing pattern. Meanwhile, different pattern is shown in Fig. 4. It shows the velocity and temperature profiles are decreasing as magnetic parameter increases at the top moving lid area where the effects of magnetic retards the flow of the fluid.

The influence of heat generation/absorption on the velocity, temperature and nanoparticle volume fraction profile at constant $n = 0.6$, $M = 5$ and $\kappa = 0.2$ is presented in Fig. 6 – 7. We investigate heat absorption $(Q = -0.2)$, heat generation $(Q = 0.2)$ and no heat generation/absorption $(Q = 0)$. It demonstrates that the temperature profile enhances and nanoparticle volume fraction declines due to rising heat generation/absorption parameter. Interestingly, velocity profile shows slight increases at $0.3 \le x \le 0.7$.

Lastly, Fig. 8 – 9 reveal the effects of chemical reaction parameter on the velocity, temperature and nanoparticle volume fraction profiles at constant $n = 0.6$, $M = 5$ and $Q = -0.2$. The results show that increasing chemical reaction parameter can enhance the nanoparticle volume fraction profile.

CONCLUSION

In this paper, steady two-dimensional mixed convective flow of power-law nanofluid in lid-driven cavity with the presence of magnetic field, heat generation/absorption and chemical reaction effects was considered. The dimensionless governing equations were solved using finite element method in FEniCS. It was found that:

- Increasing power-law index lead to decreasing temperature profile while velocity profile increases at $x < 0.5$ and decreases at $x > 0.5$ in the middle of cavity $(y = 0.5)$.
- Velocity and temperature profiles decrease as magnetic parameter increases near the moving lid.
- Influence of rising the heat generation/absorption parameter can enhance temperature profile but decline nanoparticle volume fraction profile.
- As chemical reaction parameter increases, nanoparticle volume fraction profile increase.

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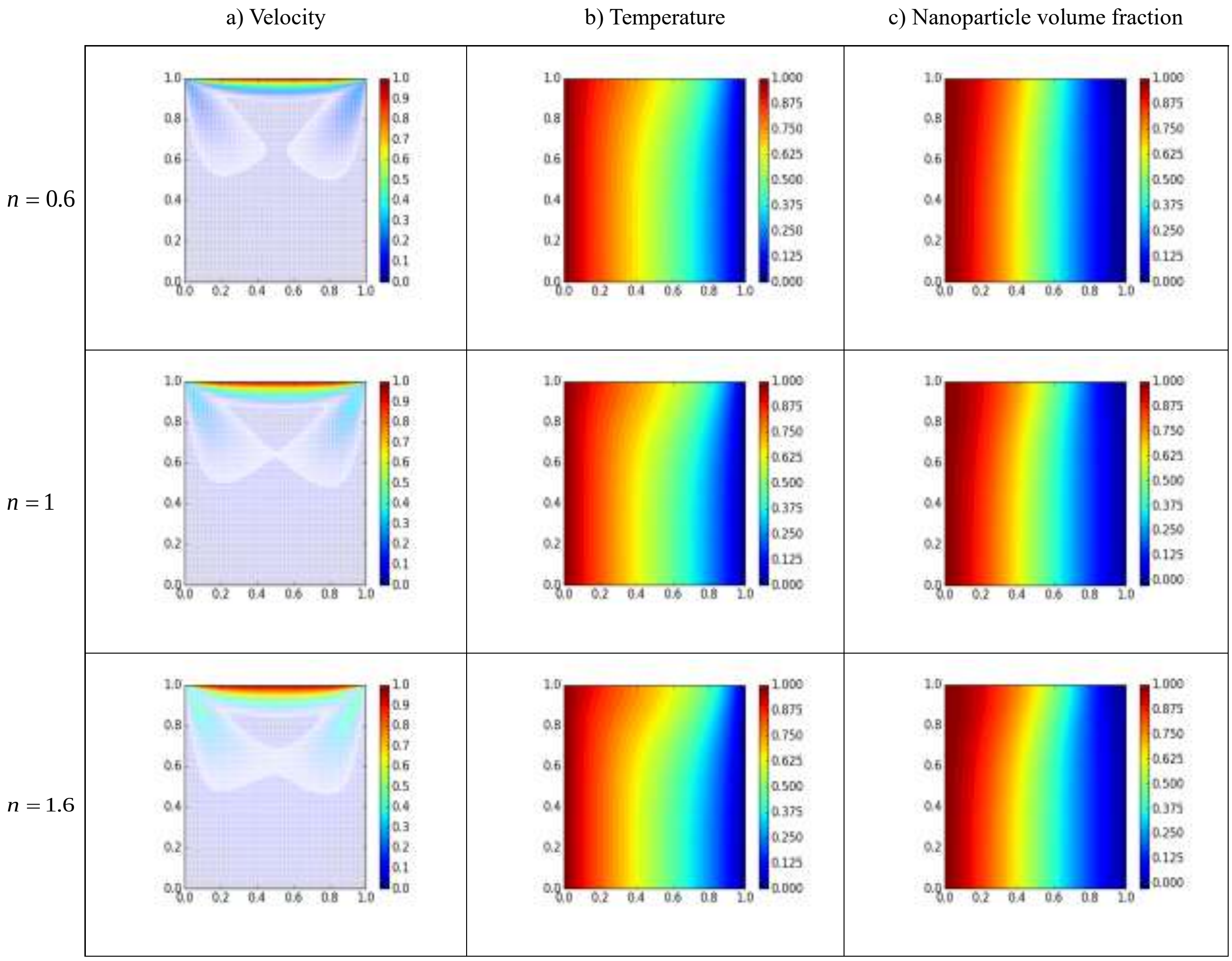
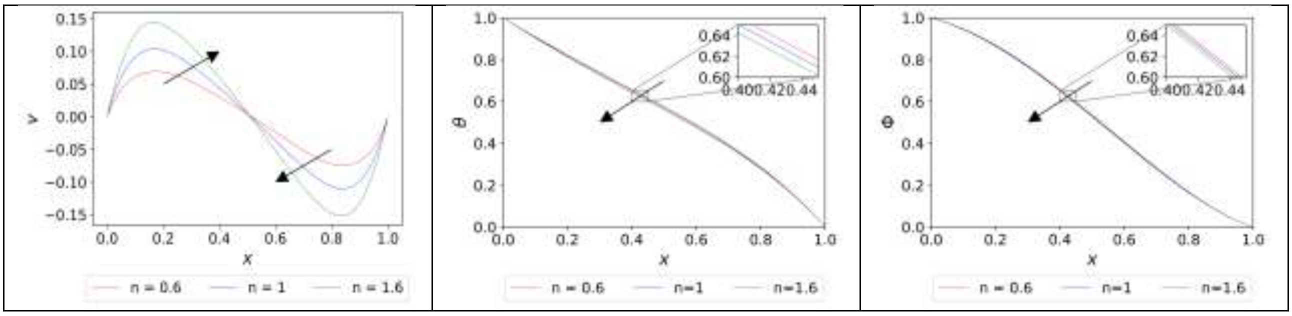
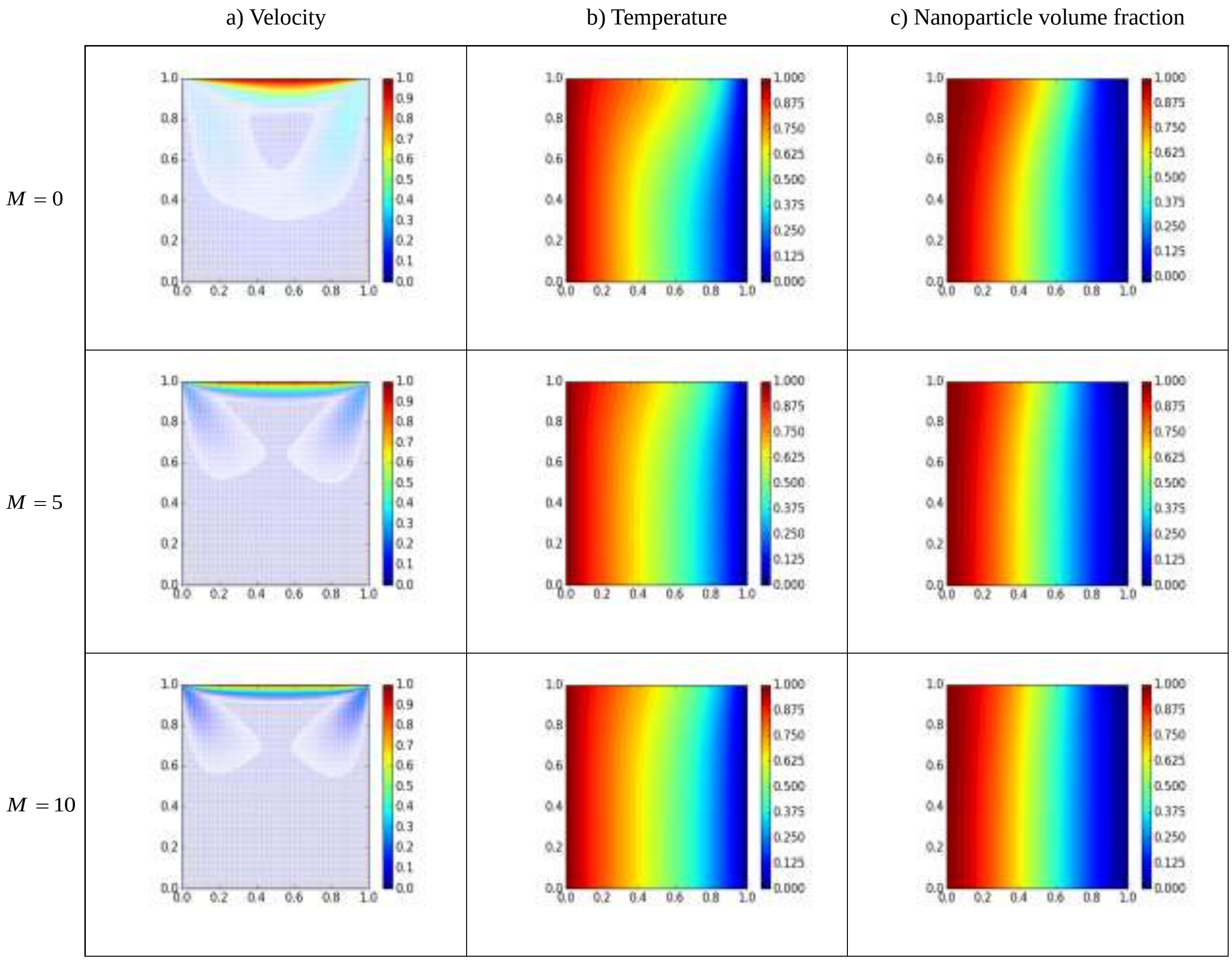


Fig. 2 Comparison of a) velocity, b) temperature and c) nanoparticle volume fraction profiles for different power-law indexes at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, M = 5, Q = -0.2, \kappa = 0.2$



a) Velocity b) Temperature c) Nanoparticle volume fraction

Fig. 3 Comparison of a) velocity, b) temperature and c) nanoparticle volume fraction profiles for different power-law indexes in the middle of cavity $(y = 0.5)$ at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, M = 5, Q = -0.2, \kappa = 0.2$

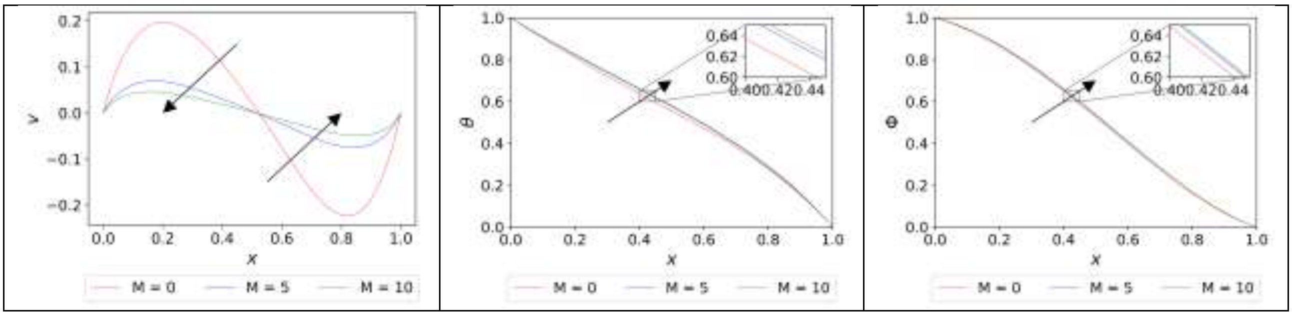

a) Velocity b) Temperature c) Nanoparticle volume fraction

$M = 0$

$M = 5$

$M = 10$

Fig. 4 Effect of magnetic parameter on a) velocity, b) temperature and c) nanoparticle volume fraction profiles at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, n = 0.6, Q = -0.2, \kappa = 0.2$



a) Velocity b) Temperature c) Nanoparticle volume fraction

Fig. 5 Effect of magnetic parameter on a) velocity, b) temperature and c) nanoparticle volume fraction profiles in the middle of cavity $(y = 0.5)$ at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, n = 0.6, Q = -0.2, \kappa = 0.2$

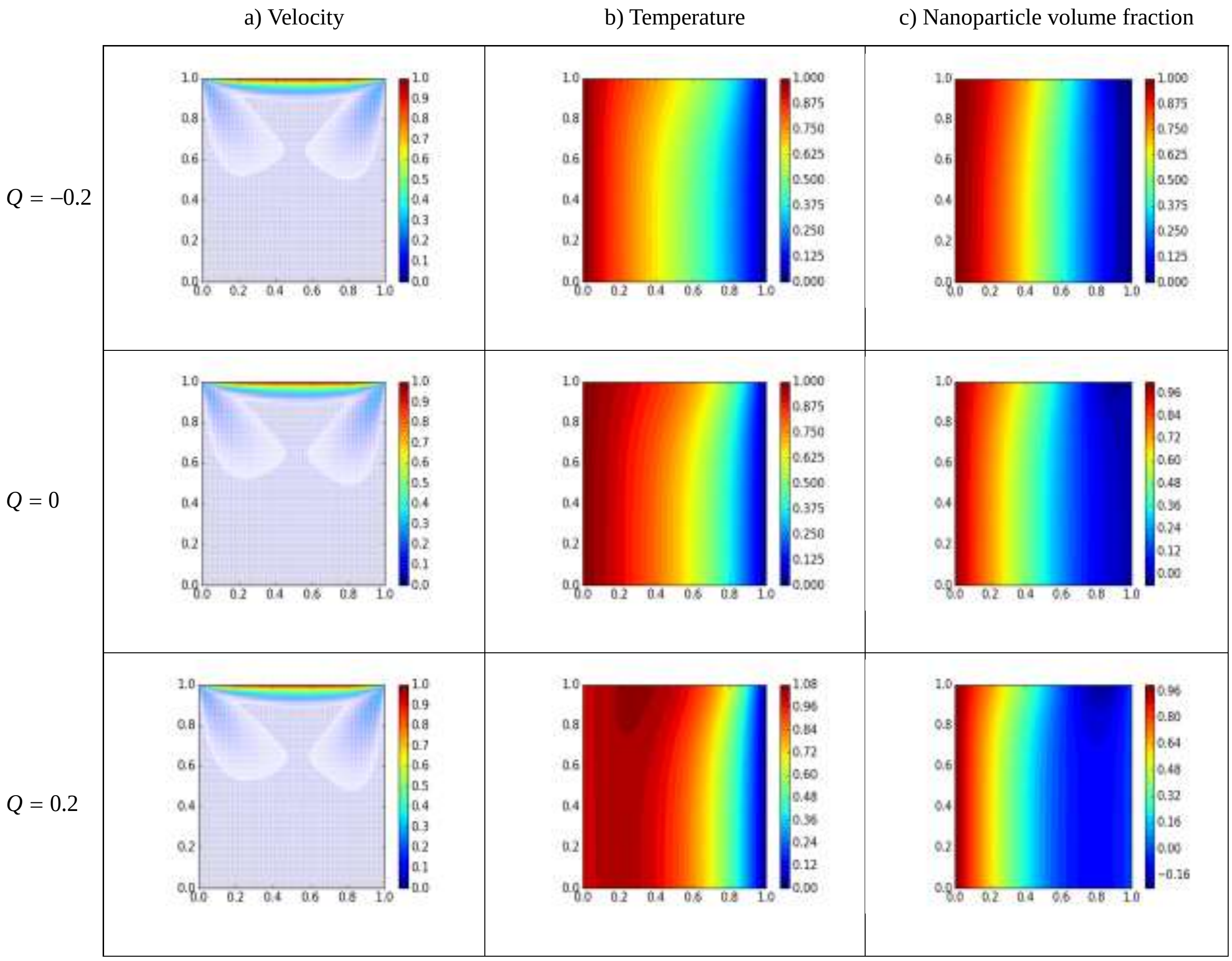
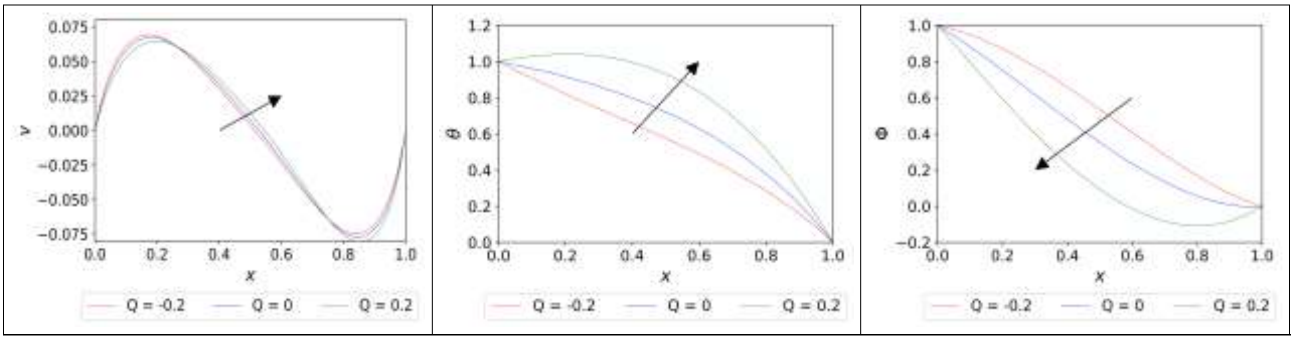
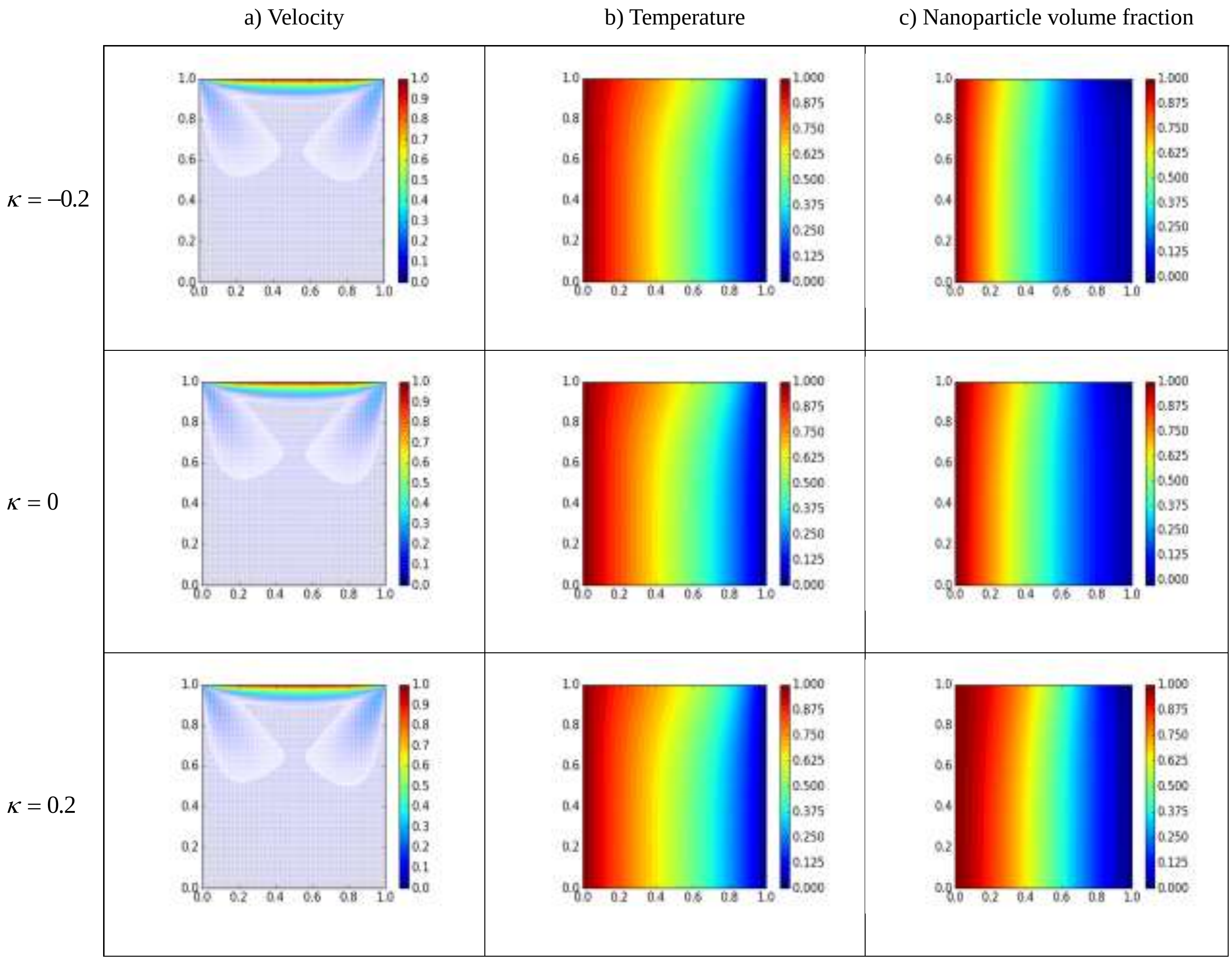


Fig. 6 Effect of heat generation/absorption parameter Q on a) velocity, b) temperature and c) nanoparticle volume fraction profiles at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, n = 0.6, M = 5, \kappa = 0.2$



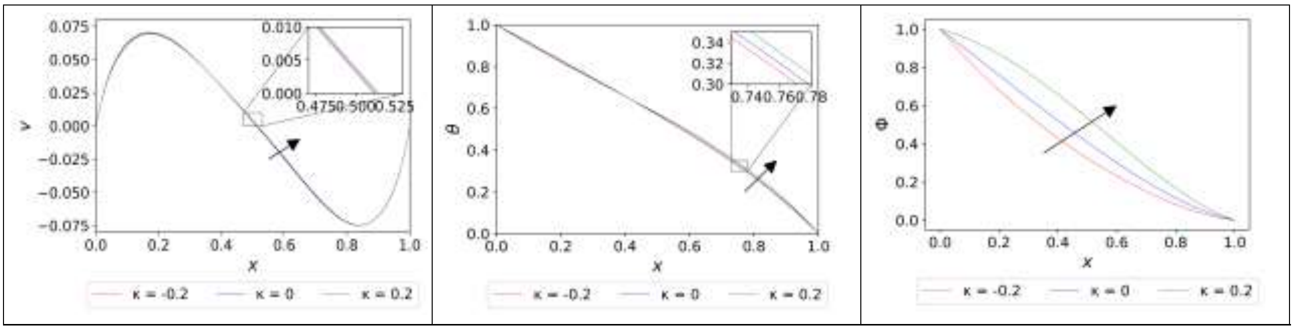
a) Velocity b) Temperature c) Nanoparticle volume fraction

Fig. 7 Effect of heat generation/absorption parameter Q on a) velocity, b) temperature and c) nanoparticle volume fraction profiles in the middle of cavity $(y = 0.5)$ at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, n = 0.6, M = 5, \kappa = 0.2$



a) Velocity b) Temperature c) Nanoparticle volume fraction

Fig. 8 Effect of chemical reaction parameter κ on a) velocity, b) temperature and c) nanoparticle volume fraction profiles at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, n = 0.6, M = 5, Q = -0.2$



a) Velocity b) Temperature c) Nanoparticle volume fraction

Fig. 9 Effect of chemical reaction parameter κ on a) velocity, b) temperature and c) nanoparticle volume fraction profiles in the middle of cavity $(y = 0.5)$ at $Ri = 1, \Pr = 1, Nb = Nt = Nr = 0.1, Le = 1, n = 0.6, M = 5, Q = -0.2$

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