MJFAS MALAYSIAN JOURNAL OF FUNDAMENTAL AND APPLIED SCIENCES
PRINT ISSN: 2289-5981 | ONLINE ISSN: 2289-599X

RESEARCH ARTICLE

Ordered discrete and continuous Z-numbers

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Article history
Received 13 May 2019
Revised 17 December 2019
Accepted 5 February 2020
Published Online 16 August 2020

Abstract

Both discrete and continuous Z-numbers are pairs of discrete and continuous fuzzy numbers. Even though the later are ordered, this do not simply imply the discrete and continuous Z-numbers are ordered as well. This paper proposed the idea of ordered discrete and continuous Z-numbers, which are necessary properties for constructing temporal Z-numbers. Linear ordering relation, $\prec$, is applied between set of discrete or continuous Z-numbers and any arbitrary ordered subset of $\mathbb{R}$ to obtain the properties.

Keywords: *Z-number, Discrete Z-number, Continuous Z-number, Relation, Lattice*

INTRODUCTION

Real-world information is flawed, and natural language is often used to represent this feature. Such information is often characterized by fuzziness, which implies that soft constraints are imposed on the values of variables of interest. Furthermore, reliability is another essential property of information. Any estimation of values of interest, be it precise or soft, are subject to the confidence with regards to sources of information. Thus, fuzziness from the one side and partial reliability form the other side are strongly associated to each other [1]. To discuss this concept, Zadeh in [2] introduced the concept of Z-number as a formal description of such information. Basically, the concept of Z -number relates to the issue of reliability of information. A Z-number is an ordered pair of fuzzy numbers (A, B). It is associated with a real valued uncertain variable X, with the first component, A, playing the role of a fuzzy restriction $(R(X))$ on the values which X can take, written as X is A such that A is a fuzzy. The second component B is a measure of reliability (certainty) of the first component [2].

B. Kang et al. [3] proposed an approach of dealing with Z-numbers which naturally arises in the areas of decision making, control, regression analysis and others. The approach is based on transforming a Z-number into fuzzy number on the basis of fuzzy expectation of the fuzziness. The advantage of this approach is its low analytical and computational complexity, which allows for a wide spectrum of its applications. Unfortunately, converting Z-number to fuzzy leads to significant loss of original information and reducing the benefit of using Z-number-based information in the first place.

The authors in [4] developed some basics for direct computation with Z-number, by suggesting a general and computationally effective approach to deal with discrete Z-number. The authors provided motivation to use discrete Z-numbers as an alternative to the continuous one, based on the fact that natural language-based information has a discrete framework and it is not required to decide upon a reasonable assumption to use some type of probability distributions. Furthermore, it has lower computational complexity than that with continuous Z-numbers. Some basic theoretical aspects of arithmetic operations over discrete Z-numbers such as addition, subtraction, multiplication, division, square root of a Z-number, and other operations are proposed as well as a series of numerical examples are provided by them to illustrate the validity of the suggested approach.

A mathematical property called ordered, is required for constructing temporal discrete Z-numbers. Consider the set of complex numbers, $\mathbb{C}$. It is not ordered naturally but when the relation $||: \mathbb{C} \longrightarrow \mathbb{R}$ is employed on $\mathbb{C}$ such that $|C| = |a + ib| = \sqrt{a^2 + b^2} \in \mathbb{R}$, then the ordered property is deduced indirectly. Fig. 1 shows the coordinates of complex number.

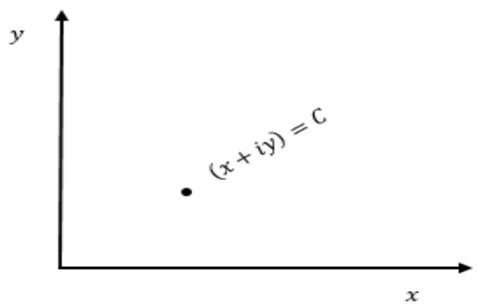

Fig. 1 A complex number.

Similarly, discrete Z-number is an ordered pair of discrete fuzzy numbers, however, this does not guarantee that discrete Z-number is an ordered set too. This paper proves that both discrete and continuous Z-numbers can be ordered by applying a linear ordering relation $\prec$ between set of discrete or continuous Z-numbers and any arbitrary ordered subset of $\mathbb{R}$. The rest of the paper is organized as follows: Section 2 contains some basic definitions related to this work; the concepts of ordered discrete and continuous Z-number are revealed in Section 3; a sample of the implementation is presented in Section 4; and finally, the conclusion is drawn in Section 5.

PRELIMINARIES

Here are some important definitions which are essential in this work.

Definition 1. 1 [7] The relation $\prec$ on $X \times X$ is a partial ordering on X if it satisfies the following properties:

1. (Reflexivity) $x \prec x$ for every x ∈ X.
2. (antisymmetry) If $x_1 \prec x_2$ and $x_2 \prec x_1$, then $x_1 = x_2$.
3. (transitivity) If $x_1 \prec x_2$ and $x_2 \prec x_3$, then $x_1 \prec x_3$.

A pair $(X, \prec)$ is called a partially ordered set. A partially ordered set $(X, \prec)$ is said to be totally ordered (also called linearly ordered), provided that for every $x_1, x_2 \in X$ and $x_1 \neq x_2$, either $x_1 \prec x_2$ or $x_2 \prec x_1$. A partial order $\prec$ is then said to be a linearly ordered.

Definition 2.2 [7] A partially ordered set in which every pair of element has the greatest lower bound and the least upper bound is called a lattice.

Definition 2.2 [7] A lattice (Z, ∨, ∧) is a distributive lattice if the following additional identity holds for all $a, b, c \in Z, a \wedge (b \vee c) = (a \wedge b) \vee c$ also $a \vee (b \wedge c) = (a \vee b) \wedge c$.

Definition 2.4 [8] A fuzzy number A of the real line R with membership function $\mu_A: \mathbb{R} \longrightarrow [0,1]$ is a discrete fuzzy number if its support is finite, i.e. there exist $\{x_1, \dots, x_n\} \in R$ with $x_1 < x_2 < \cdots < x_n$, such that $supp(A) = \{x_1, \dots, x_n\}$ and there exist natural numbers s, t with $1 \leq s \leq t \leq n$ satisfying the following conditions:

1. $\mu_A(x_i) = 1$ for any natural number i with $s \leq i \leq t$
2. $\mu_A(x_i) \leq \mu_A(x_j)$ for each natural number i, j with $1 \leq i \leq j \leq s$
3. $\mu_A(x_i) \geq \mu_A(x_j)$ for each natural number i, j with $t \leq i \leq j \leq n$.

Definition 3. [5] A continuous fuzzy number is a fuzzy subset A of the real line $\mathbb{R}$ with membership function $\mu_A: \mathbb{R} \longrightarrow [0,1]$ which possesses the following properties:

1. A is a normal fuzzy set.
2. A is a convex fuzzy set.
3. α-cut A^α is a closed interval for every $\alpha \in (0,1]$.
4. The support of A, *supp(A)* is bounded.

A continuous fuzzy number A with the membership function defined as

$$\mu_A = \begin{cases} \dfrac{x-a}{b-a} & if\ a \leq x < b \\ 1 & if\ b \leq x \leq c \\ \dfrac{d-x}{d-c} & if\ c < x \leq d \\ 0 & otherwise \end{cases}$$

is referred to as trapezoidal fuzzy number and is denoted as (a, b, c, d). A special case of trapezoidal fuzzy number is a triangular fuzzy number (TFN) A with membership function defined as

$$\mu_A = \begin{cases} \dfrac{x-a}{b-a} & if\ a \leq x < b \\ \dfrac{c-x}{c-b} & if\ b \leq x \leq b \\ 0 & otherwise \end{cases}$$

and denoted as (a, b, c).

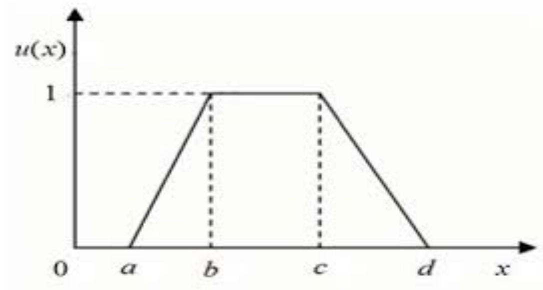

Fig. 2 Trapezoidal fuzzy number.

Definition 2.3 [4] A discrete Z-number is an ordered pair $Z = (A, B)$ where A is a discrete fuzzy number playing a role as a fuzzy constraint on values that a random variable X may take:

$$X\ is\ A$$

and B is a discrete fuzzy number with a membership function $\mu_B: \{b_1, \dots, b_n\} \longrightarrow [0,1]$, $\{b_1, \dots, b_n\} \subset [0,1]$, playing a role of a fuzzy constraint on the probability measure of A:

$P(A)\ is\ B$.

Aliev et al. in [5] defined continuous Z-number by using continuous fuzzy number.

Definition 2.3 [5] A continuous Z-number is an ordered pair $Z = (A, B)$ where A is a continuous fuzzy number playing a role as a fuzzy constraint on values that a random variable X may take:

$$X \text{ is } A$$

and B is a continuous fuzzy number with a membership function $\mu_B: [0,1] \longrightarrow [0,1]$ playing a role of a fuzzy constraint on the probability measure of A:

$$P(A) \text{ is } B$$

ORDERED Z-NUMBER

In [6], the concept of minimum and maximum of both discrete and continuous Z-number was introduced and denoted as MIN and MAX, respectively. They showed that for the discrete Z-number, the triple (Z_D,MIN,MAX) is a distributive lattice, where Z_D represents the set of discrete Z-numbers whose support is a sequence of consecutive natural numbers. The term MIN and MAX serve as *meet* and *joint* of Z_D which implies immediately that discrete Z-number is partially ordered. Similarly, the triple (Z_C,MIN,MAX) is also a distributive lattice, where Z_C represents the set of continuous Z-numbers support, which is a bounded set of natural numbers. Since set of natural numbers is well-ordered and has a least element, hence, continuous Z-number is partially ordered. However, [9] did not show explicitly that (Z_D,MIN,MAX) is a distributive lattice. Therefore, in this paper the relation $\prec$ on Z-number (discrete or continuous) is shown to be partially ordered in Theorem 3.1.

A discrete or continuous Z-number can be ordered using two different methods. The first one is by using the method proposed by Kang B in [3], which is, converting discrete or continuous Z-number to a discrete or continuous generalized fuzzy number. However, this method may lead to sufficient loss of original information. The second method, which is the most preferable, is by creating a relation between set of Z-number (discrete or continuous) and any arbitrary ordered subset in $\mathbb{R}$ as follows.

Definition 3.1 Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be two Z-numbers (discrete or continuous). Then $Z_1 = Z_2$ if and only if $A_1 = A_2$ and $B_1 = B_2$, namely, $\mu_{A_1}(x) = \mu_{A_2}(x)$ and $\mu_{B_1}(x) = \mu_{B_2}(x)$, respectively.

Theorem 3.1 The relation $(\bar{Z}_D, \prec) \Leftrightarrow (G, \leq)$ is well-defined.

Proof Consider two sets $(\bar{Z}_D, \prec)$and $(G, \leq)$ with relation $(\bar{Z}_D, \prec) \Leftrightarrow (G, \leq)$ for $G \subseteq \mathbb{R}$ and $(\bar{Z}_D, \prec)$ means a set of Z-numbers (discrete or continuous) with binary operation $\prec$. The relation $\Leftrightarrow$ is well defined due to its tautology as shown in Table 2, where T:= $True$ and F:= $False$.

Table 2 Trues table.

$(\bar{Z}_D, \prec)$	$(G, \leq)$	$\Leftrightarrow$
T	**T**	**T**
T	F	F
F	T	F
F	**F**	**F**

Both sides must be exactly the same in order the relation $\Leftrightarrow$ to be true.

■

Theorem 3.2. The set $([-1, 0], \leq)$ is partially ordered.

Proof. Consider $([-1, 0], \leq)$.

It is reflexive since $x \leq x, \forall x \in [-1, 0]$.

It is antisymmetry since $x \leq y$ and $y \leq x$ then $x = y$ for $x, y, z \in [-1,0]$.

It is transitive since $x \leq y$ and $y \leq z$ then $x \leq z$ for $x, y, z \in [-1,0]$. Hence, $([-1, 0], \leq)$ is a partially ordered.

■

Corollary 3.1 The relation $(\bar{Z}_D, \prec) \Leftrightarrow ([-1, 0], \leq)$ is well defined.

Proof By replacing $G = [-1, 0]$ in Theorem 3.1, the proposed relation $(\bar{Z}_D, \prec) \Leftrightarrow ([-1, 0], \leq)$ is well defined.

■

Theorem 3.3 The set $(\bar{Z}_D, \prec)$ with relation defined as $(\bar{Z}_D, \prec) \Leftrightarrow ([-1, 0], \leq)$ is a partially ordered set.

Proof The relation $(\bar{Z}_D, \prec) \Leftrightarrow ([-1, 0], \leq)$ is well defined by Corollary 3.1. Furthermore, $([-1, 0], \leq)$ is partially ordered by Theorem 3.2.

The reflexive property holds for $(\bar{Z}_D, \prec)$ by invoking the reflexivity of $([-1, 0], \leq)$. In orther words, $Z_1 = Z_1, \forall Z_1 \epsilon (\bar{Z}_D, \prec)$.

Similarly, for antisymmetry and transitive properties for $(\bar{Z}_D, \prec)$, by invoking the antisymmetry and transitive properties of $([-1, 0], \leq)$.

Hence, $(\bar{Z}_D, \prec)$ with relation defined as $(\bar{Z}_D, \prec) \Leftrightarrow ([-1, 0], \leq)$ is a partially ordered set. ■

The relation $\Leftrightarrow$ is well defined between $(\bar{Z}_D, \prec)$ and $(G, \prec)$, as proven in Theorem 3.1. Obviously, similar relation is well defined when it is replaced by $(\bar{Z}_D \times \mathrm{G}, \prec)$ such that

$$\prec : (\bar{Z}_D \times \mathrm{G}, \prec) \longrightarrow G \ni (Z_1, g_1) \prec (Z_2, g_2) \Leftrightarrow g_1 \prec g_2 .$$

Theorem 3.4 Let $\bar{Z}_D$ be a set of discrete Z-numbers and $\prec$ be a linear ordering relation. The set $(\bar{Z}_D, \prec)$ is said to be totally ordered, by creating a relation between $\bar{Z}_D$ and any arbitrary ordered set in $\mathbb{R}$.

Proof Let H be any arbitrary ordered set in $\mathbb{R}$, namely $(H, \prec) \subset (\mathbb{R}, \prec)$. Consider $(\bar{Z}_D \times \mathrm{H}, \prec)$ where $Z_1, Z_2, Z_3 \in \bar{Z}_D$ and $h_1, h_2, h_3 \in H$. The relation $\prec$ is define as $\prec : (\bar{Z}_D \times \mathrm{H}, \prec) \longrightarrow H \ni (Z_1, h_1) \prec (Z_2, h_2) \Leftrightarrow h_1 \prec h_2$. Now we need to show that it is reflexive, antisymmetry and transitive for any $(Z_1, h_1), (Z_2, h_2), (Z_3, h_3) \in \bar{Z}_D \times H$.

1. Reflexive: $(Z_1, h_1) \prec (Z_1, h_1)$ is true since $(H, \prec)$ is linearly ordered.
2. Transitive: Suppose $(Z_1, h_1) \prec (Z_2, h_2)$ and $(Z_2, h_2) \prec (Z_1, h_1)$, this implies that $h_2 = h_1$ since $(H, \prec)$ is linearly ordered. Therefore, $(Z_1, h_1) = (Z_2, h_2)$.
3. Antisymmetry: Suppose $(Z_1, h_1) \prec (Z_2, h_2)$ and $(Z_2, h_2) \prec (Z_3, h_3)$, this implies that $h_1 \prec h_3$ since $(H, \prec)$ is linearly ordered. Therefore, $(Z_1, h_1) \prec (Z_3, h_3)$.

Thus, $(\bar{Z}_D \times \mathrm{H}, \prec)$ is partially ordered, which implies that $(\bar{Z}_D, \prec)$ is also partially ordered.

Next, we want to show that $(\bar{Z}_D \times \mathrm{H}, \prec)$ is totally ordered. For any two distinct elements $(Z_1, h_1), (Z_2, h_2) \in (\bar{Z}_D \times \mathrm{H})$, i.e. $(Z_1, h_1) \neq (Z_2, h_2)$. Since H is totally ordered, there exist $h_1 \neq h_2$ such that $h_1 \prec h_2$ or $h_2 \prec h_1$. This implies that $(Z_1, h_1) \prec (Z_2, h_2)$ or $(Z_2, h_2) \prec (Z_1, h_1)$. Therefore, $(\bar{Z}_D \times \mathrm{H}, \prec)$ is totally ordered, which implies that $(\bar{Z}_D, \prec)$ must be totally ordered too.

■

Similar proof for Theorem 3.4 can be adopted for continuous Z-number. Therefore, we state the theorem for the case without its proof as follows.

Theorem 3.5 Let $\bar{Z}_C$ be a set of discrete Z-numbers and $\prec$ be a linear ordering relation. The set $(\bar{Z}_C, \prec)$ is said to be totally ordered, by creating a relation between $\bar{Z}_C$ and any arbitrary ordered set in $\mathbb{R}$.

The following two definitions are motivated from Kosanovic's definition of ordered fuzzy set in [10]. Therefore, an ordered discrete Z-number can be defined as:

Definition 3.2 Let Z_D be a discrete Z-number and let $\bar{Z}_D$ be a set of discrete Z-numbers, i.e. $: Z_D \in \bar{Z}_D$, the pair $(Z_D, \prec)$ is called an ordered discrete Z-number, if there exist a relation $\prec$, such that $(\bar{Z}_D, \prec)$ is totally ordered.

An ordered continuous Z-number is defined as:

Definition 3.3 Let Z_C be a continous Z-number and let $\bar{Z}_C$ be a set of continuous Z-numbers, i.e. $: Z_C \in \bar{Z}_C$, the pair $(Z_C, \prec)$ is called an ordered continous Z-number, if there exist a relation $\prec$, such that $(\bar{Z}_C, \prec)$ is totally ordered.

The concept of temporal discrete Z-number is an example of ordered discrete Z-number, which is discussed in the following section

IMPLEMENTATION

Basically, a temporal discrete Z-number is a discrete Z-number created from a universal set whose elements are ordered in time, whereby the proposed ordered discrete Z-number is used in the construction of temporal discrete Z-number. All the content of this section is fully discussed in [11].

Definition 4.1 Let (F, d_F) and (T, d_T) be metric spaces, where $(T, \prec)$ is a linearly ordered set with minimal element $t_0 \in T$. Let $S_t \subset F \times T$ be an augmented trajectory of a dynamic motion $g \in F^T$ defined for all $t \in T$. The relation $\prec'$ on $S_t \times S_t$, generated by $g(\cdot)$, is called a temporal ordering on S_t, and is defined as $\forall\, (Z_1, t_1), (Z_2, t_2) \in S_t \; (Z_1, t_1) \prec' (Z_2, t_2) \Leftrightarrow t_1 \prec \mathrm{t}$, where Z_1

and Z_2 are ordered discrete Z-numbers. For any set $K_t \subseteq S_t$, a pair $(K_t, \prec')$ is said to be a temporal set on S_t.

Definition 4.2 Let S_t be an augmented dynamic trajectory with appropriate temporal ordering $\prec'$. Let $(K_t, \prec')$ be a temporal set on S_t. A discrete Z-number in the universe K_t is called a temporal discrete Z-number which is denoted as $Z^t = (A^t, B^t)$.

Fig. 3 illustrates the relationship between the augmented trajectory S_t, temporal set K_t and the temporal discrete Z-number Z^t .

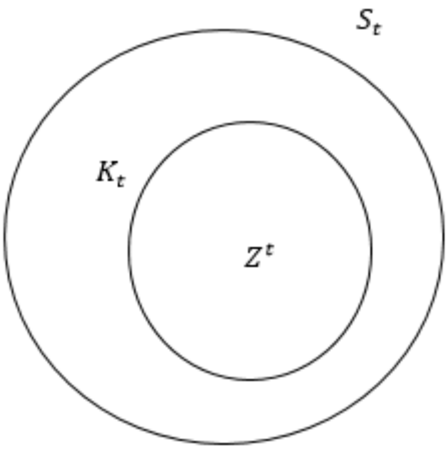

Fig. 3 Relation between S_t, K_t and Z^t.

The following Lemma, theorem and corollary lead to temporal discrete Z-numbers as a class of ordered discrete Z-numbers.

Lemma 4.1. Let $S_t$be an augmented trajectory, then every temporal ordering $\prec'$ on S_t is a partial ordering on S_t.

Proof. Let S_t be an augmented trajectory with the temporal ordering $\prec'$. Based on the Definition 31 of temporal ordering, the relation $\prec'$ on $S_t \times S_t$ generated by $g \in F^T$ has the characteristic such that

$$(Z_1, t_1) \prec' (Z_2, t_2), \Leftrightarrow t_1 \prec t_2$$

Now, we want to show that its reflexive, antisymmetry and transitive for any $(Z_1, t_1), (Z_2, t_2), (Z_3, t_3) \in S_t$ where $t_1, t_2, t_3 \in T$.

1. Reflexive: $(Z_1, t_1) \prec' (Z_1, t_1)$ is true since $(T, \prec)$ is linearly ordered.
2. Antisymmetry: Suppose $(Z_1, t_1) \prec' (Z_2, t_2)$ and $(Z_2, t_2) \prec' (Z_1, t_1)$, this impliest $t_1 \prec t_2$ and $t_2 \prec t_1 \Rightarrow t_1 = t_2$ since $(T, \prec)$ is linearly ordered. Therefore, $(Z_1, t_1) = (Z_2, t_2)$.
3. Transitive: Suppose $(Z_1, t_1) \prec' (Z_2, t_2)$ and $(Z_2, t_2) \prec' (Z_3, t_3)$, this implies $t_1 \prec t_2$ and $t_2 \prec t_3 \Rightarrow t_1 \prec t_3$ since $(T, \prec)$ is linearly ordered. Therefore, $(Z_1, t_1) \prec' (Z_3, t_3)$.

Hence the temporal ordering $\prec'$ on S_t is a partial ordering on S_t.

Theorem 4.2. Let S_t be an augmented trajectory, then every temporal ordering $\prec'$ on S_t is linearly ordering on S_t.

Proof. By Lemma 4.1 the temporal ordering $\prec'$ on S_t is a partial ordering on S_t. For any distinct elements of S_t i.e. $(Z_1, t_1) \neq (Z_2, t_2)$, then there exist $g(t_1) = Z_1$ and $g(t_2) = Z_2$ when $t_1 \neq t_2$. Since $(T, \prec)$ is linearly ordered, then $t_1 \prec t_2$ or $t_2 \prec t_1$. This implies that (Z_1, t_1) must precedes (Z_2, t_2) i.e. $(Z_1, t_1) \prec' (Z_2, t_2)$ or $(Z_2, t_2) \prec' (Z_1, t_1)$. Hence the temporal ordering $\prec'$ on S_t is a linearly ordering on S_t.

Corollary 4.3. Every temporal discrete Z-number is an ordered discrete Z-number.

Proof. By Lemma 4.1 and Theorem 4.2 the pair $(S_t, \prec')$ is linearly ordered. Furthermore, by Definition 4.1 of temporal ordering $\prec'$ is defined as $(Z_1, t_1) \prec' (Z_2, t_2), \Leftrightarrow t_1 \prec t_2 \ \forall \ (Z_1, t_1), (Z_2, t_2) \in S_t$ where Z_1, Z_2 are ordered discrete Z-numbers. For any $R_t \subseteq S_t$ where $(R_t, \prec')$ is a temporal set on S_t, which is linearly ordered. By Definition 4.2, a discrete Z-number say $Z \in R_t$ is called a temporal discrete Z-number. Therefore, this means that Z must be an ordered discrete Z-number by Definition 4.1. Hence, we can simply say that by Lemma 4.1, Theorem 4.2, Definition 4.1, and 4.2, every temporal discrete Z-number is an ordered discrete Z-number.

The detailed derivation of temporal discrete Z-number and its implementation procedure are presented in [11], whereby some of the data used are obtained from [12] to illustrate the procedure for analyzing EEG signal of an epileptic seizure.

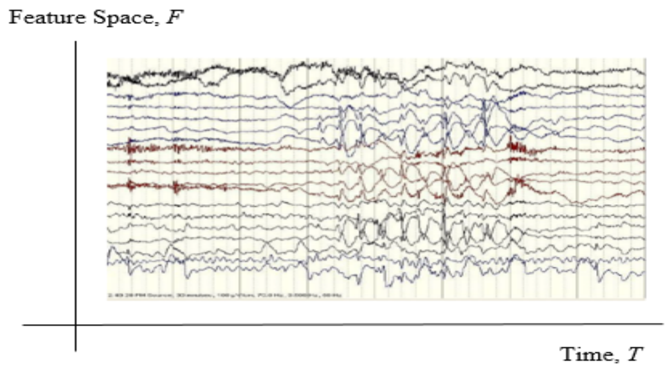

Fig. 4 EEG signal of an epileptic seizure.

Numerical example:

Some of the data used are taken from [12]. Let consider an EEG data set of an epileptic seizure which is given in Table 2. By applying Z-number clustering algorithm one can partition the data set in to clusters which are represented by membership function of temporal discrete Z-number.

Table 2 Fragment of EEG data set of seizure.

$x_{i,1}$	$x_{i,2}$
...	...
1.05	0.774906
1.10	0.822311
1.15	0.874949
1.20	0.933029
1.25	0.996711
1.30	1.066098
1.35	1.141221
1.40	1.22203
1.45	1.308371
1.50	1.399982
1.55	1.496474
...	...
4	7.386384

Firstly, in order to obtain a type-2 temporal fuzzy set cluster, fuzzy fuzzifier is used as shown in Fig. 5.

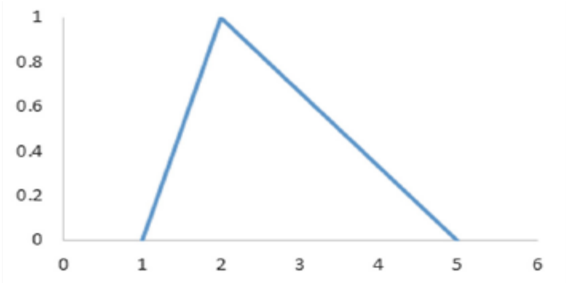

Fig. 5 Fuzzy fuzzifier membership function.

A type-2 membership function of one of the clusters obtain is described by Fig. 6, say *cluster 2.*

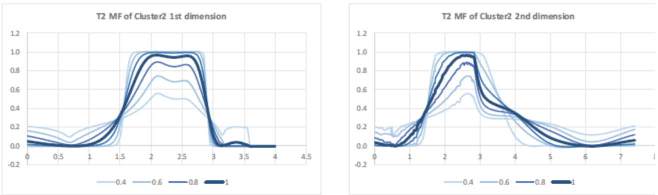

Fig. 6 Type-2 data-to-cluster membership function for x and y dimension of cluster 2.

The membership function of the first component of temporal discrete Z-number, i.e. A^t is obtained as a centroid of type-2 data-to-cluster membership function as shown in Fig. 5.

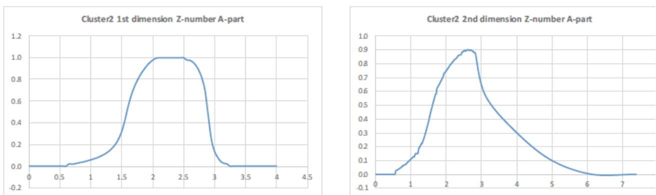

Fig. 7 Membership function of A^t for x and y dimension.

The second component of temporal discrete Z-number, i.e. B^t is determined by constructing a probability density function using the obtained membership function of A^t. Fig. 8 demonstrates the probability density function.

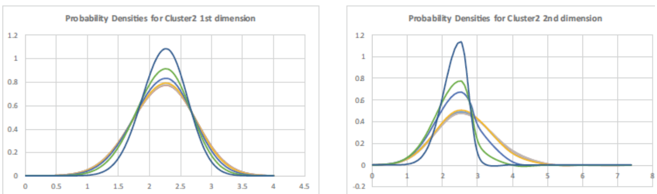

Fig. 8 Probability density function for x and y dimension.

Lastly, by computing the probability measure for A^t, the membership function of B^t is constructed and demonstrated in Fig. 9.

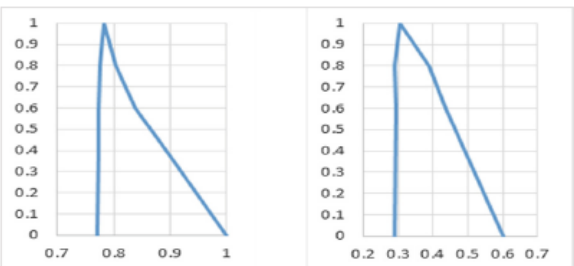

Fig. 9 Membership function of B^t for x and y dimension.

Supposed the membership functions of A^t and B^t for x dimension are represented as follows

$$A^t = {}^{0}/_{0} + {}^{0.3}/_{1.5} + {}^{1}/_{2.2} + {}^{0.1}/_{3} + {}^{0}/_{0}$$

and

$$B^t = {}^{0.8}/_{0.77} + {}^{1}/_{0.79} + {}^{0.9}/_{0.8} + {}^{0.4}/_{0.9} + {}^{0}/_{1}$$

Therefore, the membership functions are used to determine the measure of uncertainty for Z^t in x dimension with respect to the time of occurrence.

The numerical example illustrates the implementation procedure of applying temporal discrete Z-number to analyze EEG signal data of epileptic seizure and finally to determine the measure of uncertainty with respect to time of occurrence.

CONCLUSION

Even though both discrete and continuous Z-numbers are pairs of discrete and continuous fuzzy numbers, however they not simply imply discrete and continuous Z-numbers are ordered immediately as fuzzy numbers with respect to their membership values. A complex number is an example such case. In other words, both discrete and continuous Z-numbers cannot be ordered on their own. This paper proposed the idea of ordered discrete and continuous Z-number by creating a relation between set of discrete or continuous Z-numbers and any arbitrary ordered subset of $\mathbb{R}$.

The proposed structure is successfully used to construct temporal discrete Z-number with the purpose to analyze electroencephalographic signal of an epileptic seizure.

ACKNOLEDGEMENT

This work is financially supported by GUP Tier 1 (13H17), FRGS (4F756) and UTM COE Grant, 04G05.

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