Ordered discrete and continuous Z-numbers

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Abstract
Both discrete and continuous Z-numbers are pairs of discrete and continuous fuzzy numbers. Even though the later are ordered, this do not simply imply the discrete and continuous Z-numbers are ordered as well. This paper proposed the idea of ordered discrete and continuous Z-numbers, which are necessary properties for constructing temporal Z-numbers. Linear ordering relation, ≺, is applied between set of discrete or continuous Z-numbers and any arbitrary ordered subset of ℝ to obtain the properties.

Keywords: Z-number, Discrete Z-number, Continuous Z-number, Relation, Lattice

INTRODUCTION
Real-world information is flawed, and natural language is often used to represent this feature. Such information is often characterized by fuzziness, which implies that soft constraints are imposed on the values of variables of interest. Furthermore, reliability is another essential property of information. Any estimation of values of interest, be it precise or soft, are subject to the confidence with regards to sources of information. Thus, fuzziness from the one side and partial reliability form the other side are strongly associated to each other [1]. To discuss this concept, Zadeh in [2] introduced the concept of Z-number as a formal description of such information. Basically, the concept of Z-number relates to the issue of reliability of information. A Z-number is an ordered pair of fuzzy numbers \((A, B)\). It is associated with a real valued uncertain variable \(X\), with the first component \(A\), playing the role of a fuzzy restriction \((R(X))\) on the values which \(X\) can take, written as \(X\ is \ A\) such that \(A\ is a zoning\). The second component \(B\) is a measure of reliability (certainty) of the first component [2].

B. Kang et al. [3] proposed an approach of dealing with Z-numbers which naturally arises in the areas of decision making, control, regression analysis and others. The approach is based on transforming a Z-number into fuzzy number on the basis of fuzzy expectation of the fuzziness. The advantage of this approach is its low analytical and computational complexity, which allows for a wide spectrum of its applications. Unfortunately, converting Z-number to fuzzy leads to significant loss of original information and reducing the benefit of using Z-number-based information in the first place.

The authors in [4] developed some basics for direct computation with Z-number, by suggesting a general and computationally effective approach to deal with discrete Z-number. The authors provided motivation to use discrete Z-numbers as an alternative to the continuous one, based on the fact that natural language-based information has a discrete framework and it is not required to decide upon a reasonable assumption to use some type of probability distributions. Furthermore, it has lower computational complexity than that with continuous Z-numbers. Some basic theoretical aspects of arithmetic operations over discrete Z-numbers such as addition, subtraction, multiplication, division, square root of a Z-number, and other operations are proposed as well as a series of numerical examples are provided by them to illustrate the validity of the suggested approach.

A mathematical property called ordered, is required for constructing temporal discrete Z-numbers. Consider the set of complex numbers, \(C\). It is not ordered naturally but when the relation \(\|: C \rightarrow \mathbb{R}\) is employed on \(C\) such that \(|C| = |a + ib| = \sqrt{a^2 + b^2} \in \mathbb{R}\), then the ordered property is deduced indirectly. Fig. 1 shows the coordinates of complex number.
Similarly, discrete Z-number is an ordered pair of discrete fuzzy numbers, however, this does not guarantee that discrete Z-number is an ordered set too. This paper proves that both discrete and continuous Z-numbers can be ordered by applying a linear ordering relation < between set of discrete or continuous Z-numbers and any arbitrary ordered subset of \( \mathbb{R} \). The rest of the paper is organized as follows: Section 2 contains some basic definitions related to this work; the concepts of ordered discrete and continuous Z-number are revealed in Section 3; a sample of the implementation is presented in Section 4; and finally, the conclusion is drawn in Section 5.

PRELIMINARIES

Here are some important definitions which are essential in this work.

**Definition 1.1** [7] The relation \( < \) on \( X \times X \) is a partial ordering on \( X \) if it satisfies the following properties:

1. (Reflexivity) \( x < x \) for every \( x \in X \).
2. (antisymmetry) If \( x_1 < x_2 \) and \( x_2 < x_1 \), then \( x_1 = x_2 \).
3. (transitivity) If \( x_1 < x_2 \) and \( x_2 < x_3 \), then \( x_1 < x_3 \).

A pair \( (X, <) \) is called a partially ordered set. A partially ordered set \( (X, <) \) is said to be totally ordered (also called linearly ordered), provided that for every \( x_1, x_2 \in X \) and \( x_1 \neq x_2 \), either \( x_1 < x_2 \) or \( x_2 < x_1 \). A partial order \( < \) is then said to be a linearly ordered.

**Definition 2.2** [7] A partially ordered set in which every pair of element has the greatest lower bound and the least upper bound is called a lattice.

**Definition 2.3** [7] A lattice \( (Z, \lor, \land) \) is a distributive lattice if the following additional identity holds for all \( a, b, c \in Z \):

\[
(a \lor b) \land c = a \lor (b \land c)
\]

**Definition 2.4** [8] A fuzzy number \( A \) of the real line \( \mathbb{R} \) with membership function \( \mu_A: \mathbb{R} \rightarrow [0,1] \) is a discrete fuzzy number if its support is finite, i.e. there exist \( \{x_1, \ldots, x_n\} \in \mathbb{R} \) with \( x_1 < x_2 < \cdots < x_n \), such that \( \text{supp}(A) = \{x_1, \ldots, x_n\} \) and there exist natural numbers \( s, t \) with \( 1 \leq s \leq t \leq n \) satisfying the following conditions:

1. \( \mu_A(x_i) = 1 \) for any natural number \( i \) with \( s \leq i \leq t \).
2. \( \mu_A(x_i) \leq \mu_A(x_j) \) for each natural number \( i, j \) with \( 1 \leq i \leq j \leq s \).
3. \( \mu_A(x_i) \geq \mu_A(x_j) \) for each natural number \( i, j \) with \( 1 \leq i \leq j \leq n \).

**Definition 3.** [5] A continuous fuzzy number is a fuzzy subset \( A \) of the real line \( \mathbb{R} \) with membership function \( \mu_A: \mathbb{R} \rightarrow [0,1] \) which possesses the following properties:

1. \( A \) is a normal fuzzy set.
2. \( A \) is a convex fuzzy set.
3. \( \alpha \)-cut \( A^\alpha \) is a closed interval for every \( \alpha \in (0,1] \).
4. The support of \( A \), \( \text{supp}(A) \), is bounded.

A continuous fuzzy number \( A \) with the membership function defined as

\[
\mu_A(x) = \begin{cases} 
\frac{x-a}{b-a} & \text{if } a \leq x < b \\
1 & \text{if } b \leq x \leq c \\
\frac{d-x}{d-c} & \text{if } c < x \leq d \\
0 & \text{otherwise}
\end{cases}
\]

is referred to as a trapezoidal fuzzy number and is denoted as \((a, b, c, d)\).

A special case of trapezoidal fuzzy number is a triangular fuzzy number (TFN) \( A \) with membership function defined as

\[
\mu_A(x) = \begin{cases} 
\frac{x-a}{b-a} & \text{if } a \leq x < b \\
\frac{c-x}{c-b} & \text{if } b \leq x \leq c \\
0 & \text{otherwise}
\end{cases}
\]

and denoted as \((a, b, c)\).

**Fig. 2** Trapezoidal fuzzy number.

**Definition 2.3** [4] A discrete Z-number is an ordered pair \( Z = (A, B) \) where \( A \) is a discrete fuzzy number playing a role as a fuzzy constraint on values that a random variable \( X \) may take:

\[
X \text{ is } A
\]

and \( B \) is a discrete fuzzy number with a membership function \( \mu_B: \{b_1, \ldots, b_n\} \rightarrow [0,1] \), \( \{b_1, \ldots, b_n\} \subseteq [0,1] \), playing a role of a fuzzy constraint on the probability measure of \( A \):

\[
P(A) \text{ is } B
\]

Aliev et al. in [5] defined continuous Z-number by using continuous fuzzy number.

**Definition 2.3** [5] A continuous Z-number is an ordered pair \( Z = (A, B) \) where \( A \) is a continuous fuzzy number playing a role as a fuzzy constraint on values that a random variable \( X \) may take:

\[
X \text{ is } A
\]

and \( B \) is a continuous fuzzy number with a membership function \( \mu_B: [0,1] \rightarrow [0,1] \) playing a role of a fuzzy constraint on the probability measure of \( A \):

\[
P(A) \text{ is } B
\]

ORDERED Z-NUMBER

In [6], the concept of minimum and maximum of both discrete and continuous Z-number was introduced and denoted as MIN and MAX, respectively. They showed that for the discrete Z-number, the triple \((Z_D, \text{MIN}, \text{MAX})\) is a distributive lattice, where \( Z_D \) represents the set of discrete Z-numbers whose support is a sequence of consecutive natural numbers. The term MIN and MAX serve as meet and joint of \( Z_D \) which implies immediately that discrete Z-number is partially ordered. Similarly, the triple \((Z_C, \text{MIN}, \text{MAX})\) is also a distributive lattice, where \( Z_C \) represents the set of continuous Z-numbers support, which is a bounded set of natural numbers. Since set of natural numbers is well-ordered and has a least element, hence, continuous Z-number is partially ordered. However, [9] did not show explicitly that \((Z_C, \text{MIN}, \text{MAX})\) is a distributive lattice. Therefore, in this paper the relation \( < \) on Z-number (discrete or continuous) is shown to be partially ordered in Theorem 3.1.

A discrete or continuous Z-number can be ordered using two different methods. The first one is by using the method proposed by Kang B in [3], which is, converting discrete or continuous Z-number to a discrete or continuous generalized fuzzy number. However, this method may lead to sufficient loss of original information. The second method, which is the most preferable, is by creating a relation between
set of Z-number (discrete or continuous) and any arbitrary ordered subset in $\mathbb{R}$ as follows.

**Definition 3.1** Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be two Z-numbers (discrete or continuous). Then $Z_1 \equiv Z_2$ if and only if $A_1 = A_2$ and $B_1 = B_2$, respectively, namely, $\mu_{A_1}(x) = \mu_{A_2}(x)$ and $\mu_{B_1}(x) = \mu_{B_2}(x)$, respectively.

**Theorem 3.1** The relation $(Z_D, \prec)$ is well-defined.

**Proof** Consider two sets $(Z_D, \prec)$ and $(G, \leq)$ with relation $(Z_D, \prec) \Longleftrightarrow (G, \leq)$ for $G \subseteq \mathbb{R}$ and $(Z_D, \prec)$ means a set of Z-numbers (discrete or continuous) with binary operation $\prec$. The relation $\Longleftrightarrow$ is well defined due to its tautology as shown in Table 2, where $T := True$ and $F := False$.

<table>
<thead>
<tr>
<th>$(\tilde{Z}_D, \prec)$</th>
<th>$(G, \leq)$</th>
<th>$\Longleftrightarrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
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<td>$F$</td>
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<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

Both sides must be exactly the same in order the relation $\Longleftrightarrow$ to be true.

**Theorem 3.2** The set $([-1, 0], \leq)$ is partially ordered.

**Proof** Consider $([-1, 0], \leq)$. It is reflexive since $x \leq x, \forall x \in [-1, 0]$. It is antisymmetric since $x \leq y$ and $y \leq x$ then $x = y$ for $x, y, z \in [-1, 0]$. It is transitive since $x \leq y$ and $y \leq z$ then $x \leq z$ for $x, y, z \in [-1, 0]$. Hence, $([-1, 0], \leq)$ is a partially ordered set.

**Corollary 3.1** The relation $(\tilde{Z}_D, \prec)$ is well-defined.

**Proof** By replacing $G = [-1, 0]$ in Theorem 3.1, the proposed relation $(\tilde{Z}_D, \prec)$ is well defined.

**Theorem 3.3** The set $(\tilde{Z}_D, \prec)$ with relation defined as $(\tilde{Z}_D, \prec)$ is partially ordered set.

**Proof** The relation $(\tilde{Z}_D, \prec)$ is partially ordered by Corollary 3.1. Furthermore, $([-1, 0], \leq)$ is partially ordered by Theorem 3.2. The reflexive property holds for $(\tilde{Z}_D, \prec)$ by invoking the reflexivity of $([-1, 0], \leq)$. In other words, $Z_1 \equiv Z_1, \forall Z_1 \in (\tilde{Z}_D, \prec)$. Similarly, for antisymmetry and transitive properties for $(\tilde{Z}_D, \prec)$, by invoking the antisymmetry and transitive properties of $([-1, 0], \leq)$.

Hence, $(\tilde{Z}_D, \prec)$ with relation defined as $(\tilde{Z}_D, \prec)$ is partially ordered set.

**Theorem 3.4** Let $Z_D$ be a set of discrete Z-numbers and $\prec$ be a linear ordering relation. The set $(\tilde{Z}_D, \prec)$ is said to be totally ordered, by creating a relation between $Z_D$ and any arbitrary ordered set in $\mathbb{R}$.

**Proof** Let $H$ be any arbitrary ordered set in $\mathbb{R}$, namely $(H, \prec) \subseteq (\mathbb{R}, \prec)$. Consider $(\tilde{Z}_D \times H, \prec)$ where $Z_1, Z_2, Z_3 \in \tilde{Z}_D$ and $h_1, h_2, h_3 \in H$. The relation $\prec$ is defined as $:\prec : (\tilde{Z}_D \times H, \prec) \Rightarrow H \ni (Z_1, h_1) \prec (Z_2, h_2) \Leftrightarrow h_1 < h_2$. Now we need to show that it is reflexive, antisymmetry and transitive for any $(Z_1, h_1), (Z_2, h_2), (Z_3, h_3) \in Z_D \times H$.

1. Reflexive: $(Z_1, h_1) \prec (Z_2, h_1)$ is true since $(H, \prec)$ is linearly ordered.
2. Transitive: Suppose $(Z_1, h_1) \prec (Z_2, h_2)$ and $(Z_2, h_2) \prec (Z_3, h_3)$, this implies that $h_2 < h_1$ since $(H, \prec)$ is linearly ordered. Therefore, $(Z_1, h_1) \prec (Z_2, h_3)$.
3. Antisymmetry: Suppose $(Z_1, h_1) \prec (Z_2, h_2)$ and $(Z_2, h_2) \prec (Z_3, h_3)$, this implies that $h_1 < h_3$ since $(H, \prec)$ is linearly ordered. Therefore, $(Z_1, h_1) \prec (Z_3, h_3)$.

Thus, $(\tilde{Z}_D \times H, \prec)$ is partially ordered, which implies that $(\tilde{Z}_D, \prec)$ is also partially ordered.

Next, we want to show that $(\tilde{Z}_D \times H, \prec)$ is totally ordered. For any two distinct elements $(Z_1, h_1), (Z_2, h_2) \in (\tilde{Z}_D \times H)$, i.e. $(Z_1, h_1) \neq (Z_2, h_2)$. Since $H$ is totally ordered, there exist $h_1 \neq h_2$ such that $h_1 < h_2$ or $h_2 < h_1$. This implies that $(Z_1, h_1) \prec (Z_2, h_2)$ or $(Z_2, h_2) \prec (Z_1, h_1)$. Therefore, $(\tilde{Z}_D \times H, \prec)$ is totally ordered, which implies that $(\tilde{Z}_D, \prec)$ must be totally ordered too.

**Theorem 3.5** Let $Z_D$ be a set of discrete Z-numbers and $\prec$ be a linear ordering relation. The set $(\tilde{Z}_C, \prec)$ is said to be totally ordered, by creating a relation between $Z_C$ and any arbitrary ordered set in $\mathbb{R}$.

**Proof** Consider two sets $Z_C$ and let $\mu_B(x)$ be any arbitrary ordered set in $\mathbb{R}$.

**Definition 3.2** Let $Z_D$ be a discrete Z-number and let $Z_D$ be a set of discrete Z-numbers, i.e. $Z_D \in Z_D$, the pair $(Z_D, \prec)$ is called an ordered discrete Z-number, if there exist a relation $\prec$, such that $(\tilde{Z}_D, \prec)$ is totally ordered.

An ordered continuous Z-number is defined as:

**Definition 3.3** Let $Z_C$ be a continuous Z-number and let $Z_C$ be a set of continuous Z-numbers, i.e. $Z_C \in Z_C$, the pair $(\tilde{Z}_C, \prec)$ is called an ordered continuous Z-number, if there exist a relation $\prec$, such that $(\tilde{Z}_C, \prec)$ is totally ordered.

The concept of temporal discrete Z-number is an example of ordered discrete Z-number, which is discussed in the following section

**IMPLEMENTATION**

Basically, a temporal discrete Z-number is a discrete Z-number created from a universal set whose elements are ordered in time, whereby the proposed ordered discrete Z-number is used in the construction of temporal discrete Z-number. All the content of this section is fully discussed in [11].

**Definition 4.1** Let $(F, d_F)$ and $(T, d_T)$ be metric spaces, where $(T, \prec)$ is a linearly ordered set, such that $t_0 \in T$. Let $S_t \subseteq F \times T$ be an augmented trajectory of a dynamic motion $g \in F^T$ defined for all $t \in T$. The relation $\prec$ on $S_t \times S_t$, generated by $g(t)$, is called a temporal ordering on $S_t$, and is defined as $\forall (Z_1, t_1), (Z_2, t_2) \in S_t \times S_t, \Rightarrow t_1 \prec t_2 \Rightarrow (Z_1, t_1) \prec (Z_2, t_2)$.
and \( Z \) are ordered discrete \( Z \)-numbers. For any set \( K_t \subseteq S_t \), a pair \((K_t, \prec')\) is said to be a temporal set on \( S_t \).

**Definition 4.2** Let \( S_t \) be an augmented dynamic trajectory with appropriate temporal ordering \( \prec' \). Let \((K_t, \prec')\) be a temporal set on \( S_t \). A discrete \( Z \)-number in the universe \( K_t \) is called a temporal discrete \( Z \)-number which is denoted as \( Z_t = (A_t, B_t) \).

Fig. 3 illustrates the relationship between the augmented trajectory \( S_t \), temporal set \( K_t \) and the temporal discrete \( Z \)-number \( Z_t \).

![Fig. 3 Relation between \( S_t \), \( K_t \) and \( Z_t \) (Image)](image)

The following Lemma, theorem and corollary lead to temporal discrete \( Z \)-numbers as a class of ordered discrete \( Z \)-numbers.

**Lemma 4.1.** Let \( S_t \) be an augmented trajectory, then every temporal ordering \( \prec' \) on \( S_t \) is a partial ordering on \( S_t \).

**Proof.** Let \( S_t \) be an augmented trajectory with the temporal ordering \( \prec' \). Based on the Definition 3 of temporal ordering, the relation \( \prec' \) on \( S_t \times S_t \) generated by \( g \in F_T \) has the characteristic such that \( (Z_1, t_1) \prec' (Z_2, t_2) \iff t_1 < t_2 \).

Now, we want to show that its reflexive, antisymmetric and transitive for any \((Z_1, t_1), (Z_2, t_2), (Z_3, t_3) \in S_t \) where \( t_1, t_2, t_3 \in T \).

1. Reflexive: \((Z_1, t_1) \prec' (Z_1, t_1)\) is true since \((T, \prec)\) is linearly ordered.
2. Antisymmetric: Suppose \((Z_1, t_1) \prec' (Z_2, t_2)\) and \((Z_2, t_2) \prec' (Z_1, t_1)\), this implies \( t_1 < t_2 \) and \( t_2 < t_1 \Rightarrow t_1 = t_2 \) since \((T, \prec)\) is linearly ordered. Therefore, \((Z_1, t_1) = (Z_2, t_2)\).
3. Transitive: Suppose \((Z_1, t_1) \prec' (Z_2, t_2)\) and \((Z_2, t_2) \prec' (Z_3, t_3)\), this implies \( t_1 < t_2 \) and \( t_2 < t_3 \Rightarrow t_1 < t_3 \) since \((T, \prec)\) is linearly ordered. Therefore, \((Z_2, t_2) \prec' (Z_3, t_3)\).

Hence the temporal ordering \( \prec' \) on \( S_t \) is a partial ordering on \( S_t \).

**Theorem 4.2.** Let \( S_t \) be an augmented trajectory, then every temporal ordering \( \prec' \) on \( S_t \) is linearly ordered.

**Proof.** By Lemma 4.1 and Theorem 4.2 the pair \((S_t, \prec')\) is linearly ordered. Furthermore, by Definition 4.1 of temporal ordering \( \prec' \) is defined as \( (Z_1, t_1) \prec' (Z_2, t_2) \iff t_1 < t_2 \) \( \forall (Z_1, t_1), (Z_2, t_2) \in S_t \) where \( Z_1, Z_2 \) are ordered discrete \( Z \)-numbers. For any \( R_t \subseteq S_t \) where \((R_t, \prec')\) is a temporal set on \( S_t \), which is linearly ordered. By Definition 4.2, a discrete \( Z \)-number say \( Z \in R_t \) is called a temporal discrete \( Z \)-number. Therefore, this means that \( Z \) must be an ordered discrete \( Z \)-number by Definition 4.1. Hence, we can simply say that by Lemma 4.1, Theorem 4.2, Definition 4.1, and 4.2, every temporal discrete \( Z \)-number is an ordered discrete \( Z \)-number.

The detailed derivation of temporal discrete \( Z \)-number and its implementation procedure are presented in [11], whereby some of the data used are obtained from [12] to illustrate the procedure for analyzing EEG signal of an epileptic seizure.

![Fig. 4 EEG signal of an epileptic seizure (Image)](image)

**Numerical example:** Some of the data used are taken from [12]. Let consider an EEG data set of an epileptic seizure which is given in Table 2. By applying \( Z \)-number clustering algorithm one can partition the data set into clusters which are represented by membership function of temporal discrete \( Z \)-number.

<table>
<thead>
<tr>
<th>( x_{i1} )</th>
<th>( x_{i2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05</td>
<td>0.774906</td>
</tr>
<tr>
<td>1.10</td>
<td>0.822311</td>
</tr>
<tr>
<td>1.15</td>
<td>0.874949</td>
</tr>
<tr>
<td>1.20</td>
<td>0.933029</td>
</tr>
<tr>
<td>1.25</td>
<td>0.996711</td>
</tr>
<tr>
<td>1.30</td>
<td>1.066098</td>
</tr>
<tr>
<td>1.35</td>
<td>1.141221</td>
</tr>
<tr>
<td>1.40</td>
<td>1.22203</td>
</tr>
<tr>
<td>1.45</td>
<td>1.308371</td>
</tr>
<tr>
<td>1.50</td>
<td>1.399982</td>
</tr>
<tr>
<td>1.55</td>
<td>1.496474</td>
</tr>
<tr>
<td>4</td>
<td>7.386384</td>
</tr>
</tbody>
</table>

Firstly, in order to obtain a type-2 temporal fuzzy set cluster, fuzzy fuzzifier is used as shown in Fig. 5.
A type-2 membership function of one of the clusters obtain is described by Fig. 6, say cluster 2.

The second component of temporal discrete Z-number, i.e. $B^t$ is determined by constructing a probability density function using the obtained membership function of $A^t$. Fig. 8 demonstrates the probability density function.

Lastly, by computing the probability measure for $A^t$, the membership function of $B^t$ is constructed and demonstrated in Fig. 9.

Supposed the membership functions of $A^t$ and $B^t$ for $x$ dimension are represented as follows

$$A^t = 0/0 + 0.3/1.5 + 1/2.2 + 0.1/3 + 0/0$$

and

$$B^t = 0.8/0.77 + 1/0.79 + 0.9/0.8 + 0.4/0.9 + 0/1$$

Therefore, the membership functions are used to determine the measure of uncertainty for $Z^t$ in $x$ dimension with respect to the time of occurrence.

The numerical example illustrates the implementation procedure of applying temporal discrete Z-number to analyze EEG signal data of epileptic seizure and finally to determine the measure of uncertainty with respect to time of occurrence.

CONCLUSION

Even though both discrete and continuous Z-numbers are pairs of discrete and continuous fuzzy numbers, however they not simply imply discrete and continuous Z-numbers are ordered immediately as fuzzy numbers with respect to their membership values. A complex number is an example such case. In other words, both discrete and continuous Z-numbers cannot be ordered on their own. This paper proposed the idea of ordered discrete and continuous Z-number by creating a relation between set of discrete or continuous Z-numbers and any arbitrary ordered subset of $\mathbb{R}$.

The proposed structure is successfully used to construct temporal discrete Z-number with the purpose to analyze electroencephalographic signal of an epileptic seizure.

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