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Graph Dynamics of Fuzzy Autocatalytic Set of Fuzzy Graph Type-3 of a Clinical Waste Incineration Process

Sumarni Abu Bakar[1], Tahir Ahmad[2,*] and Sabariah Baharun[3]

[1,2]Department of Mathematics, Faculty of Science & [2]Ibnu Sina Institute, Nanotechnology Research Alliance Universiti Teknologi Malaysia, 81310 Skudai, Johor, MALAYSIA.
[3]Malaysian-Japan International Institute of Technology (MJIIT),Universiti Teknologi Malaysia, International Campus, Jalan Semarak, 54100 Kuala Lumpur, MALAYSIA

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ABSTRACT

Fuzzy Autocatalytic Set of Fuzzy Graph Type-3 (FACS) has been successfully implemented in modeling clinical waste incineration process. Six important variables identified in the process are represented as nodes and the catalytic relationships are represented by fuzzy edges in the graph. However, in this paper, graph dynamics of FACS is further investigated using left Perron vector of its transition matrix of fuzzy graph of FACS. This paper will highlight two important variables in the incineration process with regards to the actual process.

| Fuzzy Autocatalytic Set | Fuzzy Graph | Incineration Process | Transition Matrix |

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1. INTRODUCTION

The emergence of fuzzy graph to autocatalytic sets has instigated a new concept named Fuzzy Autocatalytic Set (FACS) [1, 2]. It is defined as a subgraph where each of whose nodes has at least one incoming link with membership value $\mu(e_i) \in (0,1]$, $\forall e_i \in E$. A clinical waste incineration process in Malacca [1, 2] is modelled using this concept where it incorporates fuzzy graph of type-3, G_F^3, where both the vertex and edge sets are crisp, but the edges have fuzzy heads and tails. As for incineration process, six variables that play vital roles in the process are waste (v_1), fuel (v_2), oxygen (v_3), carbon dioxide (v_4), carbon monoxide(v_5) and other gases including water (v_6). The vertices of the graph correspond to the variables and a directed link from vertex *i* to vertex *j* indicates that variable *i* catalyzes the production of variable *j*. When G_F^3 is specifically considered in the construction of FACS, the description of its fuzzy head, fuzzy tail and fuzzy edges connectivity of the edges are given in [1, 2].

Dynamic model of the incineration process using Perron-Frobenius eigenvector (PFE) of its adjacency matrix which resulted on the evolution of variables on a longer time scale had been discussed [1] such that the dynamic model provided information of the by- products of the incineration process which are CO_2 and H_2O and other pollutants. In the following sections we present the graph dynamic of FACS using left Perron vector of transition matrix for fuzzy graph of FACS.

**Corresponding author at:*
E-mail addresses: tahir@utm.my (Tahir Ahmad)

2. TRANSITION MATRIX FOR FUZZY GRAPH OF FACS

The definition of transition matrix for fuzzy graph of FACS is given as below.

Definition 1. [3]
Suppose $G_{FT3}(V, E)$ is a no loop FACS of Fuzzy Graph of type-3. The transition matrix for fuzzy graph of type-3 of FACS is P^*, with $P^*(u, v)$ is fuzzy value represents strength of moving from vertex *u* to vertex *v* as

$$P^*(u,v) = \begin{cases} \dfrac{\mu(u,v)}{d_{out}(u)} & if \quad (u,v) \in E \\ \\ 0 & otherwise \end{cases}$$

where for a v ertex *u*, the out-degree of *u* is $d_{out}(u) = \sum \mu(u,t)$ and $\mu(u,v)$ is the ordinary membership value of an edge from *u* to *v*. The matrix is shown to be stochastic, irreducible and aperiodic which in turn ensures the existence of steady state vector. In order to proof the stochasticity of P^*, we introduced a definition of stochastic matrix.

Definition 2.[8]
A stochastic matrix is a square non-negative matrix whose row sums equal one.

The properties of ordinary transition matrix can be obtained in [8]. This can be extended to our transition matrix for fuzzy graph of FACS, P^*. Now, we can show that P^* is stochastic and the proof are given in the following lemma.

Lemma 1
Transition matrix for fuzzy graph of FACS, P^* is stochastic.

Proof:
Let $G_{FT3}(V,E)$ be a graph of FACS. From Definition 1 of transition matrix for fuzzy graph of FACS, it shows that $P^*(i,j)=\dfrac{\mu(i,j)}{d_{out}(i)}$ *if* $(i,j)\in E$ and zero if $(i,j)\notin E$.
We also know that $P_{ii}=0$ which means the strength of connection is zero since there is no-loop existed in the graph or there is no connection from vertex *i* to itself. Since $P^*(i,j)=\dfrac{\mu(i,j)}{d_{out}(i)}$, therefore

$$\sum_{j=1}^{n} P^*(i,j)=\sum_{j=1}^{n}\frac{\mu(i,j)}{d_{out}(i)}$$
$$=\frac{1}{d_{out}(i)}\sum_{j=1}^{n}\mu(i,j)$$
$$=\frac{1}{d_{out}(i)}\left(d_{out}(i)\right) \qquad \text{since} \quad d_{out}(i)=\sum_{j}\mu(i,j)$$
$$=1$$

Thus $\sum_{j=1}^{n}P^*(i,j)=1 \ \ \forall i=1,...,n$. From Definition 2, it shows that transition matrix for fuzzy graph of FACS, P^* is stochastic.
Next, we show that P^* is irreducible.

Lemma 2.
Transition matrix for fuzzy graph of FACS, P^* is irreducible.

Proof:
Let $G_{FT3}(V,E)$ be a graph of FACS. From Corollary 1 [10], the corresponding adjacency matrix of FACS is irreducible. Therefore, from Lemma 3.1 [9], it shows that transition matrix of FACS, P^* is also irreducible.
Next, we show that P^* is aperiodic.

Lemma 3.
Transition matrix for fuzzy graph of FACS, P^* is aperiodic.
Proof:
It fulllfilled Corrollary 2 [10], therefore P^* is aperiodic

As for incineration process, the transition matrix for fuzzy graph of FACS is as follow.

$$P^*=\begin{bmatrix} 0 & \frac{0.00001}{1.00001} & \frac{0.15615}{1.00001} & \frac{0.51632}{1.00001} & \frac{0.00001}{1.00001} & \frac{0.32752}{1.00001} \\ 0 & 0 & 0 & 0.6800400 & 0.0000100 & 0.3199500 \\ 0.06529 & 0 & 0 & 0.6356300 & 0.0000200 & 0.2990600 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix P^* above, it shows that:

(1) P^* is a stochastic matrix since each row sums equal one and no value is less than 0.
(2) P^* is a regular transition matrix since for $k=6$, $\left((P^*)^k\right)_{i,j}>0,\ \forall i,j=1,...,n$.
(3) P^* is irreducible since FACS is strongly connected.
(4) The largest eigenvalue of P^* is $\lambda=1$. It is also known as right eigenvalue.

Long term behaviour of variables of incineration process described from its steady state vector is discussed [3, 4, 5]. As for $\lambda=1$, the left eigenvector is left Perron vector, $\boldsymbol{\varphi}$ of P^*. It is calculated by solving eigenvector problem or by finding its steady state vector of P^*[4]. It is also known as stationary distribution vector or PageRank vector, $\boldsymbol{\pi}$ [6]. PageRank computes a ranking of nodes in a graph based on the structure of its links. The idea of PageRank is that $\boldsymbol{\pi}(i)$ can be interpreted as the “importance” of *i*. Thus $\boldsymbol{\pi}$ defines a linear order on the vertices by treating $i\le j$ if $\boldsymbol{\pi}(i)\le\boldsymbol{\pi}(j)$. The importance of the variables represented as vertices in the graph can be measured by its PageRank or its left Perron vector.

3. METHODOLOGY

The network of the interaction between variables in the incineration process is represented by 6×6 adjacency matrix for fuzzy graph of FACS. However this variable is dynamic in nature in which it is wiped out in the process due to it is not functioning and a new network is evolved after certain time *t*. Thus, graph dynamics represent dynamical behavior of the variables of the incineration process [1]. FACS of fuzzy graph of type-3 is also represented by 6×6 transition matrix in which it described transition of one variable to the other variables in the incineration process [3]. The incineration process is evolved over time to a stationary state which is described by its steady state of left Perron vector. The likelihood of variables to move to a particular variable can be determined from its left Perron vector on each phase of the graph dynamics. The least value of j^{th} element of the row of its left Perron vector indicates the least chance of moving from other variables to the variable *j*. Thus variable *j* represented as node j in the graph of FACS of incineration process is considered least important and therefore the existence is insignificant thus can be omitted from that phase. In contrary, the highest value of j^{th} element of the row of its left Perron vector indicates the most important variable to the incineration process.

In this section, a procedure to determine the importance of the variables based on graph dynamics procedure is presented. The procedure is adopted and modified from Sabariah [1]. The modified procedure involved dynamical variables, $V = (v_1,...,v_n)$; $n=1,...,6$ where v_i stands for the chance of transition of other variables to the i^{th} variable of FACS of the incineration process is summarized as follows:

Step 1: Keeping G_F with n variables fixed and represented by transition matrix for fuzzy graph of FACS, P^*, v_i is evolved for a specified time t, which is large enough for v_i to converge to its steady state. At this particular state, v_i is no longer altered whereby, V can be represented by its left Perron vector. $V_i \equiv v_i(t)$ such that $\sum_{i=1}^{6} V_i = 1$

Step 2: The set L of nodes i with the least value of V_i is determined, i.e. $L = \{i \in S | V_i = \min_{j \in S} V_j,\ S = \{1,2,...,n\}\}$. This is the set of least important nodes, identifying transition value of a variable in the steady state vector at a specific time, t. The least the value of V_i, the least important is the node i. One of the least important node is chosen randomly and is removed from the system along with its links leaving a graph with s-1 variables.

Step 3: Graph G_F is now reduced to s-1 variables. A transition matrix $(s-1)\times(s-1)$ for fuzzy graph of FACS is reconstructed. Other nodes and links of G_F remain unchanged. All these $v_i\,(v_i > 0)$ is once again evolved to a new steady state at a consequent time t, in which $\sum_{i=1}^{n-1} v_i = 1$.

The steps are repeated until it produced 2×2 matrix.

This procedure is illustrated in Figure 3.1. At step 1, the variables evolved according to the connectivity among them until it reach a steady state. The steady state is taken as its left Perron vector. At step 2, variable j with the least value, V_j is removed from the graph together with its links. The removed variable is considered to be least important variable at that particular time, t, with assumptions for our model:

i) The value represent the likelihood for other variables to transit (move) to that particular variable.

ii) The smaller the value of V_j, the less likelihood for other variables to transit to variable j. Thus variable j is insignificant at that particular time t.

This process is then repeated until a graph with at least two nodes is attained. The whole process occurred at discretely sparse intervals labeled by $n=0,1, 2,...k$.

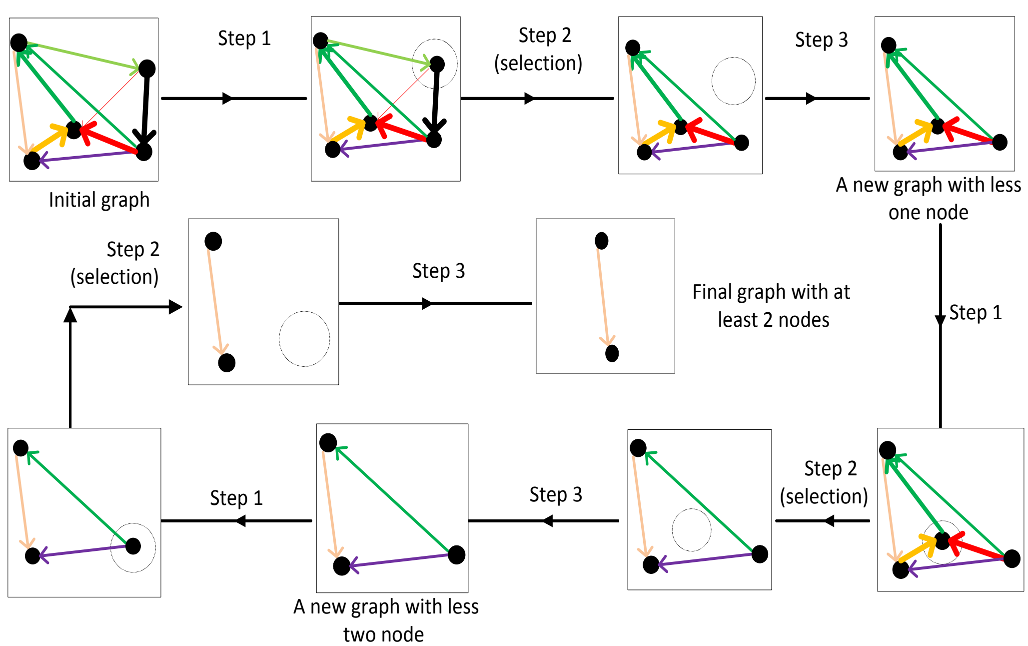

Fig. 1 Schematic diagram of graph dynamic procedure

4. IMPLEMENTATION & DISCUSSION

Six main variables, namely waste, fuel, O_2, CO_2, CO, H_2O & other pollutants interact among themselves as soon as the incineration process started and given in [1, 2]. The initial graph of FACS of an incineration process is as follow.

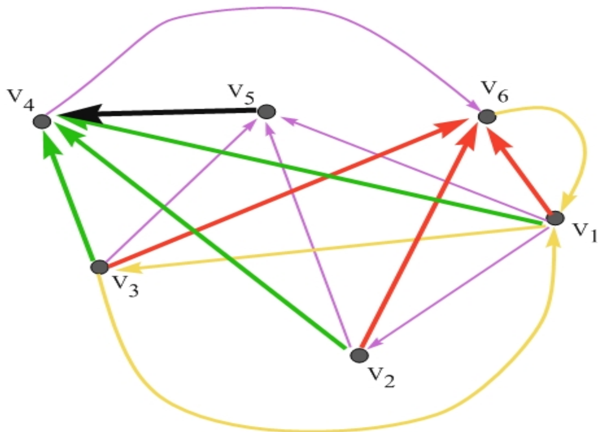

Fig. 2 $G_{FT3}(V, E)$ for the clinical waste incineration process

The different color signifies the different range of membership value for the fuzzy edge connectivity. The greater the value of connectivity between the vertices, the thicker is the link between them. From the graph in Figure 3.2, it is strongly connected where each node in the graph has access to every other node.

At $n=0$, it is anticipated that v_i evolved until it reached a particular time whereby all variables reached its steady state. This phenomenon is represented by its left Perron vector, φ of P^*. The least important variable corresponds to the least value of the element in the left Perron vector. It was then removed from the system together with their links to give way to the remaining variables for their interaction which leads to the second update of the process ($n=1$). A new graph of the remaining variables, now denoted by G_{F_1} is an induced graph of FACS. The steps given in the procedure is then repeated until $n=4$ where only two variables remain in the induced graph. The expected update on each stage of the incineration process given by this dynamic model through the procedure is shown in the following Figure 3.3. The update of variables is given in Table 4.1.

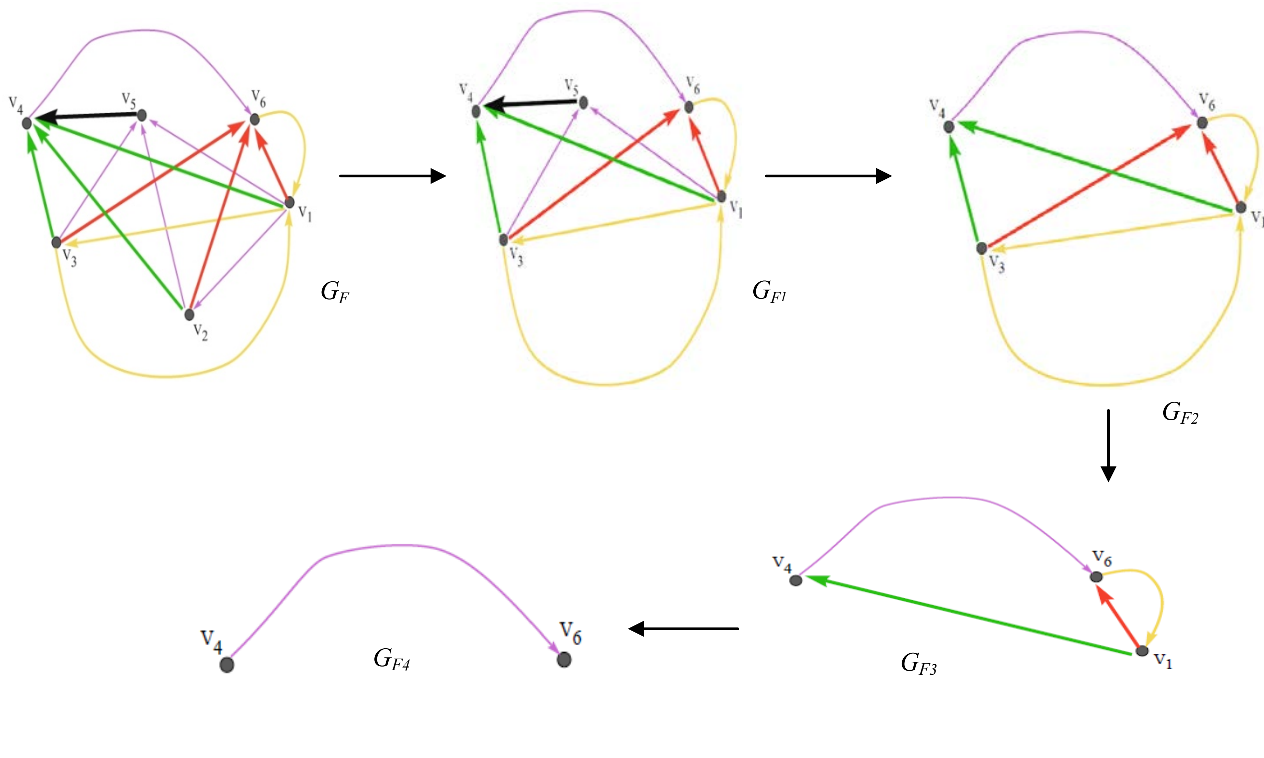

Fig. 3 Updates of graph of FACS of an incineration process

Table 1 Updates of variable for graph of FACS of an incineration process

Updates	Variables in graph	Left Perron vector	v_i deleted
G_F	v_1 = Waste v_2 = Fuel $v_3 = O_2$ $v_4 = CO_2$ $v_5 = CO$ $v_6 = H_2O$ & other pollutants	$\varphi = \begin{bmatrix} 0.36200364 \\ 0.362110019 \times 10^{-6} \\ 0.056543479 \\ 0.222912591 \\ 4.75200599 \times 10^{-6} \\ 0.358421916 \end{bmatrix}$	v_2 = Fuel
G_{F_1}	v_1 = Waste $v_3 = O_2$ $v_4 = CO_2$ $v_5 = CO$ $v_6 = H_2O$ & other pollutants	$\varphi = \begin{bmatrix} 0.362116565152852 \\ 0.056544501648618 \\ 0.222909406502631 \\ 0.000004752055685 \\ 0.358424774640213 \end{bmatrix}$	$v_5 = CO$
G_{F_2}	v_1 = Waste $v_3 = O_2$ $v_4 = CO_2$ $v_6 = H_2O$ & other pollutants	$\varphi = \begin{bmatrix} 0.362117053792066 \\ 0.056545143401065 \\ 0.222912655265575 \\ 0.358425147541286 \end{bmatrix}$	$v_3 = O_2$
G_{F_3}	v_1 = Waste $v_4 = CO_2$ $v_6 = H_2O$ & other pollutants	$\varphi = \begin{bmatrix} 0.382867513611615 \\ 0.234264972776769 \\ 0.382867513611615 \end{bmatrix}$	$v_4 = CO_2$
G_{F_4}	v_1 = Waste $v_6 = H_2O$ & other pollutants	unavailable	

The graph dynamics are shown by the deletion of the variable which is regarded as least important during time, t. The explanation on the deletion of the variables is given in Section 3. From the table, Fuel was the first variable to be deleted followed by CO, O_2 and then CO_2 at time t_1, t_2, t_3 and t_4 respectively.

However, the existence of waste and H_2O & other pollutants throughout time t indicates the important of these two variables. Their existence could be explained as follows:

i) Waste

- At the beginning of the incineration process, waste is vital input and therefore initiated the process. The process may not possible without waste.
- At the end of the incineration process, waste is totally transformed into ash but it is still considered as part of waste.

ii) H_2O & other pollutants

- About 23% of waste constitutes of water as shown by Green (1992)[1]. The level of water content in the waste play a significant role in the burning of the waste. The higher the water content in the waste, the more energy needed to incinerate the waste.
- H_2O & other pollutants are one of the by-products of combustion of waste [7].

4. CONCLUSION

We present the implementation of graph dynamic procedure using left Perron vector via transition matrix for a fuzzy graph of FACS. The result shows that Waste and H_2O & other pollutants are the most important variables in the incineration process whereas the other five variables are deleted at each stage consecutively. The dynamic model provides information of by-product and the importance of the variables in the incineration process

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REFERENCES

[1] B. Sabariah, Modelling of Clinical Waste Incinerator Process Using Novel Fuzzy Autocatalytic Set: Manifestation of Mathematical Thinking, PhD Thesis, Universiti Teknologi Malaysia, 2005,

[2] A. Tahir, B. Sabariah, A, Khairil, Journal of Mathematical Chemistry., 47 (2010) ,4, 1263 – 1273.

[3] A.B. Sumarni, A. Tahir, B. Sabariah, Journal of Advances and Applications in Mathematical Sciences., 5(2010)1, 25-38.

[4] A.B. Sumarni, A. Tahir, B. Sabariah, WSEAS Journal of Transactions in Mathematics., 9(2010)8

[5] A.B. Sumarni, A. Tahir,B. Sabariah, Proceedings of the 11th WSEAS Conference on Fuzzy Systems., (2010),139-144.

[6] C. C. Balázs, M. J. Raphaël, D. B. Vincent, LNAI 6331(2010),89-103.

[7] C.R. Bruner, Handbook of Incineration System, New York, USA, (1991).

[8] J. Kemeny, J.L. Snell, Markov random fields and their applications: American Mathematical Society, 1960

[9] K. Costello, Random walks on Directed Graph. retrieved at www.math.ucsd.edu/~phorn/math261/10_19_notes.pdf , 2005

[10] A.B. Sumarni, A Tahir, B. Sabariah, International Journal of Mathematical Sciences. (2010) (accepted for publication)