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Sufficient conditions for new integral transformation

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ABSTRACT

By using the integral transformation for meromorphic functions, an integral transformation on the class A of analytic functions in the unit disk is defined. Some sufficient conditions for this transformation to be in the some known subclasses are derived. Furthermore, new function on the class of meromorphic functions in the punctured open unit disk is introduced. Finally, starlikeness conditions for this function are pointed out.

| analytic function | meromorphic function | integral transformation | starlike | convex | close -to-convex function |

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1. INTRODUCTION

Let Σ denote the class of functions of the form

$$f(z)=\frac{1}{z}+\sum_{n=0}^{\infty}a_n z^n, \qquad (1.1.1)$$

which are analytic in the punctured open unit disk

$$U^*=\{z\in C: 0<|z|<1\}=U\setminus\{0\},$$

where U is the open unit disk $U=\{z\in C:|z|<1\}$.

We say that a function $f\in\Sigma$ is the meromorphic starlike of order α $(0\le\alpha<1)$, and belongs to the class $\Sigma^*(\alpha)$, if it satisfies the inequality:

$$-\Re\left(\frac{zf'(z)}{f(z)}\right)>\alpha.$$

A function $f\in\Sigma$ is the meromorphic convex function of order α $(0\le\alpha<1)$ if f satisfies the following inequality:

$$-\Re\left(\frac{zf''(z)}{f'(z)}+1\right)>\alpha,$$

and we denote this class by $\Sigma_k(\alpha)$.

Let A denoted the class of functions f normalized by $f(z)=z+\sum_{n=2}^{\infty}a_n z^n$, which are analytic in the open unit disc U.

Let $S^*(\alpha)$, $K(\alpha)$, $C(\alpha)$, and UCV denote the subclasses of A consisting of functions which are, respectively, starlike, convex and close-to-convex of order α $(0\le\alpha<1)$, and uniformly convex function. Thus, we have

$$S^*(\alpha)=\left\{f: f\in A \text{ and } \Re\left(\frac{zf'(z)}{f(z)}\right)>\alpha,\quad 0\le\alpha<1;\ z\in U\right\},$$

$$K(\alpha)=\left\{f: f\in A \text{ and } \Re\left(\frac{zf''(z)}{f'(z)}+1\right)>\alpha,\quad 0\le\alpha<1;\ z\in U\right\},$$

$$C(\alpha)=\left\{f: f\in A \text{ and } \Re\left(\frac{f'(z)}{g'(z)}\right)>\alpha,\quad 0\le\alpha<1;\ z\in U,\ g(z)\in K\right\},$$

and

$$UCV=\left\{f: f\in A \text{ and } \Re\left(\frac{zf''(z)}{f'(z)}+1\right)>\left|\frac{zf''(z)}{f'(z)}\right|,\quad z\in U\right\}.$$

In addition, let $N(\beta)$ be the subclass of A consisting of functions f which satisfy the inequality:

$$\Re\left(\frac{zf''(z)}{f'(z)}+1\right)<\beta,\qquad z\in U,\ \beta>1.$$

This class was introduced and studied by (Owa and Srivastava, 2002).

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The study of integral operators finds an important place in the field of Geometric Function Theory not only in the past, even recently. The Alexander transformation was introduced by Alexander (1915). The surprisingly close analytic connection between the well-known two subclasses of the class of univalent functions, namely the class of starlike functions and the class of convex functions was first discovered by Alexander by using his transformation. Later Libera (1965) introduced an integral operator which generalized by Bernardi (1969). Recently, new frontiers of integral operators are designed to stimulate interest among the young researchers and new readers to the area of geometric function theory (cf., e.g., [1, 4-5, 7-11]).

These works motivate some authors to follow similar approach for $f \in \Sigma$ (see [2-3, 6]).

The authors in [2] introduced the following integral Operator

Definition 1.1.1. (Mohammed, Darus, 2010) Let $n \in N$, $\gamma_i > 0$, $i \in \{1,\dots,n\}$. We define the integral operator: $H_n(f_1, f_2, \dots, f_n): \Sigma^n \to \Sigma$ by

$$H_n(f_1, f_2, \dots, f_n)(z) = \frac{1}{z^2}\int_0^z (u\, f_1(u))^{\gamma_1} \dots (u\, f_n(u))^{\gamma_n}\, du.$$

(1.1.2)

For the sake of simplicity, we will write $H_n(z)$ instead of $H_n(f_1, f_2, \dots, f_n)(z)$.

Now using the integral operator $H_n(z)$, we introduce the following new two analytic functions.

Definition 1.1.2. Let $n \in N$, $\gamma_i > 0$, $i \in \{1,\dots,n\}$. *For* $f_i \in \Sigma$, we define the following integral operator,

$$F(z) = z^2 H_n(z) = \int_0^z (u\, f_1(u))^{\gamma_1} \dots (u\, f_n(u))^{\gamma_n}\, du. \qquad (1.1.3)$$

where $H_n(z)$ is the integral operator defined as in (1.1.2).

Remark 1.1.3. From (1.1.2), we have

$$H(z) = \frac{1}{z} + A_0 + A_1 z + \cdots,$$

and (1.1.3) yields,

$$F(z) = z + A_0 z^2 + A_1 z^3 + \cdots, \qquad F \in A. \qquad (1.1.5)$$

Definition 1.1.4. Let $H_n(z)$ be the integral operator defined as in (1.1.2). We define the following function,

$$\Theta(z) = 2H_n(z) + zH_n'(z). \qquad (1.1.6)$$

Remark 1.1.5 From (1.1.2) and (1.1.6), we observe that $\Theta(z) \in \Sigma$, $z \in U^*$.

In order to derive our main results, we have to recall here the following lemmas.

Lemma 1.1.6 (Cho, Owa, 2001) Let $f \in \Sigma$ satisfies $f'(z)f(z) \neq 0$ in U^* and

$$\Re\left(\frac{zf'(z)}{f(z)} - \frac{zf''(z)}{f'(z)}\right) < 2 - \beta, \quad z \in U,$$

then

$$-\Re\left(\frac{zf'(z)}{f(z)}\right) > \frac{1}{3-2\beta}, \quad z \in U,$$

that is $f \in \Sigma^*\left(\frac{1}{3-2\beta}\right)$, *where* $\frac{1}{2} \le \beta < 1.$

Lemma 1.1.7 (Singh, Singh, 1982) If $f \in A$ satisfies

$$\Re\left(\frac{zf''(z)}{f'(z)} + 1\right) < \frac{3}{2}, \quad z \in U,$$

then $f \in S^*$.

Lemma 1.1.8 (Miller, Mocanu, 1985) If $f \in A$ satisfies

$$\left|1 + \frac{zf''(z)}{f'(z)}\right| < 2, \quad z \in U.$$

then $f \in S^*$.

Lemma 1.1.9 (Owa et al. 2002) If $f \in A$ satisfies

$$\Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \frac{3\alpha+1}{2(\alpha+1)}, \; z \in U, \quad 0 \le \alpha < 1,$$

then

$$\Re\{f'(z)\} > \frac{\alpha+1}{2},$$

or equivalently,

$$f \in C\left(\frac{\alpha+1}{2}\right), \; z \in U.$$

Lemma 1.1.10 (Ravichandran, 2002) If $f \in A$ satisfies

$$\left|\frac{zf''(z)}{f'(z)}\right| < \frac{1}{2}, \quad z \in U.$$

then $f \in UCV$.

2. Main results

2.1 The integral operator $F(z)$ on the classes $S^*(\alpha)$, $C(\alpha)$, UCV and $N(\beta)$

In this section, we investigate the conditions under which the integral operator $F(z)$ defined in (1.1.3) to be in the classes S^*, $C(\alpha)$, UCV and $N(\beta)$.

Our first result is the following:

Theorem 2.1.1 For $i \in \{1,2,\dots,n\}$, let $\gamma_i \in R, \gamma_i > 0$ *and*

$$\sum_{i=1}^{n}\gamma_i < \frac{2\beta-3}{4(\beta-1)}, \quad \frac{1}{2} \le \beta < 1. \tag{2.1.1}$$

If $f_i \in \Sigma$ *satisfies*

$$\Re\left(\frac{zf_i'(z)}{f_i(z)} - \frac{zf_i''(z)}{f_i'(z)}\right) < 2-\beta, \quad z \in U, \tag{2.1.2}$$

then $F \in S^*$.

Proof. From (1.1.3) we obtain

$$F'(z) = (zf_1(z))^{\gamma_1}\dots(zf_n(z))^{\gamma_n},$$

and

$$F''(z) = \gamma_1\left(\frac{f_1'}{f_1} + \frac{1}{z}\right) + \dots + \gamma_n\left(\frac{f_n'}{f_n} + \frac{1}{z}\right)F'(z).$$

Therefore

$$\frac{F''(z)}{F'(z)} = \sum_{i=1}^{n}\gamma_i\left(\frac{f_i'(z)}{f_i(z)} + \frac{1}{z}\right)$$

By multiplying the above equation with z yield

$$\frac{zF''(z)}{F'(z)} = \sum_{i=1}^{n}\gamma_i\left(\frac{zf_i'(z)}{f_i(z)} + 1\right)$$

This is equivalent to

$$\frac{zF''(z)}{F'(z)} + 1 = \sum_{i=1}^{n}\gamma_i\left(\frac{zf_i'(z)}{f_i(z)}\right) + \sum_{i=1}^{n}\gamma_i + 1. \tag{2.1.3}$$

Considering (2.1.2), (2.1.3) and applying Lemma 1.1.6, we find

$$\Re\left(\frac{zF''(z)}{F'(z)} + 1\right) < \frac{2(\beta-1)}{2\beta-3}\sum_{i=1}^{n}\gamma_i + 1. \tag{2.1.4}$$

Using the hypothesis (2.1.1) it follows from (2.1.4) that

$$\Re\left(\frac{zF''(z)}{F'(z)} + 1\right) < \frac{3}{2}.$$

Hence by Lemma 1.1.7, we get $F \in S^*$. This completes the proof.

Theorem 2.1.2 *For* $i \in \{1,2,\dots,n\}$, let $\gamma_i \in R$ *and* $\gamma_i > 0$. *If* $f_i \in \Sigma$ *satisfies*

$$\left|\frac{zf_i'(z)}{f_i(z)} + 1\right| < \frac{1}{n\gamma_i}, \tag{2.1.5}$$

then $F \in S^*$.

Proof. From (2.1.3), one get

$$\left|\frac{zF''(z)}{F'(z)} + 1\right| \le \sum_{i=1}^{n}\gamma_i\left|\frac{zf_i'(z)}{f_i(z)} + 1\right| + 1$$

$$= \gamma_1\left|\frac{zf_1'(z)}{f_1(z)} + 1\right| + \dots + \gamma_n\left|\frac{zf_n'(z)}{f_n(z)} + 1\right| + 1,$$

and (2.1.5) yield,

$$\left|\frac{zF''(z)}{F'(z)} + 1\right| < \gamma_1\left(\frac{1}{n\gamma_1}\right) + \dots + \gamma_n\left(\frac{1}{n\gamma_n}\right) + 1 = 2$$

Therefore,

$$\left|\frac{zF''(z)}{F'(z)} + 1\right| < 2.$$

Applying Lemma 1.1.8., we get $F \in S^*$.

Letting $n = 1$, $\gamma_1 = \gamma$ and $f_1 = f$ in Theorem 2.1.2, we have

Corollary 2.1.3 let $\gamma \in R$, $\gamma > 0$. *If* $f \in \Sigma$ *satisfies*

$$\left|\frac{zf'(z)}{f(z)} + 1\right| < \frac{1}{\gamma},$$

then $\int_0^z (uf(u))^{\gamma}\,du \in S^*$.

Letting $\gamma = 1$, in Corollary 2.1.3, we have

Corollary 2.1.4 *If* $f \in \Sigma$ *satisfies*

$$\left|\frac{zf'(z)}{f(z)} + 1\right| < 1$$

then $\int_0^z uf(u)\,du \in S^*$.

Theorem 2.1.5 *For* $i \in \{1,2,\dots,n\}$, let $\gamma_i \in R$ *and* $\gamma_i > 0$. *If* $f_i \in \Sigma$ *satisfies*

$$\Re\left\{\frac{zf_i'(z)}{f_i(z)}\right\} > \frac{\alpha-1}{2(\alpha+1)n\gamma_i} - 1,$$

(2.1.6)

then $F \in C\left(\frac{1+\alpha}{2}\right)$, *where* $0 \le \alpha < 1$.

Proof. From (2.1.3) and (2.1.6), we obtain

$$\Re\left\{\frac{zF''(z)}{F'(z)}+1\right\}=\gamma_1\left(\frac{zf_1'(z)}{f_1(z)}\right)+\cdots+\gamma_n\left(\frac{zf_n'(z)}{f_n(z)}\right)+\sum_{i=1}^{n}\gamma_1+1.$$
$$>\gamma_1\left(\frac{\alpha-1}{2(\alpha+1)n\gamma_1}-1\right)+\cdots$$
$$+\gamma_n\left(\frac{\alpha-1}{2(\alpha+1)n\gamma_n}-1\right)+\sum_{i=1}^{n}\gamma_1+1$$
$$=\frac{3\alpha+1}{2(\alpha+1)}.$$

Hence by Lemma 1.1.9, we get $F\in C\left(\frac{1+\alpha}{2}\right)$.

Letting $n=1,\ \gamma_1=\gamma$ and $f_1=f$ in Theorem 2.1.5, we have:

Corollary 2.1.6 let $\gamma\in R,\ \gamma>0.$ *If* $f\in\Sigma$ *satisfies*

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\}>\frac{\alpha-1}{2(\alpha+1)\gamma}-1,$$

then $\int_0^z (uf(u))^{\gamma}du\in C\left(\frac{\alpha+1}{2}\right),$ *where* $0\le\alpha<1.$

Letting $\gamma=1,$ in Corollary 2.1.6, we have

Corollary 2.1.7 If $f\in\Sigma$ *satisfies*

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\}>-\frac{\alpha+3}{2(\alpha+1)},$$

then $\int_0^z u\,f(u)\,du\in S^*.$

Theorem 2.1.8 *For* $i\in\{1,2,\ldots,n\}$, let $\gamma_i\in R$ *and* $\gamma_i>0.$ *If* $f_i\in\Sigma$ *satisfies*

$$\left|\frac{zf_i'(z)}{f_i(z)}+1\right|<\frac{1}{2n\gamma_i}, \qquad (2.1.6)$$

then $F\in UCV$.

Proof. Using (2.1.3), one get

$$\left|\frac{zF''(z)}{F'(z)}\right|\le\sum_{i=1}^{n}\gamma_i\left|\frac{zf_i'(z)}{f_i(z)}+1\right|$$
$$=\gamma_1\left|\frac{zf_1'(z)}{f_1(z)}+1\right|+\cdots+\gamma_n\left|\frac{zf_n'(z)}{f_n(z)}+1\right|.$$

Then (2.1.6) yield,

$$\left|\frac{zF''(z)}{F'(z)}\right|\le\gamma_1\left(\frac{1}{2n\gamma_1}\right)+\cdots+\gamma_n\left(\frac{1}{2n\gamma_n}\right).$$
$$=\frac{1}{2n}+\cdots+\frac{1}{2n}=\frac{1}{2}.$$

Applying Lemma 1.1.10., we get $F\in UCV$.

Letting $n=1,\ \gamma_1=\gamma$ and $f_1=f$ in Theorem 2.1.8, we have

Corollary 2.1.9 Let $\gamma\in R,\ \gamma>0.$ *If* $f\in\Sigma$ *satisfies*

$$\left|\frac{zf'(z)}{f(z)}+1\right|<\frac{1}{2\gamma},$$

then $\int_0^z (u\,f(u))^{\gamma}du\in UCV.$

Letting $\gamma=1,$ in Corollary 2.1.9, we have

Corollary 2.1.10 *If* $f\in\Sigma$ *satisfies*

$$\left|\frac{zf'(z)}{f(z)}+1\right|<\frac{1}{2},$$

then $\int_0^z u\,f(u)\,du\in UCV.$

Theorem 2.1.11 *For* $i\in\{1,2,\ldots,n\}$, let $\gamma_i\in R$ *and* $\gamma_i>0.$ *If* $f_i\in\Sigma$ *satisfies* (2.1.2), then

$F\in N(\gamma),\ \ \gamma=1+\frac{2(\beta-1)}{2\beta-3}\sum_{i=1}^{n}\gamma_i.$

Proof. From (2.1.3) we have

$$\frac{zF''(z)}{F'(z)}+1=\sum_{i=1}^{n}\gamma_1\left(\frac{zf_i'(z)}{f_i(z)}\right)+\sum_{i=1}^{n}\gamma_i+1.$$

Taking the real part of the above expression, we get

$$\Re\left\{\frac{zF''(z)}{F'(z)}+1\right\}=\sum_{i=1}^{n}\gamma_i\Re\left(\frac{zf_i'(z)}{f_i(z)}\right)+\sum_{i=1}^{n}\gamma_i+1.$$

Since f_i satisfies (2.1.2), then applying Lemma 1.1.6, we find that

$$\Re\left\{\frac{zF''(z)}{F'(z)}+1\right\}<1+\frac{2(\beta-1)}{2\beta-3}\sum_{i=1}^{n}\gamma_i.$$

Since $1+\frac{2(\beta-1)}{2\beta-3}\sum_{i=1}^{n}\gamma_i>1,$ therefore, we get the assertion result.

2.2 Starlikeness of the function $\Theta(z)$

In this section we place conditions for the function $\Theta(z)$ which is defined in (1.1.6), to be in the class $\Sigma^*(\alpha)$.

Theorem 2.2.1 *For* $i\in\{1,2,\ldots,n\}$, *let* $\gamma_i>0,$

$0\le\alpha_i<1,$ *and* $0\le 1-\sum_{i=1}^{n}\gamma_i(1-\alpha_i)<1.$ *If* $f_i\in\Sigma^*(\alpha_i),$

then the function $\Theta(z)$ *given by (1.1.6) belong to* $\Sigma^*(\mu)$, *where* $\mu = 1-\sum_{i=1}^{n}\gamma_i(1-\alpha_i)$.

Proof. A successive differentiation of $H_n(z)$, which is defined in (1.1.2), we get

$$z\Theta(z) = z^2 H_n'(z) + 2zH_n(z) = (z f_1(z))^{\gamma_1} \ldots (zf_n(z))^{\gamma_n}.$$

(2.2.1)

Using (2.2.1), we get

$$\frac{z\Theta'(z)}{\Theta(z)} = \sum_{i=1}^{n}\gamma_i \frac{zf_i'(z)}{f_i(z)} + \sum_{i=1}^{n}\gamma_i - 1. \qquad (2.2.2)$$

Taking the real part of both terms of (2.2.2), we get

$$\Re\left(\frac{z\Theta'(z)}{\Theta(z)}\right) = \sum_{i=1}^{n}\gamma_i\left(\Re\frac{zf_i'(z)}{f_i(z)}\right) + \sum_{i=1}^{n}\gamma_i - 1.$$

Since $f_i \in \Sigma^*(\alpha_i)$, we receive

$$-\Re\left(\frac{z\Theta'(z)}{\Theta(z)}\right) > 1-\sum_{i=1}^{n}\gamma_i(1-\alpha_i).$$

But by the hypothesis, $0 \le 1-\sum_{i=1}^{n}\gamma_i(1-\alpha_i) < 1$. Thus

$$\Theta(z) \in \Sigma^*(\mu),\ \mu = 1-\sum_{i=1}^{n}\gamma_i(1-\alpha_i).$$

Let $n=1,\ \gamma_1=\gamma,\ \alpha_1=\alpha$ and $f_1 = f$ in Theorem 2.2.1, we get

Corollary 2.2.2 *let* $\gamma > 0,\ 0 \le \alpha < 1,$ *and* $0 \le 1-\gamma(1-\alpha) < 1$. *If* $f \in \Sigma^*(\alpha)$, *then the function*

$$\phi(z) = z^{\gamma-1}(f(z))^{\gamma}, \qquad (2.2.3)$$

is in the class $\Sigma^*(\mu),\ \mu = 1-\gamma(1-\alpha)$.

Proof. Differentiating (2.2.3) logarithmically, we have

$$\frac{\phi'(z)}{\phi(z)} = \frac{\gamma-1}{z} + \gamma\frac{f'(z)}{f(z)}.$$

By multiplying the above expression with z, we obtain

$$\frac{z\phi'(z)}{\phi(z)} = \gamma - 1 + \gamma\frac{zf'(z)}{f(z)}.$$

This is equivalent to

$$-\frac{z\phi'(z)}{\phi(z)} = 1-\gamma-\gamma\frac{zf'(z)}{f(z)}. \qquad (2.2.4)$$

Taking the real part of both terms of (2.2.4), we get

$$-\Re\left\{\frac{z\phi'(z)}{\phi(z)}\right\} = 1-\gamma+\gamma\left\{-\Re\frac{zf'(z)}{f(z)}\right\}.$$

Since $f \in \Sigma^*(\alpha),\ 0 \le \alpha < 1,$ we receive

$$-\Re\left\{\frac{z\phi'(z)}{\phi(z)}\right\} = 1-\gamma(1-\alpha).$$

So, $\phi(z)$ is in the class $f \in \Sigma^*(\mu),\ \mu = 1-\gamma(1-\alpha)$.

Theorem 2.2.3 *For* $i \in \{1,2,\ldots,n\}$, *let* $\gamma_i \in R,\ \gamma_i > 1,$ *and* $\sum_{i=1}^{n}\gamma_i \le n+1$. *If* $f_i \in \Sigma^*\left(\frac{1}{\gamma_i}\right)$, *then the function* $\Theta(z)$ *given by (1.1.6) belong to* $\Sigma^*(0)$.

Proof. Since $f_i \in \Sigma^*\left(\frac{1}{\gamma_i}\right)$, then (2.2.2), yield

$$-\Re\left(\frac{z\Theta'(z)}{\Theta(z)}\right) = \gamma_1\frac{1}{\gamma_1} + \cdots + \gamma_n\frac{1}{\gamma_n} - \sum_{i=1}^{n}\gamma_i + 1$$

$$> n+1-\sum_{i=1}^{n}\gamma_i.$$

In accordance with the hypothesis, we obtain

$$-\Re\left(\frac{z\Theta'(z)}{\Theta(z)}\right) > 0,$$

So, Θ is a starlike function.

Theorem 2.2.4 *For* $i \in \{1,2,\ldots,n\}$, *let* $\gamma_i \in R,\ \gamma_i > 0,$ *and* $\sum_{i=1}^{n}\gamma_i \le 1$. *If* $f_i \in \Sigma^*(0)$, *then the function* $\Theta(z)$ *given by (1.1.6) is starlike by order* $1-\sum_{i=1}^{n}\gamma_i$.

Since the proof is similar to the proof of theorem 2.2.3, it will be omitted.

3 CONCLUSION

Integral operators have been of interests to many in recent years and will continue as far as research is concerned. Many new results and properties are obtained with different types and styles of operators being defined.

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