Understanding the Shift of Correlation Structure in Major Currency Exchange Rate

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Received 30 November 2012, Revised 30 January 2012, Accepted 2 April 2012, Available online 28 April 2012

ABSTRACT

In this paper, we study the high dimensional correlation structure of financial market. Correlation structure can be considered as a complex system that relates each variable to the others in terms of correlation. To analyze such complex system, minimum spanning tree is constructed to simplify the network. A case study will be presented and a conclusion will be highlighted.

1. INTRODUCTION

The stability of the covariance structure is a major aspect in multivariate analysis. The importance of covariance matrix stability has been shown in many areas. For example, financial market and real estate industry are among the areas that consider the stability of covariance structure as an important tool. See Eichholtz [1], Schindler [2] and Stephen [3]. The stability of covariance structure has been used to determine the allocation of international real estate securities investments as can be seen in Eichholtz [1]. Besides that, it provides good estimates in the ex-ante modelling process Stephen [3]. In this paper, we focused on the further analysis that should be done if the situation of unstable covariance matrix occurs in financial market. One of the ways to explain why this situation occurred is to monitor the structure of its corresponding correlation matrix. For that purpose, we test the equality of correlation structure, i.e., to test whether there is a shift or not in correlation structure that caused the instability of covariance structure.

We learn from the literature that there exist many different methods available to test that shift. See, for example, Jennrich’s test [2], [4] and [5] and Box M’s test [4]. Here, we use the most commonly used method, namely, Jennrich’s test. In case there is a shift in correlation structure, minimum spanning tree (MST) approach will be used to understand that shift and to determine the root causes of this situation.

The rest of the paper is organized as follows. In the next section, we present the methodology of Jennrich’s test and then, in Section 3, we use MST to analyze which variables that causes the shift in correlation structure. This paper will be closed with the conclusion in the last section.

2. TEST THE SHIFT IN CORRELATION MATRICES

Correlation matrices of foreign exchange rate time series are analyzed for 55 world currencies, retrieved from Pacific Exchange Rate Service (http://fx.sauder.ubc.ca/EUR/analysis.html). Let P1 and P2 are the correlation matrices of first quarter in year 2000 and second quarter in 2000, respectively. The first quarter consists of the data from January 2000 until April 2000 and second quarter consists of the data from May 2000 until July 2000. To test the hypothesis $H_0: P_1 = P_2$ versus $H_1: P_1 \neq P_2$, we use Jennrich’s statistic [5],

\[
J = \frac{1}{2} \text{tr}(Z^T\theta \gamma^2 \gamma S^2 \gamma \theta^T Z) \tag{1}
\]

where

(i) \( Z = \bar{R}^{-1}(R_1 - R_2) \left( \frac{n_1 n_2}{n_1 + n_2} \right) \),

(ii) \( \bar{R} = \frac{\bar{R}_1 + \bar{R}_2 n_2}{n_1 + n_2} \) is the pooled correlation matrix.

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(iii) \( S = (s_{ij}) = (\delta_{ij} + \overline{\tau}^y) \).

In (1), \( R_1 \) and \( R_2 \) represent sample correlation matrix of first quarter in year 2000 and second quarter in year 2000, respectively; \( \delta_{ij} \) is the Kronecker delta, i.e., \( \delta_{ij} = 1 \) for \( i = j \), otherwise \( \delta_{ij} = 0 \) and \( \overline{\tau}^y \) are the elements of \( \overline{R}^{-1} \), the inverse of \( \overline{R} \). Jennrich [5] shows that the statistical test (1) is asymptotically \( \chi^2 \) distributed with degree of freedom \( k = (p(p-1)/2) \) where \( p \) is dimension of the correlation matrix. Therefore, \( H_0 : P_1 = P_2 \) is rejected at level of significance \( \alpha \) if \( J \) exceeds \( \chi^2_{\alpha,k} \), the \((1-\alpha)\)-th quantile of \( \chi^2 \) distribution.

Based on the above tables, we obtain \( J = 1562.303 \) and \( \chi^2_{\alpha,k} = 1396.5 \) for \( \alpha = 0.05 \). Evidently, we reject the null hypothesis which means that those two correlation matrices are shifted. In the next section, we analyze how those correlation matrices differ to each other and which variables are responsible to that situation.

3. INTERPRETATION & DISCUSSION

Since the null hypothesis is rejected, by using MST [6], we analyze those two sample correlation matrices to explain why those two population correlation matrices are shifted. This analysis, in general, is started by transforming correlation matrix into distance matrix [6]. Based on distance matrix, we construct a MST, as suggested Kruskal Jr [7], by using Kruskal’s algorithm provided in Matlab. From MST, we construct the adjacent matrix to obtain the network topology of all variables. To visualize that MST, we use Pajek software [8] and [9]. The interpretation of that network will be delivered by using the degree centrality measure [10] and [11].

3.1 Minimum spanning tree

MST is a subgraph that connect all the currencies (nodes) whose total weight, i.e., total distance is minimal. Figure 1 shows the corresponding MST with Figure 1.0 for first quarter in year 2000 and Figure 1.0(b) for second quarter in year 2000. These figures show the most important relationship among all currencies in terms of MST.

![Fig. 1 MST of first quarter in year 2000 (a) and second quarter in year 2000 (b)](image)

Figure 1.0 shows that the currencies in the circle are the major world currencies in year 2000, mentioned by Madueme [12]. They are United Kingdom Pounds (GBP), Canadian Dollar (CAD), Euro (EUR), India Rupees (INR) and Japanese Yen (JPY). In terms of MST, for the first quarter, we learn that all the major currencies do not influenced the other currencies except for INR and EUR. While, for the second quarter, only EUR do influenced other currency. This conditions make those two correlation matrices are different to each others.

3.2 Degree centrality

Degree centrality indicates the connectivity of currencies (nodes). It provides information on how many number of edges incident upon a given node. It can be used to measure the importance of any particular nodes. The application of degree centrality can be found in many areas of research. See, for example, in E-Commerce [10] and social network [13]. This measure is defined by [14],

\[
C_{\text{Degree}}(N_i) = \sum_{j=1}^{p} a_{ij}
\]

where \( a_{ij} \) is the element in \( i \)-th row and \( j \)-th column of an adjacent matrix and \( N_i \) is the \( i \)-th node.

In Table 1.0, we present the degree centrality of each currency for first and second quarters in year 2000. Based on this table, a more attractive MST can be constructed. This is showed in Figure 2.0 where the size of each node corresponds to its degree centrality.
From Table 1.0 and Figure 2.0, for the first quarter, USD has the highest number of connections, i.e., eight connections in network. While, for the second quarter, SDD has the highest number of connections, i.e., seven connections in network. The higher the number of connections the more influential of a particular variable. However, to interpret this degree centrality, we will consider the first largest currency until the fifth largest currencies of the degree centrality which are appear in both quarters. They are USD, SAR, SDD, DKK, HKD, MYR and SGD.
4. CONCLUSION

From the MST in Figure 1.0, for the first quarter, we learn that only two of the major world currencies which are INR and EUR do influenced other currency and for the second quarter, only EUR do influenced other currency. This means, in January 2000 until July 2000, the five major world currencies do not give much impact to the others currencies.

According to degree centrality, the number of currencies directly related to SDD increases from first to second quarters while those that relate with USD, SAR, DKK, MYR and SGD decrease. These currencies are responsible for the inequality of first and second quarters in terms of degree centrality.

In conclusion, India Rupees (INR), Euro (EUR), United States Dollar (USD), Saudi Arabian Riyal (SAR), Sudanese Dinar (SDD), Danish Krone (DKK), Malaysian Ringgit (MYR) and Singapore Dollar (SGD) are the currencies that responsible to the shift of correlation matrices of foreign exchange rate in first and second quarters of year 2000.

ACKNOWLEDGEMENT

We acknowledge financial support from the Ministry of Higher Education, via FRGS vote number 4F013 and Research University Grant (RUG) vote number Q.J13000.7126.02H18. The authors would like to thank Universiti Teknologi Malaysia for the opportunity to do this research. The first author also thanks Universiti Malaysia Pahang and Ministry of Higher Education for the scholarship. Special thanks go to the reviewers for the comments and suggestions.

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