MJFAS MALAYSIAN JOURNAL OF FUNDAMENTAL AND APPLIED SCIENCES
PRINT ISSN: 2289-5981 | ONLINE ISSN: 2289-599X

RESEARCH ARTICLE

The role of an option-implied distribution in improving an asset allocation model

Hafizah Bahaludin, Mimi Hafizah Abdullah*

Department of Computational and Theoretical Sciences, Kulliyyah of Science, International Islamic University Malaysia, Bandar Indera Mahkota Campus, Jalan Sultan Ahmad Shah, 25200 Kuantan, Pahang, Malaysia

* *Corresponding author: mimihafizah@iium.edu.my*

Article history
Received 19 December 2018
Revised 13 February 2019
Accepted 13 March 2019
Published Online 2 February 2020

Abstract

The objective of this paper is to extend the information embedded in option-implied distribution to asset allocation model. This paper examines whether a parameter estimated from an option-implied distribution can improve a minimum-variance portfolio which consists of many risky assets. The option-implied distribution under a risk-neutral assumption is called risk-neutral density (RND) whereas a risk-world density (RWD) is calculated by incorporating a risk-premium. The computation of option-implied distributions is based on the Dow Jones Industrial Average (DJIA) index options and its constituents. The data covers the period from January 2009 until December 2015. Portfolio performance is evaluated based on portfolio volatility and Sharpe ratio. The performance of a portfolio based on an option-implied distribution is compared to a naive diversification portfolio. The empirical evidence shows that for a portfolio based on an option-implied distribution, the volatility of the portfolio is reduced and the Sharpe ratio is increased.

***Keywords*:** Option prices, option-implied distribution, asset allocation model

INTRODUCTION

Asset allocation problem discusses on how investors should allocate their capital on different assets so that their investments provide them with higher return with respect to risk. Markowitz model [1] is widely used among market participants to solve asset allocation problems. Researchers endeavour to increase the performance of Markowitz model by expanding or introducing a new parameter or factor. However, literature has pointed out a few weaknesses of an asset allocation model based on this model. Firstly, the performance of the portfolio does not perform satisfactorily in terms of portfolio volatility, Sharpe ratio, certainty-equivalent return, or turnover as compared to a naïve diversification portfolio [2]. Secondly, parameter estimation based on a historical asset price leads to an estimation error and affect the performance of an optimal portfolio [2]. Generally, parameter estimation of a portfolio model which is based on historical asset prices is considered as a backward-looking data.

Therefore, researchers are motivated to propose an alternative way to overcome the weaknesses stated above. Instead of using a backward-looking information, researchers shift their attentions to use a forward-looking information such as option prices, in order to calculate the parameters of an asset allocation model [3].

An option is a financial contract that gives the right but not an obligation to sell or to buy an underlying asset at a specified price at a maturity date. Option price has a characteristic of a forward-looking information in which the payoff function depends on the underlying asset price in the future. Thus, the distribution of option prices gives an overview of how the underlying asset prices are evolved. Due to this reason, researchers [4–6] believe that the information contains in an implied distribution is essentially a forward-looking information and provides a more accurate estimation of a parameter such as volatility. A distribution of option prices provides an expectation of market participants in the future. Previous literature highlighted that volatility extracted from a forward-looking information is better than that of a backward-looking information [7]. Thus, moments estimation such as mean, volatility, and covariance which are derived from a forward-looking information are expected to be accurate and can be used in asset allocation models.

Option-implied distributions that are extracted from option prices are known as risk-neutral density (RND) and risk-world density (RWD). RND is calculated under a risk-neutral assumption of the investors' preference. In contrast, RWD is calculated by incorporating the risk-premium of investors. Option-implied distribution is useful in forecasting the future price of an underlying asset [8–13], monetary policy purposes [14,15], and risk aversion [10,11,16,17]. However, little attention has been given to the application of an option-implied distribution to a portfolio selection. [3] estimated the parameter of asset allocation model based on an RND estimation. A cubic smoothing spline is used to estimate the RNDs and the RWD estimation, the same function used by [16]. [3] developed a portfolio with one risk-free and one risky asset.

This paper is different with that of [3] in which a fourth-order polynomial is used in RND estimation and a calibration function is used to estimate the RWD. The main contribution of this paper is to enhance the performance of a portfolio with many risky assets by extending the information embedded in an option-implied distribution. The performance of a portfolio based on option-implied distribution is then compared to a naïve diversification portfolio. To the best of our knowledge, this study is the first to extract the estimation parameter from option-implied distributions (RND, RWD-P, RWD-NP) and apply to a portfolio selection model. Plus, this paper differs from [3] in terms of the approach used in the RND and RWD estimations. In addition, this study constructs a portfolio which contains many risky assets as compared to that of [3], in which they used a portfolio of only two assets.

Data

The data used in this study were based on the historical prices of Dow Jones Industrial Average (DJIA) index and option prices of Dow Jones Industrial Average (DJIA) index option. Option prices were obtained from Optiondata.net while historical prices were obtained from Datastream database. The dataset covered the period from January 2009 until December 2015. The London Interbank Offer Rate (LIBOR) for one-month was used as the interest rate.

There were several filtering criteria imposed to the dataset before obtaining the final dataset. Firstly, this study only used out-of-the-money (OTM) of call and put options due to the liquidity reason. Secondly, this study only considered options with one-month maturity by referring to the Chicago Board Options Exchange (CBOE) calendar. Thirdly, options with the bid or ask quotes equal to zero were eliminated and that the ask quotes were greater than the bid quotes. Fourthly, option prices that violated the arbitrage condition were excluded. Lastly, only options with the lowest delta value less than or equals to 0.25 and the highest delta value equal or greater than 0.75 were used.

Generally, there are 30 companies listed in DJIA index. However, only 22 companies were constantly listed in DJIA index from the period January 2009 until December 2015 as depicted in Table 1.

Table 1 Components of DJIA index that consistently listed for the period 2009 until 2015.

	Name	Mnemonic
1	AMERICAN EXPRESS	U:AXP
2	BOEING	U:BA
3	CATERPILLAR	U:CAT
4	CHEVRON	U:CVX
5	EI DU PONT DE NEMOURS	U:DD
6	WALT DISNEY	U:DIS
7	GENERAL ELECTRIC	U:GE
8	HOME DEPOT	U:HD
9	INTERNATIONAL BUS.MCHS.	U:IBM
10	INTEL	@INTC
11	JOHNSON & JOHNSON	U:JNJ
12	JP MORGAN CHASE & CO.	U:JPM
13	COCA COLA	U:KO
14	3M	U:MMM
15	MERCK & COMPANY	U:MRK
16	MICROSOFT	@MSFT
17	PFIZER	U:PFE
18	PROCTER & GAMBLE	U:PG
19	UNITED TECHNOLOGIES	U:UTX
20	VERIZON COMMUNICATIONS	U:VZ
21	WAL MART STORES	U:WMT
22	EXXON MOBIL	U:XOM

Table 2 shows the summary statistics of the DJIA index and stock options. After filtering procedures, we obtained 83 sets of options with a constant one-month maturity from the year 2009 until 2015. The total option used for index options is 1,677 options and for stock options is 26,926 options. The stock options are the options of the 22 companies that are listed in the DJIA index.

Table 2 Summary statistics of options for DJIA index and its constituents.

	DJIA	
	Index options	**Stock options**
No of Call	943	9911
No of Put	734	17015
Total	1677	26926
Underlying asset price ($)		
Minimum	71.15	8.85
1st quartile	107.54	41.2
Median	127.42	64.55
Mean	131.88	70.78
3rd quartile	162.95	91.02
Maximum	182.99	213.21
Strike ($)		
Minimum	55	2.5
1st quartile	105	38
Median	126	61
Mean	130.1	67.69
3rd quartile	160	89
Maximum	193	235
Midprice ($)		
Minimum	0.02	0.015
1st quartile	0.26	0.06
Median	0.55	0.155
Mean	0.764	0.408
3rd quartile	1.085	0.475
Maximum	5.55	32.42
Implied Volatility		
Minimum	0.038	0.012
1st quartile	0.117	0.186
Median	0.16	0.251
Mean	0.187	0.294
3rd quartile	0.223	0.341
Maximum	0.679	0.7

METHODOLOGY

The analysis of this study consists of four stages: calculation of RND, calculation of RWD, calculation of mean, variance and covariance, and application of the estimated parameter in an asset allocation model. This subsection explains the procedure for each stage of calculation.

Risk-neutral density

Price of a call option is given by the discounted value of expected payoff on the maturity date, T with respect to the risk-neutral probability

$$C(T,K) = e^{-rT}\int_0^{\infty}\left(S_T - K,0\right)f\left(S_T\right)dS_T \qquad (1)$$

where C is the European call price; S_T is the price of the underlying asset; K is the strike price; r is the continuously compounded risk-free rate; and $f(S_T)$ is the RND function. The RND can be obtained by the second derivative of equation (1)with respect to strike prices [18]. Numerically, it can be approximated by using the following equation:

$$f(K_n) \approx e^{-rT} \frac{C_{n+1} - 2C_n + C_{n-1}}{(\Delta K)^2} \qquad (2)$$

Equation (2) can be applied based on the assumption that the option prices are continuum. However, this assumption does not exist in the real market. Thus, interpolation and extrapolation techniques are used to create continuum option prices. This paper uses a fourth-order polynomial to interpolate the option prices using the seminal works of [19]. The pseudo-points are added from the highest and lowest strike prices to extrapolate the option prices in a horizontal manner [8].

The procedures to estimate the RNDs are referred from [19]. Firstly, implied volatility of option prices is calculated by using Black-Sholes-Merton (BSM) model. The bisection method is used to calculate the implied volatility numerically. Secondly, the fourth-order polynomial interpolation takes place in the implied-volatility-strike prices space. Thirdly, 5000 points are evaluated by using an implied volatility function and are converted to call prices by using BSM model. Finally, the RNDs are obtained by using equation (2). Note that, the BSM model is only a medium to estimate the RNDs without imposing the assumption of this model.

Risk-world density

The RNDs estimation obtained is adjusted for risk premium. This study uses parametric and non-parametric calibrations to obtain the RWDs estimation.

The general framework to adjust the RNDs to RWDs can be explained as follows. RND is represented by Q and RWD is represented by P.

Let $f_Q(x)$ and $F_Q(x)$ be the risk-neutral density and cumulative distribution function of S_T, respectively, while S_T is the price of an underlying asset at maturity, T. A random variable, U is defined as $U = F_Q(S_T)$ and $C(u)$ is the calibration function. Generally, a real-world cumulative distribution function, $F_P(x)$, and a real-world density function, $f_P(x)$, can be expressed as follows:

$$F_P(x) = C(F_Q(x)), \qquad (3)$$

$$f_P(x) = \frac{dF_P(x)}{dx} = \frac{dC(F_Q(x))}{dx} = \frac{dC}{dF_Q}\frac{dF_Q}{dx} = c(F_Q(x)) f_Q(x). \qquad (4)$$

This paper uses parametric and non-parametric calibrations to calibrate the RNDs into RWDs estimation. The parametric calibration uses a beta distribution as a calibration function as recommended by [20]. The beta distribution has an advantage in which it allows to have a different shape to accurately estimate the RND. The calibration function based on a beta distribution can be defined as follows:

$$C(u) = \frac{1}{B(\alpha,\beta)} \int_0^u h^{\alpha-1}(1-h)^{\beta-1}\, dh, \qquad (5)$$

where $$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Based on equations (4) and (5), the relationship between RND and risk-world density by using parametric calibration (RWD-P) can be expressed as follows:

$$f_P(x) = \frac{F_Q(x)^{\alpha-1}(1-F_Q(x))^{\beta-1}}{B(\alpha,\beta)} f_Q(x). \qquad (6)$$

The maximum likelihood estimation is applied to estimate the parameters, α and β.

The use of non-parametric calibration is referred from [11]. There are four steps to obtain the risk-world density by using parametric calibration (RWD-NP) by using the non-parametric calibration. Firstly, extract the cumulative risk-neutral probability from the underlying asset at option's maturity, $u_t = F_Q(S_T)$. Secondly, convert u_t into $h_t = \Phi^{-1}(u_t)$, where $\Phi(u_t)$ is the cumulative distribution function of the standard normal density. Finally, the Gaussian kernel density is used to smooth the series of h_t. Equations (7) and (8) represent the Gaussian kernel density and cumulative distribution of kernel density, respectively. The computation of bandwidth is referred from the [21] where $W = 0.9\sigma_h / N^{0.2}$.

$$g(h) = \frac{1}{NW}\sum_{t=1}^{N} \phi\left(\frac{h-h_t}{W}\right), \qquad (7)$$

$$G(h) = \frac{1}{N}\sum_{t=1}^{N} \Phi\left(\frac{h-h_t}{W}\right). \qquad (8)$$

Based on Equations (4) and (7), the relationship between RND and RWD-NP can be expressed as follows :

$$f_p(x) = \frac{f_Q(x) g(h)}{\phi(h)}. \qquad (9)$$

Option-implied mean, variance, and covariance

The computations of mean and variance of the option-implied distribution are referred from Figlewski [19].

Let x be a continuous random variable with density $f(x)$. Let $y = g(x)$ be the one-to-one transformation of x such that the derivative of $x = g^{-1}(y)$ with respect to y is continuous. The $y = g(x)$ is a continuous random variable with density

$$f_Y(y) = \left|\frac{d}{dy} g^{-1}(y)\right| f_X(g^{-1}(y)) \qquad (10)$$

where $g^{-1}(y)$ is the inverse function.

The calculation of mean and variance of returns that derived from RND, RWD-P, and RWD-NP can be calculated as follows:

a) Risk-neutral density (RND)

$$f_r^{RND}(r) = S_T \quad f_X^{RND}(S_T), \qquad (11)$$

$$\mu^{RND}(r) = \int_{-\infty}^{\infty} r f_r^{RND}(r)\,dr, \qquad (12)$$

$$\text{var}^{RND}(r) = \int_{-\infty}^{\infty} (r \quad \mu^{RND})^2 f_r^{RND}(r)\,dr. \qquad (13)$$

b) Risk-world density using parametric calibration (RWD-P)

$$f_r^{RWD-P}(r) = S_T \quad f_X^{RWD-P}(S_T), \qquad (14)$$

$$\mu^{RWD-P}(r) = \int_{-\infty}^{\infty} r f_r^{RWD-P}(r)\,dr, \qquad (15)$$

$$\text{var}^{RWD-P}(r) = \int_{-\infty}^{\infty} (r \quad \mu^{RWD-P})^2 f_r^{RWD-P}(r)\,dr. \qquad (16)$$

c) Risk-world density using non-parametric calibration (RWD-NP)

$$f_r^{RWD-NP}(r) = S_T \quad f_X^{RWD-NP}(S_T), \qquad (17)$$

$$\mu^{RWD-NP}(r) = \int_{-\infty}^{\infty} r f_r^{RWD-NP}(r)\,dr, \qquad (18)$$

$$\text{var}^{RWD-NP}(r) = \int_{-\infty}^{\infty} (r \quad \mu^{RWD-P})^2 f_r^{RWD-NP}(r)\,dr. \qquad (19)$$

Option-implied covariance

The option-implied covariance is calculated using the seminal works of [22] and [23] by combining the historical correlation and option-implied volatility. The variance of a portfolio can be written as

$$\sigma_{P,t}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t} w_{j,t} \sigma_{i,t} \sigma_{j,t} \rho_{ij,t} \quad (20)$$

where $w_{i,t}$ is the weight, N is the number of stocks, and $\sigma_{i,t}$ is the volatility of stock i at time t. While, $\rho_{ij,t}$ represents the pair-wise correlation between stock i and stock j and the $\sigma_{P,t}^2$ is the variance of the portfolio at time t. Assume that the volatilities and the weight are given, the only parameter that needs to be estimated is the pair-wise correlation among the stocks, $\hat{\rho}_{ij,t}$. [23] calculated the implied-correlation matrix with the assumption that all pairwise correlations are allowed to be different. [23] showed that the relationship between historical and expected correlation by single fixed proportions can be represented as follows:

$$\hat{\rho}_{ij,t} = \rho_{ij,t} - \alpha\left(1-\rho_{ij,t}\right). \quad (21)$$

When substitute equation (21) into equation (20), it will yield

$$\sigma_{P,t}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t} w_{j,t} \sigma_{i,t} \sigma_{j,t} \left(\rho_{ij,t} - \alpha\left(1-\rho_{ij,t}\right)\right). \quad (22)$$

The correlation between two stocks is derived from one-year rolling windows of historical asset prices. The fixed proportions, α, can be computed by equation (23).

$$\alpha_t = \frac{\sigma_{P,t}^2 - \sum_{i=1}^{N-1}\sum_{j \neq i}^{N} w_{i,t} w_{j,t} \sigma_{i,t} \sigma_{j,t} \rho_{ij,t}}{\sum_{i=1}^{N-1}\sum_{j \neq i}^{N} w_{i,t} w_{j,t} \sigma_{i,t} \sigma_{j,t} (1-\rho_{ij,t})}. \quad (23)$$

The computation for each day weight uses closing market capitalization of all current index components from the previous day. According to [22], the implied covariance, Σ, can be estimated using a diagonal matrix G of standard deviation and a correlation matrix, ψ, such that

$$\Sigma = G\psi G. \quad (24)$$

Asset allocation strategies

Naïve diversification portfolio is constructed based on an equally amount of wealth $(1/N)$ across all N available stocks. This portfolio is considered as the benchmark because it exhibits a good performance portfolio even though it does not rely on any optimization model [2,22]. The benchmark portfolio uses 1/N weightage portfolios which are being calculated based on 60 months and 60 days rolling windows.

This paper employs a minimum-variance strategy to construct a portfolio with many risky assets. This strategy assumes that the expected return of each stock is the same. The minimum-variance strategy is achieved when the minimum variance of portfolio return is obtained with respect to the weight and is equal to one. The construction of a portfolio based on a minimum-variance strategy is based on two conditions: short-selling is allowed and short selling is not allowed.

Denote that $w \in R^N$ is the vector of portfolio weights invested in stocks, $\Sigma \in R^{N\times N}$ is the estimated covariance matrix. A minimum-variance strategy with short selling is allowed can be expressed as

$$\begin{aligned} &\min_w w^T \Sigma w \\ &s.t \quad w^T 1 = 1 \qquad i = 1,2,\ldots,N \end{aligned} \quad (25)$$

For a minimum-variance strategy with short selling is not allowed can be expressed as

$$\begin{aligned} &\min_w w^T \Sigma w \\ &s.t \quad w^T 1 = 1 \\ &\qquad w_i \geq 0 \qquad i = 1,2,\ldots,N \end{aligned} \quad (26)$$

The performance of the portfolio is evaluated based on portfolio's volatility and Sharpe ratio. Sharpe ratio can be calculated as follows:

$$SR = \frac{\hat{\mu}}{\hat{\sigma}} \quad (27)$$

where $\hat{\mu}$ is the return of the portfolio and $\hat{\sigma}$ is the volatility of a portfolio.

RESULTS AND DISCUSSION

This section presents the empirical findings of the performance of a portfolio based on the option-implied distribution (RND, RWD-P and RWD-NP) and is compared to a naïve portfolio. The performance of a naïve portfolio which uses 60 months and 60 days rolling windows. The portfolio volatility and Sharpe ratio are used as the benchmarks of the portfolio performance. A portfolio is considered a better performance than the other portfolio if it has a low volatility and a high value of Sharpe ratio.

The performance measurements are identified as the average of the observed monthly means (expected return of portfolio), $\hat{\mu}$, standard deviation of the portfolio, $\hat{\sigma}$, and Sharpe ratio (SR). The values in the parentheses show the p-value of a one-sided t-test for higher mean, lower standard deviation, and higher Sharpe ratio in comparison with that of the benchmark strategies. The performance of minimum-variance strategies which are based on the option-implied distribution when short-selling is allowed is presented in Table 3. The first parenthesis represents the comparison with a portfolio that uses 60 days rolling window and the second parenthesis is compared with a portfolio that uses 60 months rolling windows.

Generally, a portfolio based on an option-implied distribution provides a statistical significance difference at a 5 % level of significance compared with the benchmark portfolio. Portfolio volatilities based on RND, RWD-P, and RWD-NP moments are slightly lower than that of the benchmark portfolio value. Yet, the value of Sharpe ratio based on option-implied distribution portfolio is lower than that of the benchmark. In details, the performance of a portfolio based on RND is better than the performance of portfolios based on RWD-P and RWD-NP in terms of Sharpe ratio. However, a portfolio based on RWD-P provides the lowest volatility as compared to other portfolios.

Table 3 Performance of minimum-variance portfolio with short selling is allowed.

	$\hat{\mu}$	$\hat{\sigma}$	SR
60 days	-0.004361	0.07796	-0.055939
60 months	-0.005045	0.095877	-0.05262
RND	-0.000826	0.007048	-0.108065
	(0.000)	(0.000)	(0.970)
	(0.000)	(0.000)	(0.990)
RWD-P	-0.00307	0.00666	-0.51896
	(0.000)	(0.000)	(0.430)
	(0.000)	(0.000)	(1.000)
RWD-NP	-0.0008	0.0071	-0.1957
	(0.000)	(0.000)	(0.9650)
	(0.000)	(0.000)	(0.9070)

The performance of minimum-variance strategies that are based on option-implied distribution when short-selling is not allowed is presented in Table 4. Generally, a portfolio based on option-implied distribution has statistical significance difference compared with the benchmark portfolio. It is apparent from this table that the portfolios based on RND, RWD-P, and RWD-NP perform better with a higher expected return, lower volatility, and higher Sharpe ratio when compared with the benchmark portfolio. The results suggest that the performance of the portfolio improves when short sale constraints is imposed. This finding is consistent with [24] in which the prohibiting short sale enhances the performance of a portfolio. In details, it is apparent that the expected return of portfolios based on RWD-P and RWD-NP moments have slightly higher values than that of the portfolio based on RND moments. Portfolio based on RWD-P has the lowest volatility of 1.326 %, followed by the portfolio based on RWD-NP 1.377 %, and the portfolio based on RND has the highest volatility, 1.61%. The portfolios based on RWD-P and RWD-NP have higher Sharpe ratios than that of the portfolio based on RND moments.

Table 4 Performance of minimum-variance portfolio with short selling is not allowed.

	$\hat{\mu}$	$\hat{\sigma}$	SR
60 days	-0.004361	0.07796	-0.055939
60 months	-0.005045	0.095877	-0.05262
RND	0.0008847	0.0162725	0.0580429
	(0.000)	(0.000)	(0.000)
	(0.000)	(0.000)	(0.000)
RWD-P	0.0011962	0.0132608	0.082678
	(0.000)	(0.000)	(0.000)
	(0.000)	(0.000)	(0.000)
RWD-NP	0.0013806	0.0137706	0.0841527
	(0.0000)	(0.0000)	(0.000)
	(0.0000)	(0.0000)	(0.000)

The analysis of asset allocation by using option-implied moments which is based on a portfolio when short sales are not allowed is depicted in Table 5. Figure 1 illustrates the information contains in Table 5. The weightage of portfolio based on RND for thirteen companies are: DIS (1.11 %), GE (24.61 %), HD (0.36 %), INTC (9.11 %), JNJ (4.01 %), KO (9.84 %), MRK (0.58 %), MSFT (1.97 %), PFE (34.66 %), PG (1.80 %), VZ (8.47 %), and WMT (3.48 %). The portfolio based on RWD-P has weightage in fifteen companies as follows: DD (0.04%), DIS (1.03 %), GE (26.86 %), HD (0.53 %), INTC (10.01 %), JNJ (2.51 %), KO (6.31 %), MRK (0.63 %), MSFT (1.00 %), PFE (36.74 %), PG (1.71 %), UTX (0.08 %), VZ (11.90 %), and WMT (0.64 %). For portfolio based on RWD-NP, the fourteen companies and their weights are: DD (0.05 %), DIS (1.11 %), GE (27.77 %), HD (0.37 %), INTC (9.48 %), JNJ (3.93 %), KO (7.12 %), MRK (0.58 %), MSFT (1.99 %), PFE (36.90 %), PG (1.26 %), VZ (9.21 %), and WMT (0.23 %). The portfolios based on RND, RWD-P, and RWD-NP allocate the highest proportions to the PFE company.

CONCLUSION

This study constructs portfolios based on option-implied moments namely RND, RWD-P, and RWD-NP which consist of DJIA index components. The objective of this study is to extend the estimation parameter from option-implied distribution to asset allocation model. In addition, this study examines the performance of these portfolios and comparison is made with a naïve portfolio. The empirical results indicate that the performance of a portfolio based on option-implied moments has a significant difference when compared with the naïve portfolio. The performance of a portfolio based on option-implied distribution improves when short-sale constraints are imposed. In details, portfolios based on RWD-P and RWD-NP provide better performance with lower volatility and higher Sharpe ratio when compared with RND.

Table 5 Average weights for a minimum-variance portfolio with short-selling constraints according to companies.

Company	RND	RWD-P	RWD-NP
AXP	0.0000	0.0000	0.0000
BA	0.0000	0.0000	0.0000
CAT	0.0000	0.0000	0.0000
CVX	0.0000	0.0000	0.0000
DD	0.0000	0.0004	0.0005
DIS	0.0111	0.0103	0.0111
GE	0.2461	0.2686	0.2777
HD	0.0036	0.0053	0.0037
IBM	0.0000	0.0000	0.0000
INTC	0.0911	0.1001	0.0948
JNJ	0.0401	0.0251	0.0393
JPM	0.0000	0.0000	0.0000
KO	0.0984	0.0631	0.0712
MMM	0.0000	0.0000	0.0000
MRK	0.0058	0.0063	0.0058
MSFT	0.0197	0.0100	0.0199
PFE	0.3466	0.3674	0.3690
PG	0.0180	0.0171	0.0126
UTX	0.0000	0.0008	0.0000
VZ	0.0847	0.1190	0.0921
WMT	0.0348	0.0064	0.0023
XOM	0.0000	0.0000	0.0000

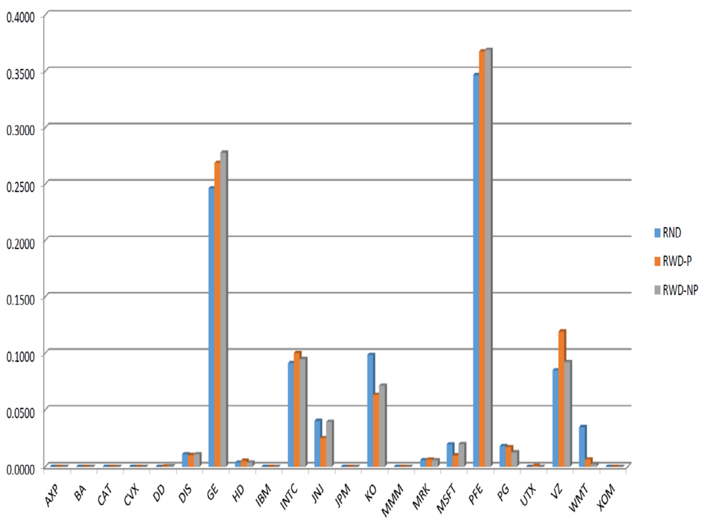

Fig. 1 Average weights for a minimum-variance portfolio with short-selling constraints according to companies.

ACKNOWLEDGEMENT

The authors thank the Ministry of Higher Education Malaysia (MOHE) under Fundamental Research Grant Scheme (FRGS15-191-0432) for financial support provided.

REFERENCES

[1] Markowitz, H. 1952. Portfolio Selection. *J. Finance* **7,** 77–91.

[2] DeMiguel, V., Garlappi, L., and Uppal, R. 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *Rev. Financ. Stud.* **22,** 1915–1953.

[3] Kostakis, A., Panigirtzoglou, N., and Skiadopoulos, G. 2011. Market timing with option-implied distributions: A forward looking approach *Manage. Sci.* **57,** 1231–1249.

[4] Bu, R., and Hadri, K. 2007. Estimating option implied risk-neutral densities using spline and hypergeometric functions AND. *Econom. J.* **10,** 216–244.

[5] Jackwerth, J. C. 1999. Option-implied risk-neutral distributions and implied binomial trees: A literature review. *J. Deriv.* **7,** 66–82.

[6] Aït-sahalia, Y., and Lo, A. W. 1998. Nonparametric estimation of state-price. *J. Finance* **53,** 499–547.

[7] Poon, A. S., and Granger, C. W. J. 2003 Forecasting volatility in financial markets: A review. *J. Econ. Lit.* **4,** 478–539.

[8] Bliss, R. R., and Panigirtzoglou, N. 2002. Testing the stability of implied probability density functions. *J. Bank. Financ.* **26,** 381–422.

[9] Esben, H., and Tsiaras, L. 2011. Density forecasts of crude-oil prices using option-implied and ARCH-type models. *J. Futur. Mark.* **31,** 727-754.

[10] Ivanova, V., and Puigvert Gutiérrez, J. M. 2014. Interest rate forecasts, state price densities and risk premium from Euribor options. *J. Bank. Financ.* **48,** 210–223.

[11] Shackleton, M. B., Taylor, S. J., and Yu, P. 2010. A multi-horizon comparison of density forecasts for the S&P 500 using index returns and option prices. *J. Bank. Financ.* **34,** 2678–2693.

[12] Kliber, P. 2014. Estimation of risk neutral measure for polish stock market. *Financ. Internet Q.* **10,** 28–37.

[13] Birru, J., and Figlewski, S. 2011. Anatomy of a meltdown: The risk neutral density for the S & P 500 in the fall of 2008. *J. Financ. Mark.* **15,** 151–80.

[14] Bahra, B. 1997. Implied risk-neutral probability density functions from option prices: Theory and application. *Bank England Work. Pap.* **66,** 1-56.

[15] Gustavo, A., Bernavides, G., and Jose Gonzalo, R. 2010. Exchange rate market expectations and central bank policy: The case of the Mexican Peso-US Dollar from 2005-2009. *J. Deriv.* **19,** 1-35.

[16] Bliss, R. R., and Panigirtzoglou, N. 2004. Option-implied risk aversion estimates. *J. Finance* **59,** 407–446.

[17] Fajardo, J., Ornelas, J. R. H., and de Farias, A. R. 2012. Estimating risk aversion, risk-neutral and real-world densities using Brazilian real currency options. *Econ. Apl.* **16,** 567–577.

[18] Breeden, D. T., and Litzenberger, R. H. 1978. Prices of state-contingent claims implicit in option prices. *J. Bus.* **51,** 621–51.

[19] Figlewski, S. 2010. Estimating the implied risk-neutral density for the US market portfolio. In: *Volatility and time series econometrics: Essays in honor of Robert Engle.* (eds.) Bollerslev, T., Russell, J. R., and Watson, M. W. Oxford, UK: Oxford University Press, pp 323–353.

[20] Fackler, P. L., and King, R. P. 1990. Calibration of option-based probability assessments in agricultural commodity markets. *Am. Agric. Econ. Assoc.* **72,** 73–83.

[21] Silverman, B. 1986. *Density estimation for statistics and data analysis* vol 26. Boca Raton, Florida: CRC press.

[22] DeMiguel, V., Plyakha, Y., Uppal, R., and Vilkov, G. 2013. Improving portfolio selection using option-implied volatility and skewness. *J. Financ. Quant. Anal.* **48,** 1813–1845.

[23] Buss, A., and Vilkov, G. 2012. Measuring equity risk with option-implied correlations. *Rev. Financ. Stud.* **25**, 3113–3140.

[24] Jagannathan, R., and Ma, T. 2003. Risk reduction in large portfolios: Why imposing the wrong constraints helps. *J. Finance* **58,** 1651–1683.