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Ranking Fuzzy Numbers by Centroid Method

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ABSTRACT

Ranking fuzzy numbers are one of the important tools in decision process. There are many methods that have been proposed by a number of researchers but most of the methods are nondiscriminating and counterintuitive. Thus, proposing a new method for ranking fuzzy numbers are very prominent. The main objective of this paper is to get better ranking results to rank generalized fuzzy numbers than existing method. This paper reviews the centroid method in ranking fuzzy numbers by several researchers. A new calculation of centroid method will be proposed in this paper. At the end of the paper, a numerical calculation and a comparison of centroid method between the proposed method and other researchers' method will be showed to check on its consistency.

| Ranking Fuzzy Numbers | Centroid | Circumcenter of Centroids |

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1. INTRODUCTION

Ranking fuzzy numbers is a base of decision-making in represents uncertain values and thus, it is one of the important research topics in fuzzy set theory. Fuzzy numbers usually represented by possibility distributions can often overlap each other in many practical situations (1). When two fuzzy numbers overlap with each other, a fuzzy number may not be considered absolutely larger than the other (2). That is, even when a fuzzy number may be considered larger than the other, it may also be considered smaller than the other. Many authors have proposed different methods for ranking fuzzy numbers since their inception in 1965, but most of the methods proposed are nondiscriminating and counterintuitive (3). Ranking fuzzy numbers only started in 1976 by Jain (4). Jain proposed a ranking fuzzy numbers for decision making in the presence of fuzzy variables by representing the ill-defined quantity as a fuzzy set. Since that, a number of researchers came up with various procedures to rank fuzzy quantities. Bortolan and Degani (5) reviewed some of ranking methods for ranking fuzzy subsets. Chen (6) proposed ranking fuzzy numbers with maximizing set and minimizing set. Delgado et. al (7) presented a procedure for ranking fuzzy numbers. Kim and Park (8) presented a method of ranking fuzzy numbers with index of optimism. Yuan (9) presented a criterion for evaluating fuzzy ranking methods. Choobineh and Li (10) presented the ranking of fuzzy sets based on the concept of existence. Cheng (11) presented a new approach for ranking fuzzy numbers by distance method. Chen and Lu (12) presented an approximate approach for ranking fuzzy numbers based on left and right dominance. Deng, Zhenfu and Qi (13) presented ranking fuzzy numbers with an area method using radius of gyration. Nejad and Mashinchi (14) presented the ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number.

Methods of ranking fuzzy numbers can be classified into four classes (1). The first method is the preference relation, where the techniques involved are degree of optimality, α-cut, Hamming distance and comparison function. The second method is the fuzzy mean and spread which is using the probability and possibility distributions technique. The third method is the fuzzy scoring. This method applied a few techniques such as proportional optimal, left and right scores, centroid index and measurement. The last method is the linguistic expression which contains the intuition and linguistic approximation techniques.

A most commonly used technique is the centroid index under fuzzy scoring classes. This method was first proposed by Yager (15) in 1981 with weighting function. Since that, a various number of researchers have also been investigated about ranking numbers using centroid method. Cheng (11) improved Yager's method by presenting the centroid index ranking method that calculates the distance of the centroid point of each fuzzy number and original point. Chu and Tsao (16) pointed out the inconsistent and counter intuition of these two indices and proposed ranking fuzzy numbers with the area between the centroid point and original point. Liang et al. (17) also proposed the ranking indices and rules for fuzzy numbers based on gravity center

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point. Wang and Lee (18) revised the method of ranking fuzzy numbers with an area between the centroid and original points. The main objective of this paper is to get better ranking results to rank generalized fuzzy numbers than existing method.

The new working method proposed in this paper is based on Circumcenter of Centroid. A figure of trapezoidal fuzzy number is illustrated and the trapezoid is split into three parts. The first, second and third parts consist of a triangle, a rectangle and a triangle respectively. Then, the centroids of each part are calculated by using the centroid equation. The combination of the centroid for each part will result in a triangle. The next step is to calculate the midpoint for each side of the new triangle to get the midpoint of a line in the triangle. The slopes for each side of the triangle are calculated followed by the calculation of the slope of perpendicular bisector. Then, the equation of the perpendicular bisectors with the slope and the midpoint are calculated to get the circumcenter equation and the equation is solved by finding the value of the *x*-axes and *y*-axes from the equation. Finally, a ranking function is calculated which is the Euclidean distance from the Circumcenter of the Centroids and the original point. Most of the ranking procedures proposed in the literature use Centroid of trapezoid as reference point, as the centroid is the balancing point of the trapezoid. But the Circumcenter of Centroids can be considered a much more balancing point as this point is equidistant from all the vertices which are centroids (3).

In this paper, the work is divided into 5 sections. Section 2 briefly introduces the basic concepts and definitions of fuzzy numbers. Section 3 presented the new proposed method. A numerical example is showed in Section 4. The comparison between the new proposed method and other researcher's methods are presented in Section 5. Finally, the conclusions of the work are presented in Section 6.

2. PRELIMINARIES

Definition 1 Let U be a set of objects (the universe) and define a mapping on U

$$A : U \to [0,1]$$
$$u \mapsto A(u).$$

Then A is called a fuzzy set on U, $A(u)$ is called the membership function of A (or called the measure that u belongs to A). For $\forall \lambda \in [0,1]$, $A_\lambda = \{u \mid u \in U, A(u) \ge \lambda\}$ is called a λ-cut of A ; Supp$A$ = $\{u \mid u \in U, A(u) > 0\}$ and $KerA = \{u \mid u \in U, A(u) = 1\}$ are respectively called the support and kernel of the fuzzy set A. If $KerA \neq \emptyset$, then A is called normal fuzzy set.

Definition 2 (Fuzzy Numbers and Membership Functions).

A fuzzy number Ã is a fuzzy subset in support $\Re$ (real number) which is both "normal" and "convex" with membership function

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{L}(x), & a \le x \le b \\ w, & b \le x \le c \\ f_{\tilde{A}}^{R}(x), & c \le x \le d \\ 0, & otherwise, \end{cases}$$

where $0 < w \le 1$ is a constant, a, b, c, d are real numbers, and $f_{\tilde{A}}^{L} : [a,b] \to [0,w], f_{\tilde{A}}^{R} : [c,d] \to [0,w]$ are two strictly monotonic and continuous functions from $\Re$ to the closed interval $[0,w]$. It is customary to write a fuzzy number as $\tilde{A} = (a,b,c,d;w)$. If $w = 1$, then $\tilde{A} = (a,b,c,d;1)$ is a normalized fuzzy number, otherwise $\tilde{A}$ is said to be a generalized or non-normal fuzzy number.

If the memberships function $f_{\tilde{A}}(x)$ is piecewise linear, then $\tilde{A}$ is said to be a trapezoidal fuzzy number. The membership function of a trapezoidal fuzzy number is given by

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{w(x-a)}{b-a}, & a \le x \le b \\ w, & b \le x \le c \\ \dfrac{w(x-d)}{c-d}, & c \le x \le d \\ 0, & otherwise, \end{cases}$$

If $w = 1$, then $\tilde{A}=(a, b, c, d; 1)$ is a normalized trapezoidal fuzzy number and $\tilde{A}$ is a generalized or nonnormal trapezoidal fuzzy number if $0 < w < 1$. The image of $\tilde{A}=(a, b, c, d; w)$ is given by $-\tilde{A} = (-d,-c,-b,-a;w)$.

As a particular case if $b = c$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A}=(a, b, d; w)$. The value of "b" corresponds with the mode or core and $[a,d]$ with the support. If w=1, then $\tilde{A}=(a, b, d)$ is a normalized triangular fuzzy number and $\tilde{A}$ is a generalized or nonnormal triangular fuzzy number if $0<w<1$.

As $f_{\tilde{A}}^{L} : [a,b] \to [0,w], f_{\tilde{A}}^{R} : [c,d] \to [0,w]$ are strictly monotonic and continuous functions, their inverse functions $g_{\tilde{A}}^{L}(y) : [0,w] \to [a,b]$ and $g_{\tilde{A}}^{R}(y) : [0,w] \to [c,d]$ are also continuous and strictly monotonic. Hence $g_{\tilde{A}}^{L}(y)$ and $g_{\tilde{A}}^{R}(y)$ are integrable on $[0,w]$.

3. PROPOSED METHOD

In this paper, a new approach method is presented which is based on Circumcenter to rank fuzzy numbers. A trapezoid ABCDSR has been illustrated by Bushan Rao and Ravi Shankar (3) as in Figure 1. Figure 1 showed a trapezoid which are split into three parts. The first part is a triangle, the second part is a rectangle and the third part is a triangle. The proposed procedure can be expressed in a series of step:

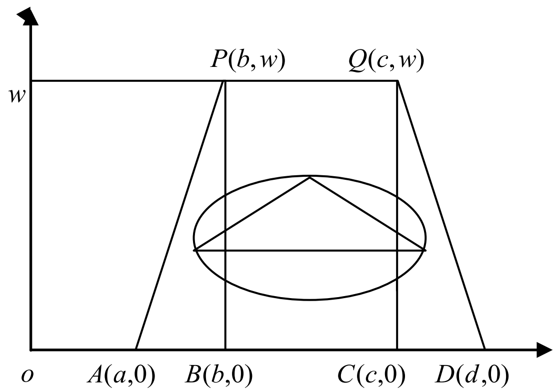

Fig. 1 Centroids of Trapezoid

Step 1 Calculate the Centroids of the three parts triangle, rectangle and triangle respectively by using the centroid equation.

Centroid for triangle :

$$\left(\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3}\right)$$

Centroid for rectangle :

$$\left(\frac{(x_1+x_2+x_3+x_4)}{4}, \frac{(y_1+y_2+y_3+y_4)}{4}\right)$$

Therefore, the centroid point of each part will be as follow:

$$M_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right)$$

$$M_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right)$$

$$M_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right)$$

Equation of the line $\overleftrightarrow{M_1M_3}$ is $y = \frac{w}{3}$ and M_2 does not lie on the line $\overleftrightarrow{M_1M_3}$. Therefore, M_1, M_2 and M_3 are non-collinear and they form a triangle.

Step 2 Calculate the midpoint of the sides M_1, M_2 and M_3 in Figure 2, which is the average of the x and y co-ordinates.

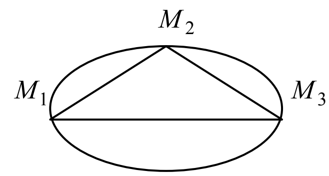

Fig. 2 Circumcenter of Centroids.

$$\text{Midpoint of a line in triangle} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Step 3 Find the slopes of the sides M_1, M_2 and M_3 using the *m* formula.

$$m = \left(\frac{y_2-y_1}{x_2-x_1}\right)$$

Step 4 Calculate the slope of the perpendicular bisector of the lines M_1, M_2 and M_3.

$$\text{Slope of perpendicular bisector} = \frac{1}{slope\, of\ line}$$

Step 5 Find the equation of the perpendicular bisectors with the slope and the midpoints.

Circumcenter equation : $y - y_1 = m(x - x_1)$

Step 6 Find the value of x and y by solving any 2 of the above 3 equations.

Since M_2 does not lie on the line $\overrightarrow{M_1M_3}$, therefore M_1, M_2 and M_3 are non-collinear. The Circumcenter of the triangle are defined as

$$S_{\widetilde{A}}(\overline{x_0}, \overline{y_0}) = \left(\frac{(2a^2+2b^2+5ab-3ac+2cb)}{3(2a+b-3c)}, \frac{w}{3}\right)$$

The ranking function of the trapezoidal fuzzy number $\widetilde{A} = (a,b,c,d;w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\widetilde{A}) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2}$ which is the Euclidean distance from the Circumcenter of the Centroids and the original point.

Let $\widetilde{A}_i$ and $\widetilde{A}_j$ be two fuzzy numbers, then

i.
ii. if $R(\widetilde{A}_i) > R(\widetilde{A}_j)$, then $\widetilde{A}_i > \widetilde{A}_j$,
iii. if $R(\widetilde{A}_i) < R(\widetilde{A}_j)$, then $\widetilde{A}_i < \widetilde{A}_j$,
iv. if $R(\widetilde{A}_i) = R(\widetilde{A}_j)$ then in this case the discrimination of fuzzy numbers is not possible.

4. NUMERICAL EXAMPLE

Let $\widetilde{A} = (0.2,0.4,0.4,0.5;1), \widetilde{B} = (0.2,0.3,0.3,0.4;1)$ and $\widetilde{C} = (0.1,0.3,0.3,0.5;1)$.

By using the new proposed method $S_{\widetilde{A}}(\overline{x_0},\overline{y_0})$, the following results are obtained :

$$S_{\widetilde{A}}(\overline{x_0},\overline{y_0}) = (-0.7333, 0.3333)$$
$$S_{\widetilde{B}}(\overline{x_0},\overline{y_0}) = (-0.9333, 0.3333)$$
$$S_{\widetilde{C}}(\overline{x_0},\overline{y_0}) = (0.3666, 0.3333)$$

The ranking function of the trapezoidal fuzzy number for $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$ are calculated using $R(\widetilde{A}) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2}$ and the results are as below:

$$R(\widetilde{A}) = 0.8055$$
$$R(\widetilde{B}) = 0.9910$$
$$R(\widetilde{C}) = 0.4955$$

Therefore, the ranking order is $\widetilde{C} < \widetilde{A} < \widetilde{B}$.

5. COMPARATIVE STUDY

Let's reconsider a numerical example by Bushan Rao and Ravi Shankar (3) using the centroid point of ranking fuzzy numbers.

Consider $\widetilde{A} = (0.1,0.2,0.4,0.5;1)$ and $\widetilde{B} = (1,1,1,1;1)$.

It can be seen from Table 1 that Cheng (11) and Chu and Tsao (16) gives the same centroids and ranking results. Cheng (11) proposed a ranking function which is the distance from centroid point and the original point using the given formula:

$$S_{\widetilde{A}}(\overline{x_0},\overline{y_0}) = \left(\frac{w(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(c^2 - b^{2)}}{3w(d - c + b - a) + 6(c - b)}, \frac{w}{3}\left(1 + \frac{(b+c) - (a+d)(1-w)}{(b+c-a-d) + 2(a+d)w}\right)\right)$$

While Chu and Tsao (16) proposed a ranking function which is the area between the centroid point and original point using the formula as below:

$$S_{\widetilde{A}}(\overline{x_0},\overline{y_0}) = \left(\frac{w(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(c^2 - b^{2)}}{3w(d - c + b - a) + 6(c - b)}, \frac{w}{3}\left(1 + \frac{b+c}{a+b+c+d}\right)\right).$$

Both Cheng (11) and Chu and Tsao (16) methods give the same ranking order: $\widetilde{A} > \widetilde{B}$.

Table 1 Comparison of various ranking methods.

Method	$S_{\widetilde{A}}(\overline{x_0},\overline{y_0})$ $S_{\widetilde{B}}(\overline{x_0},\overline{y_0})$	$R_{\widetilde{A}}$ $R_{\widetilde{B}}$	Ranking Order
Cheng (11)	(0.3, 0.5)	0.5831	$\widetilde{A} > \widetilde{B}$
	(0, 0.5)	0.5	
Chu and Tsao (16)	(0.3, 0.5)	0.5831	$\widetilde{A} > \widetilde{B}$
	(0, 0.5)	0.5	
Bushan Rao and Ravi Shankar (3)	(0.3, 0.3633)	0.4711	$\widetilde{B} > \widetilde{A}$
	(1, 0.4166)	1.0833	
Proposed method	(-0.1, 0.3333)	0.3479	$\widetilde{A} > \widetilde{B}$
	(0, 0.3333)	0.3333	

A ranking fuzzy numbers method by Bushan Rao and Ravi Shankar (3) which is based on the Circumcenter of Centroids and uses an index of optimism gives a different ranking order; $\widetilde{B} > \widetilde{A}$. The method formula is given as :

$$S_{\widetilde{A}}(\overline{x_0},\overline{y_0}) = \left(\frac{a + 2b + 2c + d}{6}, \frac{(2a + b - 3c)(2d + c - 3b) + 5w^2}{12w} \right)$$

For the new proposed method, the ranking order is $\widetilde{A} > \widetilde{B}$. The new proposed method which is also based on Circumcenter of Centroids can rank the crisp numbers as the centroid of the trapezoidal is calculated using the point of the concurrence of the triangle's three perpendicular bisectors and the center of the circumcircle.

6. CONCLUSIONS

This paper proposes a method that ranks fuzzy numbers using the simple and convenient process. A numerical example is used to investigate the effectiveness of the method. The simplified centroid formulae derived in this paper provide a very useful computational support to the applications of the two ranking approaches.

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