Malaysian Journal of Fundamental and Applied Sciences Vol.9, No.4 (2013) 180-184

ISSN 1823-626X

Malaysian Journal of Fundamental and Applied Sciences

available online at http://mjfas.ibnusina.utm.my

A New Proof on Sequence of Fuzzy Topographic Topological Mapping

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Received 8 January 2013, Revised 30 March 2013, Accepted 16 April 2013, Available online 20 May 2013

ABSTRACT

Fuzzy Topological Topographic Mapping (*FTTM*) is a model for solving neuromagnetic inverse problem. *FTTM* consists of four components and connected by three algorithms. *FTTM* version 1 and *FTTM* version 2 were designed to present *3D* view of an unbounded single current and bounded multicurrent source, respectively. In 2008, Suhana proved the conjecture posed by Liau in 2005 such that, if there exists n number of *FTTM*, then n^4-n new elements of *FTTM* will be generated from it. Suhana also developed some new definitions on geometrical features of *FTTM*, and discovered some interesting algebraic properties. In this paper, new proof on sequence of *FTTM* will be presented. In the proof, the sequence of *FTTM* is transformed into a system of differential equation.

| Fuzzy topographic topological mapping | Number Theory | Sequence | Differential Equation |

http://dx.doi.org/10.11113/mjfas.v9n4.106

1. INTRODUCTION

FTTM is a novel method for solving neuromagnetic inverse problem to determine the current source, i.e. epileptic foci. *FTTM* Version 1 is developed to present a 3-D view of an unbounded single current source [1, 2] in one angle observation (upper of a head model). It consists of three algorithms, which link between four components of the model as shown in Figure 1.

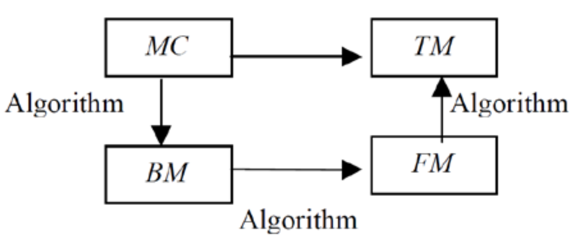

Fig. 1 *FTTM* version 1

The four components of *FTTM* are Magnetic Contour Plane (*MC*), Base Magnetic Plane (*BM*), Fuzzy Magnetic Field (*FM*) and Topographic Magnetic Field (*TM*) (Figure 1). *MC* is a magnetic field on a plane above a current source with z = 0. The plane is lowered down to *BM*, which is a plane of the current source with $z = -h$. Then the entire *BM* is fuzzified into a fuzzy environment (*FM*), where all the magnetic field readings are fuzzified. Finally, a three dimensional presentation of *FM* is plotted on *BM*. The final process is defuzzification of the fuzzified data to obtain a 3-D view of the current source (*TM*).

FTTM Version 2 is developed to present 3-D view of a bounded multi current source [3] in 4 angles of observation (upper, left, right and back of a head model). It consists of three algorithms, which link between four components of the model. The four components are Magnetic Image Plane (*MI*), Base Magnetic Image Plane (*BMI*), Fuzzy Magnetic Image Field (*FMI*) and Topographic Magnetic Image Field (*TMI*) (Figure 2).

MI is a plane above a current source with $z = 0$ containing all grey scale readings (0DN-255DN) of magnetic field. The plane is lowered down to *BMI*, which is a plane of the current source with $z = -h$. Then the entire base *BMI* is fuzzified into a fuzzy environment (*FMI*), where all the gray scale readings are fuzzified. Finally, a three dimensional presentation of *FMI* is plotted on *BMI*. The final process is defuzzification of the fuzzified data to obtain a 3-D view of the current source (*TMI*).

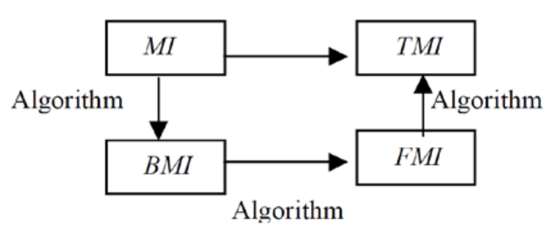

Fig. 2 *FTTM* version 2

FTTM Version 1 and *FTTM* Version 2 are specially designed to have equivalent topological structures between their components [4]. In other words, a homeomorphism

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between each component of *FTTM* Version 1 and *FTTM* Version 2 exists [5] as shown in Figure 3.

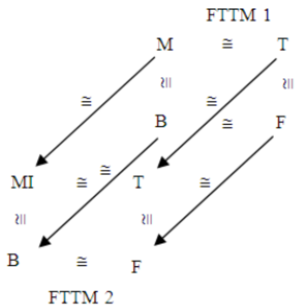


Fig. 3 Homemorphisms between *FTTM* 1 and *FTTM* 2

2. MATERIALS AND METHODS

Recall definitions on sequence, vertices, edges, vertices of *FTTM* [6].

Definition 1 *FTTM* [6]

Generally, *FTTM* can be represented as

$$FTTM = \{(M, B, F, T) : M \cong B \cong F \cong T\}$$

In order to extract some geometrical features of *FTTM*, the exact arrangement of sequence of *FTTM* is presented in Figure 4 [7].

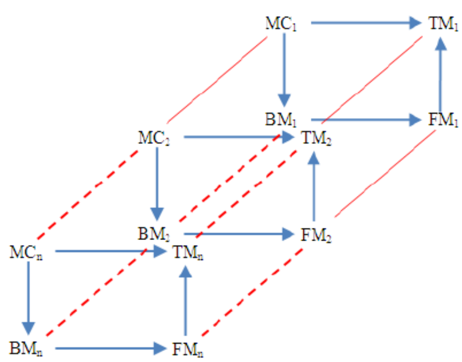


Fig. 4 Sequence of $FTTM_n$

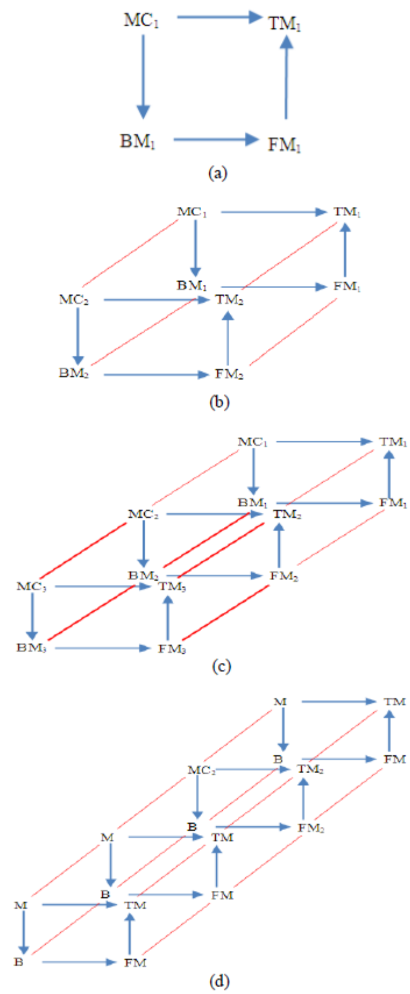
Definition 2 Sequence of *FTTM* [7]

Let $FTTM_i=(MC_i, BM_i, FM_i, TM_i)$ such that MC_i, BM_i, FM_i, TM_i are topological spaces with $MC_i \cong BM_i \cong FM_i \cong TM_i$. Sequence of $nFTTM_i$ of *FTTM* is $FTTM_1$, $FTTM_2$, $FTTM_3, \ldots, FTTM_n$ such that $MC_i \cong MC_{i+1}$, $BM_i \cong BM_{i+1}$, $FM_i \cong FM_{i+1}$, $TM_i \cong TM_{i+1}$.

$FTTM_1$, $FTTM_2$, $FTTM_3$, and $FTTM_4$ *are* illustrated in Figure 5 respectively. $FTTM_1$ can be viewed generally as a square and with *MC*, *BM*, *FM* and *TM* as vertices and the homeomorphism, namely $MC \cong BM$, $BM \cong FM$, $FM \cong TM$ and $MC \cong TM$, as edges. $FTTM_1$ has 4 vertices and 4 edges. $FTTM_2$ consists of 8 vertices, 12 edges, 6 faces and 1 cube.

Generally a cube is a combination of 2 *FTTM*. $FTTM_3$ consists of 12 vertices, 24 edges, 15 faces and 3 cubes. $FTTM_4$ has 16 vertices, 28 edges, 16 faces and 6 cubes. Consequently, some patterns of vertices, edges, faces and cubes emerge from sequences of *FTTM* as listed in the Table I.

Figure 5. **a)** $FTTM_1$, **b)**$FTTM_2$, **c)** $FTTM_3$, **d)**$FTTM_4$,



Definition 3 Sequence of Vertices of $FTTM_n$ [6]

The sequence of vertices for $FTTM_n$ which is $vFTTM_n$ are given recursively by equation

$$vFTTM_n = vFTTM_{n-1} + 4 \text{ for } n \geq 1 \text{ and } vFTTM_0 = 0. \quad (1)$$

Definition 4 Sequence of Edges of $FTTM_n$ [6]

The sequence of edges for $FTTM_n$ which is $eFTTM_n$ are given recursively by equation

$$eFTTM_n = eFTTM_{n-1} + 8 \text{ for } n > 1 \text{ and } eFTTM_1 = 4. \quad (2)$$

Definition 5 Sequence of Faces of $FTTM_n$ [6]

The sequence of face for $FTTM_n$ which is $fFTTM_n$ are given recursively by equation

$$fFTTM_n = fFTTM_{n-1} + 5 \text{ for } n > 1 \text{ and } fFTTM_1 = 1. \quad (3)$$

Definition 6 Sequence of cubes of $FTTM_n$ [6]
The sequence of cubes for $FTTM_n$ which is $FTTM_{2/1}$, $FTTM_{2/2}$, $FTTM_{2/3}$,.., are given recursively by equation

$$FTTM_{2/n} = FTTM_{2/n-1} + (n-1) \text{ for } n \geq 1 \text{ and } FTTM_{2/0} = 0. \quad (4)$$

Table 1 show the vertices, edges, faces and cubes for some sequences of *FTTM* [6].

Table 1 Vertices, Edges, Faces and Cubes for Sequence of *FTTM*

$FTTM_n$	Vertices	Edges	Faces	Cubes
$FTTM_1$	4	4	1	0
$FTTM_2$	8	12	6	1
$FTTM_3$	12	20	11	3
$FTTM_4$	16	28	16	6
$FTTM_5$	20	36	21	10
$FTTM_6$	24	44	26	15
$FTTM_7$	28	52	31	21
$FTTM_8$	32	60	36	28
$FTTM_9$	36	68	41	36
$FTTM_{10}$	40	76	46	45

From the table and definitions above, several new theorems on sequence of *FTTM* can be deduced and presented in following section.

3. A NEW PROOF ON SEQUENCE OF *FTTM*

Proving the sequence of *FTTM* can be very tedious as it involves large sequence of numbers. The previous method [8] of proving sequence of *FTTM* was by proof by construction. However this method required one to develop geometrical features for all sequence of *FTTM* [9]. However, in this section an alternative method of proving sequence of *FTTM* is presented, namely by method of differential equations [10].

3.1 Theorem 1

The Sequence of edges of $FTTM_n$; $eFTTM_n$ can be represented as

$$eFTTM_n = 8n - 4$$

Proof:
Recall Definition 4, sequence of edges can be defined as

$$eFTTM_{n-1} = eFTTM_n + 8$$

Equivalently,

$$eFTTM_{n+1} - eFTTM_n = 8 \quad (5)$$

Equation (5) can be viewed as a non-homogenous ordinary differential equation as follows

$$eFTTM_n = S_n + T_n \quad (6)$$

with S_n is the general solutions

$$S_{n+1} - S_n = 0 \quad (7)$$

and T_n is particular solutions such that

$$T_{n+1} - T_n = 8 \quad (8)$$

Equation (7) can be solved by finding the related polynomial for equation (5). In this case, the related polynomial for (5) is

$$P(r) = r - 1$$
$$=0$$

which implies $r = 1$. Solution for (7) is given by $S_n = Ar$.

$$\therefore S_n = A \quad (9)$$

To get solution for (8), let

$$T_n = Bn \quad (10)$$

Substituting T_n into (8) will gives

$$T_{n+1} - T_n = 8$$
$$B(n+1) - Bn = 8$$
$$B = 8$$
$$\therefore T_n = 8n \quad (11)$$

From (9) and (11), solution for (6) can be written as

$$eFTTM_n = 8n + A$$

From Table I, initial values for $eFTTM_1 = 4$ which implies $A = -4$.

$$\therefore eFTTM_n = 8n - 4 \qquad \square$$

3.2 Theorem 2

The Sequence of faces of $FTTM_n$; $fFTTM_n$ can be represented as

$$fFTTM_n = 5n - 4$$

Proof:
Recall Definition 5, sequence of edges can be defined as

$$fFTTM_n = fFTTM_{n-1} + 5$$

Equivalently,

$$fFTTM_{n+1} - fFTTM_n = 5 \quad (12)$$

Similarly, equation (12) can be viewed as a non-homogenous ordinary differential equation as follows

$$fFTTM_n = U_n + V_n \quad (13)$$

with S_n is the general solutions

$$U_{n+1} - U_n = 0 \quad (14)$$

and T_n is particular solutions such that

$$V_{n+1} - V_n = 5 \quad (15)$$

Equation (14) can be solved by finding the related polynomial for equation (12). In this case, the related polynomial for (12) is

$$P(r) = r - 1$$
$$= 0$$

which implies $r = 1$. Solution for (14) is given by $U_n = Ar$.

$$\therefore U_n = A \quad (16)$$

To get solution for (8), let

$$V_n = Bn \quad (17)$$

Substituting T_n into (8) will gives

$$V_{n+1} - V_n = 5$$
$$B(n+1) - Bn = 5$$
$$B = 5$$
$$\therefore V_n = 5n \quad (18)$$

From (9) and (11), solution for (6) can be written as

$$fFTTM_n = 5n + A$$

From Table I, initial value for $fFTTM_1 = 1$ which implies $A = -4$.

$$\therefore fFTTM_n = 5n - 4 \quad \square$$

3.3 Theorem 3

The Sequence of cubes of $FTTM_n$; $cFTTM_n$ can be represented as

$$cFTTM_n = \frac{n(n-1)}{2}$$

Proof:
Recall definition 6, sequence of edges can be defined as

$$cFTTM_n = cFTTM_{n-1} + n$$

Equivalently,

$$cFTTM_{n+1} - cFTTM_n = n \quad (19)$$

Equation (19) can be viewed as a non-homogenous ordinary differential equation as follows

$$cFTTM_n = W_n + Z_n \quad (20)$$

with W_n is the general solutions

$$W_{n+1} - W_n = 0 \quad (21)$$

and Z_n is particular solutions such that

$$Z_{n+1} - Z_n = n \quad (22)$$

Equation (21) can be solved by finding the related polynomial for equation (19). In this case, the related polynomial for (19) is

$$P(r) = r - 1$$
$$= 0$$

which implies $r = 1$. Solution for (21) is given by $W_n = Ar$.

$$\therefore W_n = A \quad (23)$$

To get solution for (22), let

$$Z_n = Bn^2 \quad (24)$$

Substituting T_n into (22) will gives

$$B = \frac{1}{2}$$
$$\therefore V_n = \frac{1}{2}n^2 \quad (25)$$

Using (23) and (25), solution for (20) is follows

$$cFTTM_n = A + \frac{1}{2}n^2$$

From the Table I, initial value for $cFTTM_1 = 0$ which implies $A = -\frac{1}{2}$.

$$\therefore cFTTM_n = \frac{n(n-1)}{2} \quad \square$$

4.0 CONCLUSION

The aim of this paper is to produce new proof on sequence of edges, faces and cubes of *FTTM*.

Table II, III, IV show sequences of edges, faces, and cubes of *FTTM*$_n$, respectively. As a result, the number of sequence of edges, faces, and cubes are exactly the same as defined in [6].

Table II Comparison between sequences of Edges to Definition 4

$eFTTM_n$	$eFTTM_{n-1} + 8$	$-4 + 8n$
n=1	4	4
n=2	12	12
n=3	20	20
n=4	28	28
n=5	36	36
n=6	44	44
n=7	52	52
n=8	60	60
n=9	68	68
n=10	76	76

Table III Comparison between sequences of Faces to Definition 5

$fFTTM_n$	$fFTTM_{n-1} + 5$	$-4 + 5n$
n=1	1	1
n=2	6	6
n=3	11	11
n=4	16	16
n=5	21	21
n=6	26	26
n=7	31	31
n=8	36	36
n=9	41	41
n=10	46	46

Table IV Comparison between sequences of Cubes to Definition 6

$c\mathrm{FTTM}_n$	$FTTM_{2/n-1} + (n-1)$	$\frac{n(n-1)}{2}$
n=1	0	0
n=2	1	1
n=3	3	3
n=4	6	6
n=5	10	10
n=6	15	15
n=7	21	21
n=8	28	28
n=9	36	36
n=10	45	45

ACKNOWLEDGEMENT

Author wants to thanks the members of Theoretical & Computational Modeling for Complex Systems (TCM), Department of Mathematical Sciences & Ibnu Sina Institute for Fundamental Science Studies, UTM, for their cooperation.

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